Geothermics and Geothermal Energy
Geothermal Heating Systems

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The Heating Demand

Dependencies

The actual heating demand of a building depends on

- the thermal efficiency of the building
- the climatic conditions
- the number and the behavior of the residents.

Data Requirements

Fired heating systems (oil, gas, biofuels, . . .): long-term mean power (yearly) and peak power

Geothermal and solar heating systems: time-resolved data (e. g., on a monthly scale)

Thermal Efficiency

The U value of an element of the building's surface (wall, window, ...) quantifies the heat flux density per temperature difference:

$$U = \frac{q}{T_i - T_o} \tag{1}$$

where

$$\begin{aligned} q &= \text{heat flux density} = \text{power per area } [\tfrac{\mathrm{W}}{\mathrm{m}^2}] \\ T_i &= \text{inside temperature [K]} \\ T_o &= \text{outside temperature [K]} \end{aligned}$$

Unit: $\frac{\mathrm{W}}{\mathrm{m}^2\mathrm{K}}$

The Heating Demand

Thermal Efficiency

The R value of an element of the building's surface is

$$R = \frac{1}{U} = \frac{T_i - T_o}{q} = \frac{d}{\lambda} \tag{2}$$

for a homogeneous material of thickness d and thermal conductivity λ.

Actual required heating power for the entire building:

$$P = \sum_j q_j A_j = (\underset{\text{residents}}{\underset{\uparrow}{T_i}} - \underset{\text{climate}}{\underset{\uparrow}{T_o}}) \sum_j \underset{\text{building}}{\underset{\uparrow}{U_j A_j}} \tag{3}$$

for $T_i > T_o$ where the A_j are the surface areas of the elements.

The Influence of the Climate

Total energy required for heating during a given time span:

$$E = \int\limits_{T_i > T_o} (T_i - T_o(t))\, dt \sum_j U_j A_j \qquad (4)$$

The integral

$$\mathrm{HDD} = \int\limits_{T_i > T_o} (T_i - T_o(t))\, dt \qquad (5)$$

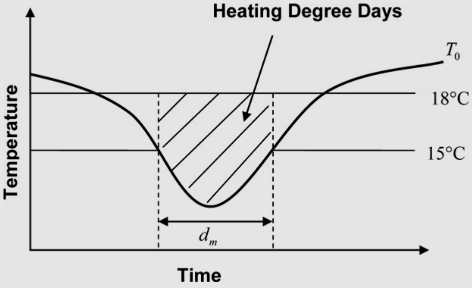
for a fixed value of T_i (independent of the specific residents' behavior) measured in units of (K days) is called number of heating degree days within the given time span.

Several slightly different ways of calculating the HDD.

The Influence of the Climate

Definition established in the EU:

- $T_i = 18°C$
- Use mean temperatures over one day periods for T_o instead of continuous time.
- Take into account only days below a heating threshold of 15°C.



Source: Global CCS Institute

The Influence of the Climate

Total energy required for heating during a given time span:

$$E = \text{HDD} \sum_j U_j A_j \tag{6}$$

- Obtained unit is W days.
- If the total energy demand, $\sum E$, and total HDD, $\sum \text{HDD}$, for one year are given instead of $\sum_j U_j A_j$:

$$E = \sum E \, \frac{\text{HDD}}{\sum \text{HDD}} \tag{7}$$

Geothermal Heat Pumps

Why?

Domestic heating systems require $T \geq 35°$C (old systems even much more)

$\Downarrow$

Cannot be achieved by shallow geothermal systems.

$\Downarrow$

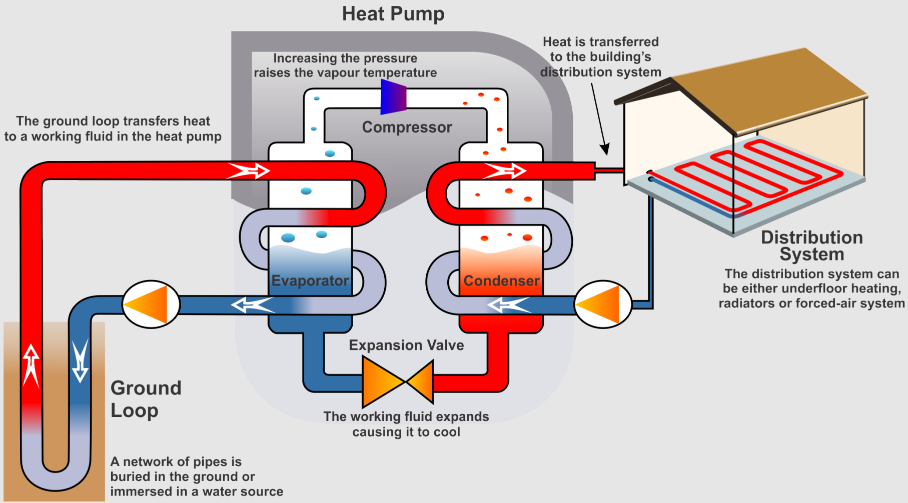
Temperature of the fluid in the heat exchanger must be increased using mechanical work.

Principle

Heat engine: heat (hot reservoir) $\rightarrow$ mechanical work + heat (cold res.)

Heat pump: heat (cold reservoir) + mechanical work $\rightarrow$ heat (hot res.)

Principle



Source: Geothermal heat-pump association of New Zealand

Fundamentals – Thermodynamics

Entropy

Source: Wehrli, Die Kunst aufzuräumen

Entropy

Definition of entropy:

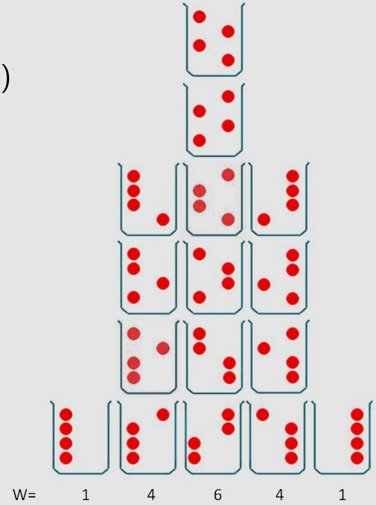
$$S = k \ln N \qquad (8)$$

where

$$k = 1.38 \times 10^{-23} \, \frac{\mathrm{J}}{\mathrm{K}}$$

= Boltzmann constant

N = number of states that cannot be distinguished



Source: Wikipedia

Entropy

- Second law of thermodynamics: Entropy of a closed system cannot decrease through time.
- Processes where the entropy is constant are reversible.
- Relationship between entropy, thermal energy and temperature in classical thermodynamics: Adding an amount of thermal energy δQ (or extracting if $\delta Q < 0$) at constant temperature T results in a change of entropy

$$\delta S = \frac{\delta Q}{T} \tag{9}$$

or in integral form

$$\delta S = \int \frac{dQ}{T} \tag{10}$$

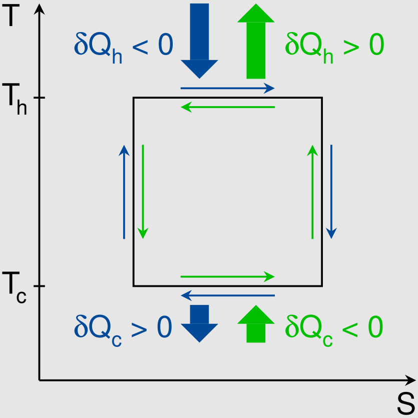
The Carnot Cycle

- Transfer of thermal energy from a “hot” reservoir of temperature T_h to a “cold” reservoir of temperature T_c yielding the maximum amount of mechanical work.
- Assumes two reservoirs of infinite capacity and a hypothetic gas.
- Only a theoretical thermodynamic cycle, not a real physical process.
- Reversible

Carnot Cycle and Inverse Carnot Cycle

Carnot cycle

Inverse Carnot cycle



Directions:

- $\rightarrow$ isothermal expansion (coupled to large reservoir)
- $\leftarrow$ isothermal compression (coupled to large reservoir)
- $\downarrow$ isentropic cooling (by rapid expansion)
- $\uparrow$ isentropic heating (by rapid compression)

The Thermodynamic Limit of the Carnot Cycles

$$\delta S \;=\; \frac{\delta Q_h}{T_h} + \frac{\delta Q_c}{T_c} \;\geq\; 0 \tag{11}$$

where

$$\begin{aligned}
\delta Q_h &= \text{thermal energy supplied to to the hot system} \\
T_h &= \text{temperature of the hot system} \\
\delta Q_c &= \text{thermal energy supplied to the cold system} \\
T_c &= \text{temperature of the cold system}
\end{aligned}$$

$\delta Q < 0$ describes extraction of energy from the system.

The Thermodynamic Limit of a Geothermal Heating System

Hot system: heating system, $\delta Q_h > 0$

Cold system: geothermal reservoir, $\delta Q_c < 0$

Thermodynamic limit of the heat pump (Eq. 11) written in terms of total power P_{tot} (to the heating system) and thermal power P_{th} (from the geothermal reservoir):

$$\frac{P_{\text{tot}}}{T_h} - \frac{P_{\text{th}}}{T_c} \geq 0 \tag{12}$$

Geothermal Heat Pumps

The Thermodynamic Limit of a Geothermal Heating System

The difference between P and P_{th} must be supplied as mechanical (electrical) power by the compressor, $P_{\text{el}} = P_{\text{tot}} - P_{\text{th}}$.

⇓

$$\frac{P_{\text{tot}}}{T_h} - \frac{P_{\text{tot}} - P_{\text{el}}}{T_c} \geq 0 \tag{13}$$

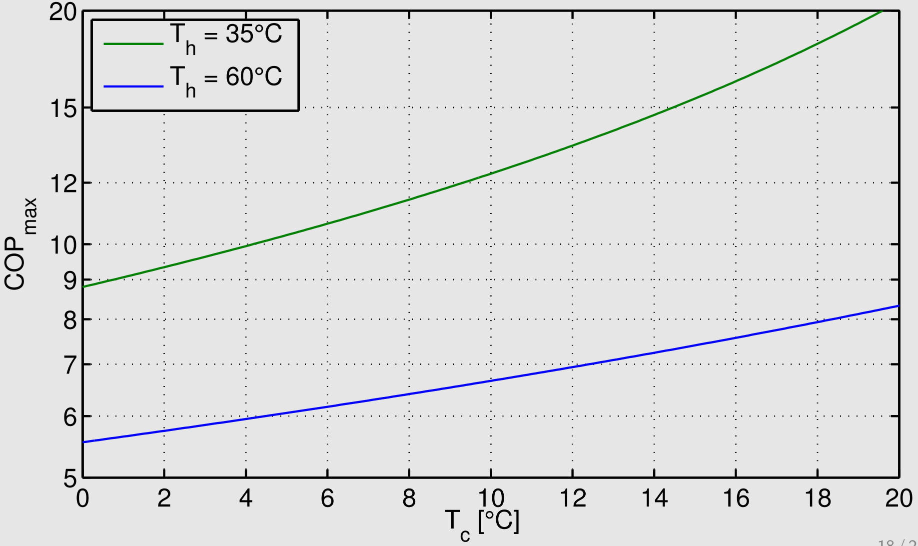
⇓

$$P_{\text{tot}} \leq \frac{T_h}{T_h - T_c} P_{el} \tag{14}$$

The Coefficient of Performance

$$\text{COP} = \frac{P_{\text{tot}}}{P_{\text{el}}} \leq \frac{T_h}{T_h - T_c} \tag{15}$$

The Upper Thermodynamic Limit of the COP



The COP of Real Heat Pumps

- Real heat pumps achieve a significantly lower performance than the thermodynamic limit, e. g., $\mathrm{COP} = 5$ is very good for $T_h = 35°\mathrm{C}$ and $T_c = 0°\mathrm{C}$ (instead of $\mathrm{COP}_{\mathrm{max}} = 8.8$).
- Data sheets with the COP for different temperatures are provided by some suppliers.
- If not, use the concept of relative efficiency.

The Relative Efficiency

General concept:

$$\eta = \eta_{\text{max}}\,\eta_{rel} \tag{16}$$

where

$$\begin{aligned} \eta &= \text{total efficiency (output/input)} \\ \eta_{\text{max}} &= \text{theoretically possible maximum efficiency} \\ \eta_{rel} &= \text{relative efficiency of the specific device} \end{aligned}$$

For a heat pump: η_{max} defined by the thermodynamic limit (inverse Carnot Cycle)

The Relative Efficiency

Two ways to apply the concept of relative efficiency to a heat pump:

Electrical power → total power:

$$\text{COP} = \frac{P_{\text{tot}}}{P_{\text{el}}} = \eta_{rel} \left(\frac{P_{\text{tot}}}{P_{\text{el}}} \right)_{\text{max}} = \eta_{rel} \frac{T_h}{T_h - T_c} \quad (17)$$

Electrical power → thermal power:

$$\frac{P_{\text{th}}}{P_{\text{el}}} = \eta_{rel} \left(\frac{P_{\text{th}}}{P_{\text{el}}} \right)_{\text{max}} = \eta_{rel} \left(\frac{P_{\text{tot}} - P_{\text{el}}}{P_{\text{el}}} \right)_{\text{max}} \quad (18)$$

$$= \eta_{rel} \left(\frac{T_h}{T_h - T_c} - 1 \right) = \eta_{rel} \frac{T_c}{T_h - T_c} \quad (19)$$

The Relative Efficiency

↓

$$\text{COP} \quad = \quad \frac{P_{\text{tot}}}{P_{\text{el}}} \;=\; 1 + \frac{P_{\text{th}}}{P_{\text{el}}} \;=\; 1 + \eta_{rel}\,\frac{T_c}{T_h - T_c} \tag{20}$$

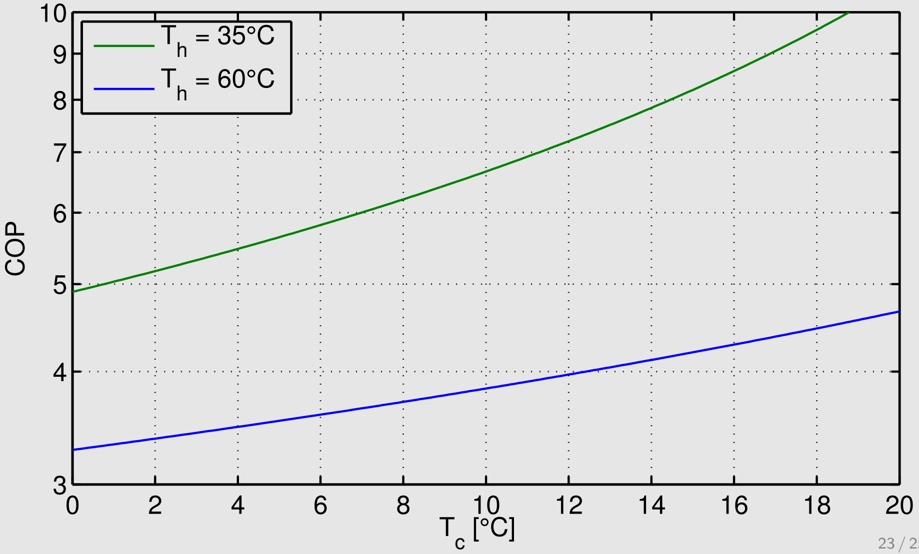
- The second concept is physically better.

↓

Use it in the following.

- Typical value: $\eta_{rel} \approx 0.5$ for good heat pumps

Typical COP of a Good Heat Pump ($\eta_{rel} = 0.5$)



Actual Energy Prices in Germany

Energy source	Price $[\frac{\text{ct}}{\text{kWh}}]$
wood	6
gas	6
oil	7
electricity	28

$\Downarrow$

Heat pump driven by electricity makes sense only if COP $\gtrapprox 4.5$ in the mean.

Alternative: gas heat pump

The Heat Pump in Geothermal Calculations

- P_{tot} given by the heating demand
- P_{th} required for the calculation of the geothermal system
- P_{el} required for calculating the costs of heating

$$P_{\text{el}} = \frac{P_{\text{tot}}}{1 + \eta_{rel} \frac{T_c}{T_h - T_c}} \tag{21}$$

$$\begin{aligned} P_{\text{th}} &= P_{\text{tot}} - P_{\text{el}} = P_{\text{tot}} - \frac{P_{\text{tot}}}{1 + \eta_{rel} \frac{T_c}{T_h - T_c}} \\ &= \frac{P_{\text{tot}}}{1 + \frac{1}{\eta_{rel}} \frac{T_h - T_c}{T_c}} \end{aligned} \tag{22}$$