Geothermics and Geothermal Energy
Closed Geothermal Systems

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Closed Geothermal Systems

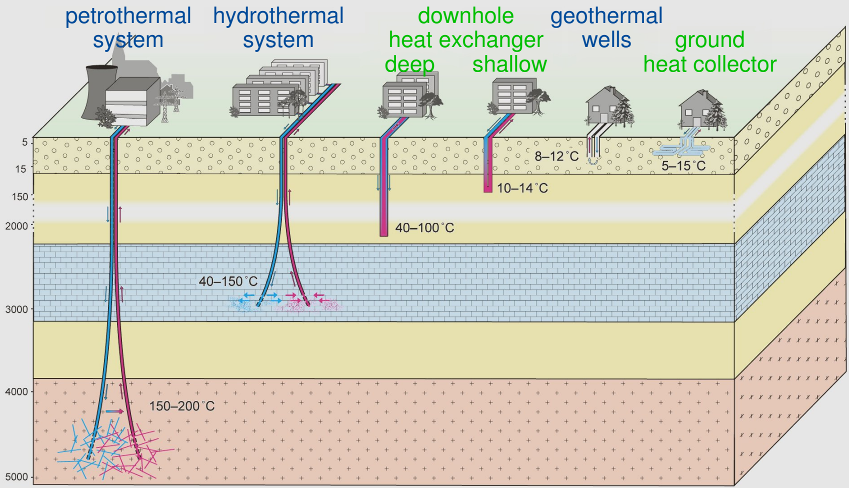
Principle

- Fluid circulates in a closed heat exchanger.
- Heat is transported to the fluid by heat conduction.
- Heat transport in the surrounding rock or soil by heat conduction; in some cases also by advection (groundwater).

Fluids

- Water or alcohol-water mixtures.
- Water has the best properties (heat capacity, thermal conductivity, viscosity) as long as $T > 0°\mathrm{C}$.

Types of Geothermal Systems



Source: Ingolstädter Kommunalbetriebe (modified)

Closed Geothermal Systems

Limitation

Heat is transported to the heat exchanger by conduction.

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Requires a temperature gradient towards the exchanger.

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Temperature in the exchanger is lower than the undisturbed subsurface temperature; temperature drop depends on

- extracted power
- thermal properties of the subsurface (mainly the thermal conductivity)
- properties of the exchanger (size, shape, material)

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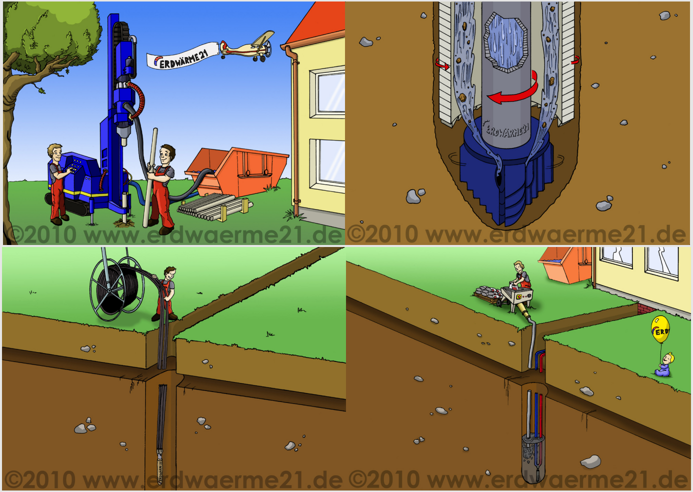
Production of electricity is economically not reasonable (so far?).

Geothermal Heat Exchangers

Downhole Heat Exchangers (Borehole Heat Exchangers)



Source: Erdwärme21

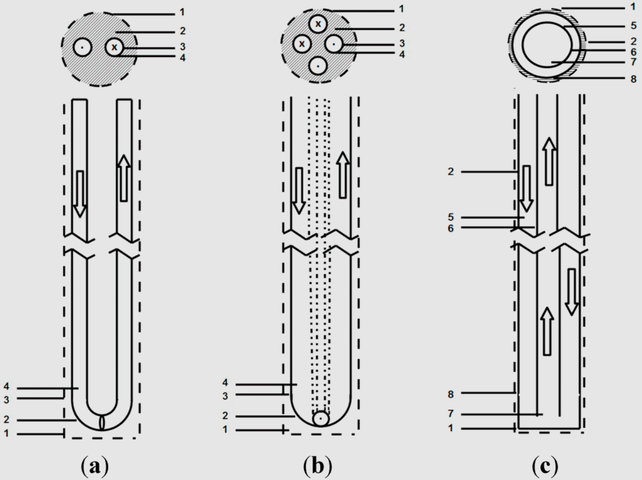
Geothermal Heat Exchangers

Downhole Heat Exchangers (Borehole Heat Exchangers)

Source: Baublog: Villa Lugana in Teltow

Geothermal Heat Exchangers

Downhole Heat Exchangers (Borehole Heat Exchangers)



(a) single U-tube
(b) double U-tube
(c) coaxial
1 borehole wall
2 grout
3 interior of U-tube
4 U-tube wall
5 annulus space
6 inner column (pipe)
7 interior of inner column (pipe)
8 external column

Source: Sliwa & Rosen (2015), *Sustainability*, 7(10), 13104, doi:10.3390/su71013104

Application of Downhole Heat Exchangers

Shallow heat exchangers ($d \lessapprox 400\,\mathrm{m}$, $T \lessapprox 25^\circ\mathrm{C}$): heating of buildings with the help of heat pumps

Deep heat exchangers ($d \gtrapprox 1000\,\mathrm{m}$, $T \gtrapprox 40^\circ\mathrm{C}$): direct heating

- down to depths of about 3000 m so far (coaxial type only)
- mostly reuse or deepen abandoned hydrocarbon boreholes
- economically still questionable

Geothermal Heat Exchangers

Ground Heat Collectors

Source: www.bauweise.net

Ground Heat Collectors

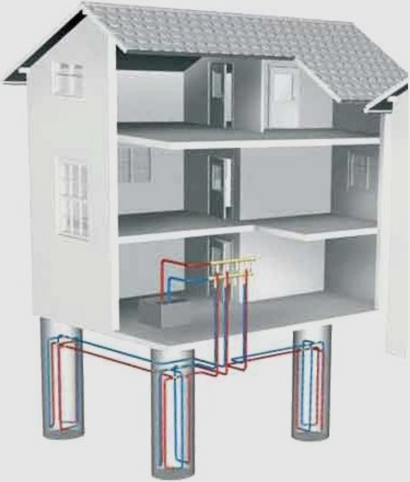
Source: Rehau AG & Co

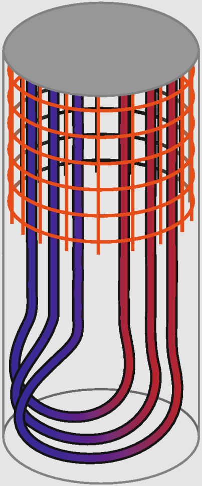
Geothermal Heat Exchangers

Geothermal Baskets

Source: Heizungsjournal

Geothermal Energy Piles





Sources: Energy Systems Research Unit, University of Strathclyde; Stober & Bucher, Geothermie

Superposition of Solutions

The heat conduction equation is linear.

$\Downarrow$

Can be solved by superposing individual components:

$$T(\vec{x}, t) = T_m(\vec{x}) + T_y(\vec{x}, t) + T_1(\vec{x}, t) + T_2(\vec{x}, t) + ... \quad (1)$$

with

$$T_m(\vec{x}) = \text{steady-state geotherm}$$
$$T_y(\vec{x}, t) = \text{natural seasonal variation}$$
$$T_i(\vec{x}, t) = \text{temperature drop caused by the } i^{\text{th}} \text{ heat exchanger}$$

We use T instead of T_i for the rest of the chapter.

Analytical Approximations

All three components can be approximated by analytical solutions of the heat conduction equation reasonably well in most cases.

$\Downarrow$

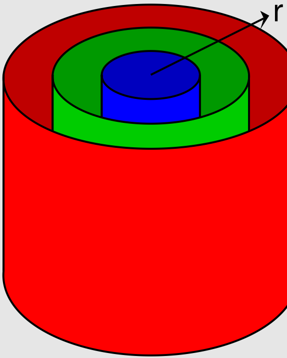
No need for numerical simulations and / or specific software

Analytical solutions use

- symmetries for reducing the spatial dimension (mainly from 3 to 1) and
- scaling properties (length vs. time).

Shallow Downhole Heat Exchangers

Cylindrical Symmetry



$T(x, y, z, t)$ only depends on $r = \sqrt{x^2 + y^2}$ and t.

Limitations:

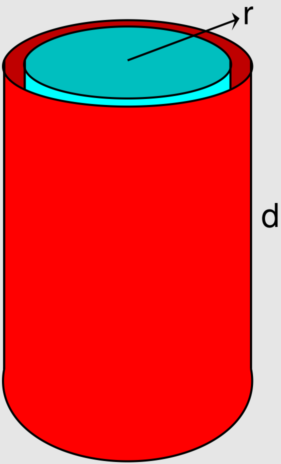
- Fluid temperature in the heat exchanger must increase with the geothermal gradient.

  ⬇

  Only applicable to shallow boreholes and to deep coaxial heat exchangers under specific conditions.
- Seasonal temperature variation in the upper region cannot be taken into account.
- Borehole length l must be much larger than $L(t) = \sqrt{\kappa t}$.

The Heat Conduction Equation for Cylindrical Symmetry

Energy balance for a cylindrical shell:



Change in thermal energy E contained in the shell:

$$\frac{\partial E}{\partial t} = \left(\pi r^2 d - \pi r^2 d\right) \rho c \frac{\partial T}{\partial t} = 2\pi r d\, q - 2\pi r d\, q \quad (2)$$

where q is the heat flux density in radial direction.

$$\rho c \frac{\partial}{\partial t} T(r,t) = -\frac{1}{r} \frac{\partial}{\partial r} (rq(r,t)) \quad (3)$$

for $r - r \to 0$

The Heat Conduction Equation for Cylindrical Symmetry

With $q = -\lambda \frac{\partial T}{\partial r}$:

$$\rho c \frac{\partial}{\partial t} T(r,t) = \frac{1}{r} \frac{\partial}{\partial r} \left(r \lambda \frac{\partial}{\partial r} T(r,t) \right) \tag{4}$$

If λ is constant:

$$\frac{\partial}{\partial t} T(r,t) = \kappa \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} T(r,t) \right) \tag{5}$$

with $\kappa = \frac{\lambda}{\rho c}$

Solution of the Heat Conduction Equation for Cylindrical Symmetry

Look for solutions where the shape of the temperature profile remains constant, while only the spatial scale changes.

$$\Downarrow$$

Look for solutions $T(r,t)$ which depend on

$$u = \frac{r}{2L(t)} = \frac{r}{2\sqrt{\kappa t}} \tag{6}$$

instead of r and t only. The factor 2 is only for convenience.

Solution of the Heat Conduction Equation for Cylindrical Symmetry

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial u}\frac{\partial u}{\partial t} = \frac{\partial T}{\partial u}\frac{r}{2\sqrt{\kappa}}\frac{-1}{2t^{\frac{3}{2}}} = \frac{\partial T}{\partial u}\frac{-u}{2t} \quad (7)$$

$$r\frac{\partial T}{\partial r} = r\frac{\partial T}{\partial u}\frac{\partial u}{\partial r} = \frac{\partial T}{\partial u}\frac{r}{2\sqrt{\kappa t}} = u\frac{\partial T}{\partial u} \quad (8)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial u}\left(u\frac{\partial T}{\partial u}\right)\frac{\partial u}{\partial r} = \frac{1}{u}\frac{\partial}{\partial u}\left(u\frac{\partial T}{\partial u}\right)\frac{1}{4\kappa t} \quad (9)$$

$$\Downarrow$$

$$\frac{\partial T}{\partial u}\frac{-u}{2t} = \kappa\frac{1}{u}\frac{\partial}{\partial u}\left(u\frac{\partial T}{\partial u}\right)\frac{1}{4\kappa t} \quad (10)$$

$$\Downarrow$$

$$\frac{\partial}{\partial u}\left(u\frac{\partial T}{\partial u}\right) = -2u\left(u\frac{\partial T}{\partial u}\right) \quad (11)$$

Solution of the Heat Conduction Equation for Cylindrical Symmetry

Solution:

$$u \frac{\partial T}{\partial u} = a\, e^{-u^2} \tag{12}$$

with an arbitrary constant a.

Extracted power per borehole length l:

$$\begin{aligned} P_l &= \frac{P}{l} = -\lim_{r \to 0} \left(2\pi r\, q(r,t)\right) = 2\pi\lambda \lim_{r \to 0} r \frac{\partial T}{\partial r} \\ &= 2\pi\lambda \lim_{u \to 0} u \frac{\partial T}{\partial u} = 2\pi\lambda \lim_{u \to 0} a\, e^{-u^2} = 2\pi\lambda a \end{aligned} \tag{13}$$

$$\Downarrow$$

$$\frac{\partial T}{\partial u} = \frac{P_l}{2\pi\lambda} \frac{e^{-u^2}}{u} \tag{14}$$

Solution of the Heat Conduction Equation for Cylindrical Symmetry

Solution for $T(u)$ with the condition $T(u) \to 0$ for $u \to \infty$:

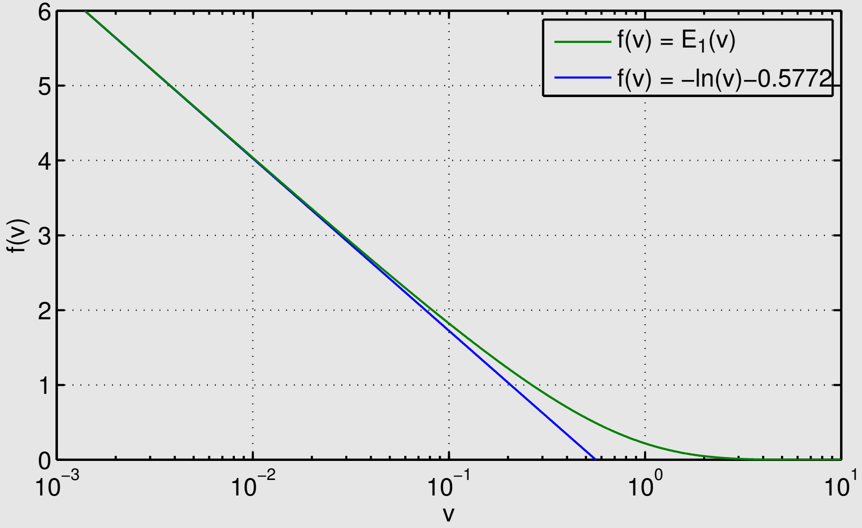
$$\begin{aligned} T(u) &= -\frac{P_l}{2\pi\lambda}\int_{u}^{\infty}\frac{e^{-\xi^2}}{\xi}d\xi = -\frac{P_l}{4\pi\lambda}\int_{u^2}^{\infty}\frac{e^{-x}}{x}dx \\ &= -\frac{P_l}{4\pi\lambda}E_1(u^2) \end{aligned} \tag{15}$$

with the function

$$E_1(v) = \int_{v}^{\infty}\frac{e^{-x}}{x}dx \tag{16}$$

which is called exponential integral.

Solution of the Heat Conduction Equation for Cylindrical Symmetry



Solution of the Heat Conduction Equation for Cylindrical Symmetry

- The exponential integral can be approximated by

$$E_1(v) \approx -\ln(v) - 0.5772 \tag{17}$$

  for $v \ll 1$.
- $E_1(v) \to 0$ for $u \to \infty$

$$\Downarrow$$

$$E_1(v) \approx \max\{-\ln(v) - 0.5772, 0\} \tag{18}$$

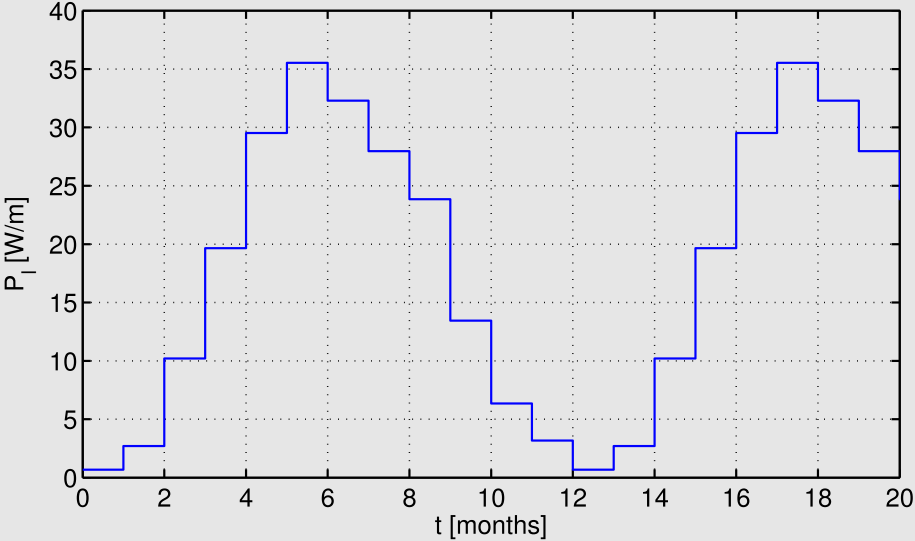
is a reasonable approximation if E_1 is not available.

Solution of the Heat Conduction Equation for Cylindrical Symmetry

Solution written in terms of r and t:

$$T(r,t) = -\frac{P_l}{4\pi\lambda} E_1\left(\frac{r^2}{4L(t)^2}\right) = -\frac{P_l}{4\pi\lambda} E_1\left(\frac{r^2}{4\kappa t}\right) \quad (19)$$

Solution for Step-Like Heat Extraction



Solution for Step-Like Heat Extraction

Superposition of several heat exchangers switched on at different times:

Month #	P_l $[\frac{W}{m}]$
1	0.7
2	2.7
3	10.2
4	19.7
5	29.5
6	35.5
7	32.3
8	28.0
9	23.8
10	13.4
11	6.3
12	3.2
...	...

Exchanger #	Starting time [mon]	P_l $[\frac{W}{m}]$
1	0	0.7
2	1	2.0
3	2	7.5
4	3	9.5
5	4	9.8
6	5	6.0
7	6	-3.2
8	7	-4.3
9	8	-4.2
10	9	-10.4
11	10	-7.1
12	11	-3.1
...	...	...

Solution for Step-Like Heat Extraction

Formally:

$$P_l(t) = \begin{cases} 0 & \text{for} \quad t < t_0 \\ P_{l,i} & \phantom{\text{for}} \quad t_{i-1} \leq t < t_i \end{cases} \tag{20}$$

$$\Downarrow$$

$$\begin{aligned} T(r,t) &= \frac{P_{l,1}}{4\pi\lambda} E_1\left(\frac{r^2}{4\kappa(t-t_0)}\right) \\ &+ \frac{(P_{l,2}-P_{l,1})}{4\pi\lambda} E_1\left(\frac{r^2}{4\kappa(t-t_1)}\right) \\ &+ \ldots \\ &+ \frac{(P_{l,n+1}-P_{l,n})}{4\pi\lambda} E_1\left(\frac{r^2}{4\kappa(t-t_n)}\right) \end{aligned} \tag{21}$$

The Thermal Resistance

If r_b is the radius of the borehole,

$$-T(r_b, t) \quad = \quad \frac{P_l}{4\pi\lambda} E_1\left(\frac{r_b^2}{4\kappa t}\right) \tag{22}$$

is the temperature drop at the wall of the borehole towards the undisturbed rock temperature.

$$\Downarrow$$

$$-T(r_b, t) \quad = \quad R_s\, P_l \tag{23}$$

where

$$R_s \quad = \quad \frac{1}{4\pi\lambda} E_1\left(\frac{r_b^2}{4\kappa t}\right) \tag{24}$$

is called thermal resistance of the surronding rock or soil $[\frac{\mathrm{m\,K}}{\mathrm{W}}]$.

The Thermal Resistance

- R_s depends on the borehole radius and on the thermal properties of the surronding rock or soil.
- R_s increases through time.
- Heat conduction from the borehole wall to the fluid through the walls of the pipes and through the filling material cause an additional temperature drop.

$\Downarrow$

Total thermal resistance

$$R = R_s + R_b \tag{25}$$

where R_b is the thermal resistance of the borehole.

The Thermal Resistance

- $L(t) \gg r_b$ for large times

$$\Downarrow$$

Borehole can be considered in steady steate.

- Borehole resistance depends on the geometry (single U-tube, double U-tube, coaxial) and on the material used for filling.
- Typical values for double U-tube heat exchangers:
  $R_b \approx 0.1 \, \frac{\mathrm{m\,K}}{\mathrm{W}}$ (standard filling)
  $R_b \approx 0.08 \, \frac{\mathrm{m\,K}}{\mathrm{W}}$ (thermally improved filling)

Including the Thermal Resistance in the Calculation

Define an apparent borehole radius r_a in such a way that

$$\frac{1}{4\pi\lambda} E_1\left(\frac{r_a^2}{4\kappa t}\right) = R = R_s + R_b = \frac{1}{4\pi\lambda} E_1\left(\frac{r_b^2}{4\kappa t}\right) + R_b \tag{26}$$

$\Downarrow$

$$\begin{aligned} 4\pi\lambda R_b &= E_1\left(\frac{r_a^2}{4\kappa t}\right) - E_1\left(\frac{r_b^2}{4\kappa t}\right) \\ &\approx \left(-\ln\left(\tfrac{r_a^2}{4\kappa t}\right) - 0.5772\right) - \left(-\ln\left(\tfrac{r_b^2}{4\kappa t}\right) - 0.5772\right) \\ &= -2\ln\left(\tfrac{r_a}{r_b}\right) \end{aligned} \tag{27}$$

$\Downarrow$

$$r_a = r_b\, e^{-2\pi\lambda R_b} \tag{28}$$

The Thermal Resistance of a Single Tube

Assume a single tube of outer radius r, wall thickness d and thermal conductivity λ_t (in general smaller than λ of the surrounding rock or soil).

$$\frac{1}{4\pi\lambda_t} E_1\left(\frac{(r_b-d)^2}{4\kappa t}\right) = R = R_s + R_b = \frac{1}{4\pi\lambda_t} E_1\left(\frac{r_b^2}{4\kappa t}\right) + R_b \quad (29)$$

$\Downarrow$

$$R_b = -\frac{1}{2\pi\lambda_t} \ln\left(\frac{r_b-d}{r_b}\right) \quad (30)$$

$\Downarrow$

or in terms of an apparent radius r_a:

$$r_a = r_b\, e^{\frac{\lambda}{\lambda_t} \ln\left(\frac{r_b-d}{r_b}\right)} = r_b \left(\frac{r_b-d}{r_b}\right)^{\frac{\lambda}{\lambda_t}} \quad (31)$$

Main Difference towards Shallow Heat Exchangers

Significant variation in temperature along the borehole

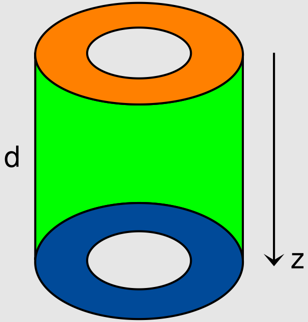
$$\Downarrow$$

Only coaxial heat exchangers can be used.

Main Field of Application

Direct (district) heating without heat pumps.

Energy Balance for the Fluid in the Heat Exchanger



Steady-state energy balance for the fluid:

$$\rho_f c_f Q T_f - \rho_f c_f Q T_f + P_l d = 0 \quad (32)$$

with

ρ_f = density of the fluid
c_f = specific heat capacity of the fluid
Q = flow rate $[\frac{m^3}{s}]$
T_f = fluid temperature

Energy Balance for the Fluid in the Heat Exchanger

$$\Downarrow$$

$$\rho_f c_f Q \frac{T_f - T_f}{d} = P_l \tag{33}$$

$$\Downarrow \quad \text{for } d \to 0$$

$$\rho_f c_f Q \frac{\partial T_f}{\partial z} = P_l \tag{34}$$

Simplest Situation: Perfectly Adjusted Flow Rate

Assume that $T_m(z)$ is the undisturbed temperature, and $\frac{T_m}{\partial z} = \text{const}$.

Adjust $Q(t)$ to $P_l(t)$ according to

$$Q = \frac{P_l}{\rho_f \, c_f \frac{\partial T_m}{\partial z}} \tag{35}$$

⬇

$$\frac{\partial T_f}{\partial z} = \frac{\partial T_m}{\partial z} \tag{36}$$

⬇

$$T_m - T_f = \text{const for all } z \tag{37}$$

⬇

Theory for shallow downhole heat exchangers can be applied.

Steady-State Solution for an Arbitrary Flow Rate

Previous solution: P_l depends on t, but not on z

General case: P_l depends on t and on z → no analytical solution

Alternative scenario: P_l depends on z, but not on t.

↓

$$T_m(z) - T_f(z) \quad = \quad R\,P_l(z) \;=\; R\rho_f c_f Q \frac{\partial}{\partial z} T_f(z) \tag{38}$$

where R is the total thermal resistance (Eq. 25).

↓

$$\frac{\partial}{\partial z}(T_f(z) - T_m(z)) \quad = \quad -\frac{T_f(z) - T_m(z)}{a} - \frac{\partial}{\partial z} T_m(z) \tag{39}$$

with $a = R\rho_f c_f Q$

Steady-State Solution for an Arbitrary Flow Rate

If $\frac{\partial}{\partial z} T_m = \text{const}$:

$$\frac{\partial}{\partial z}\left(T_f(z) - T_m(z) + a\frac{\partial}{\partial z}T_m\right) = -\frac{T_f(z) - T_m(z) + a\frac{\partial}{\partial z}T_m}{a} \quad (40)$$

$$\Downarrow$$

$$T_f(z) - T_m(z) + a\frac{\partial}{\partial z}T_m = \left(T_f(0) - T_m(0) + a\frac{\partial}{\partial z}T_m\right)e^{-\frac{z}{a}} \quad (41)$$

$$\Downarrow$$

$$T_f(z) = T_m(z) - a\frac{\partial}{\partial z}T_m + \left(T_f(0) - \left(T_m(0) - a\frac{\partial}{\partial z}T_m\right)\right)e^{-\frac{z}{a}} \quad (42)$$

Modeling Approaches

- Infinite horizontal plane
- Set of parallel pipes

Too small

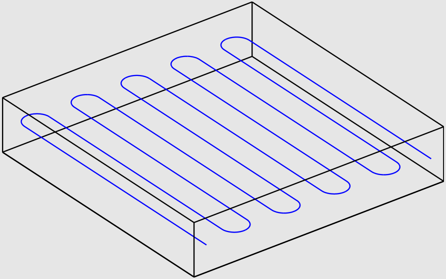
Source: www.bauweise.net

Large enough

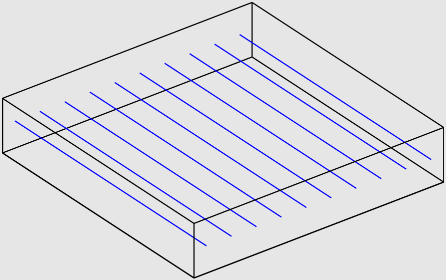
Source: Rehau AG & Co

Modeling as a Set of Parallel Pipes

Typical configuration



Simplified model



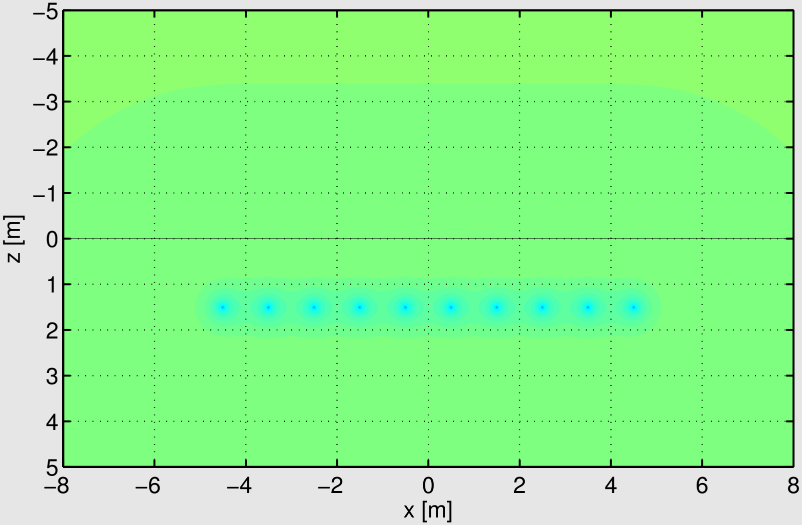
- Parallel horizontal pipes of infinite length

↓

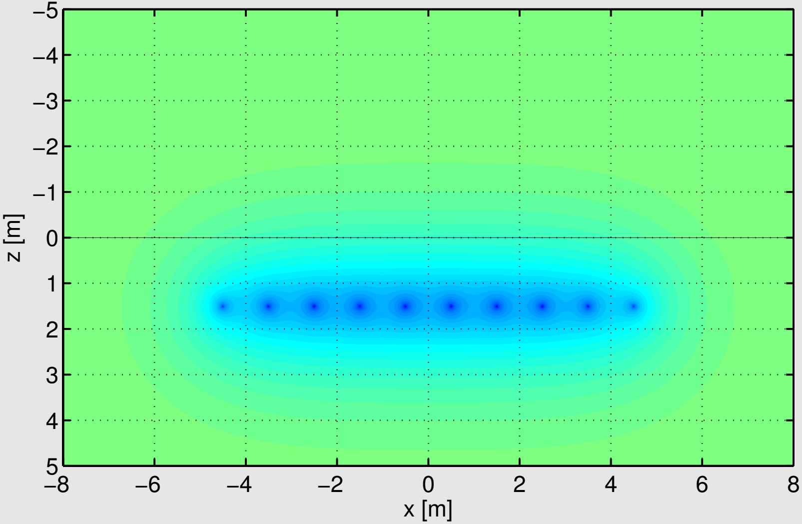
Theory of shallow downhole heat exchangers can be applied.

- Assume the same power per length for all pipes.

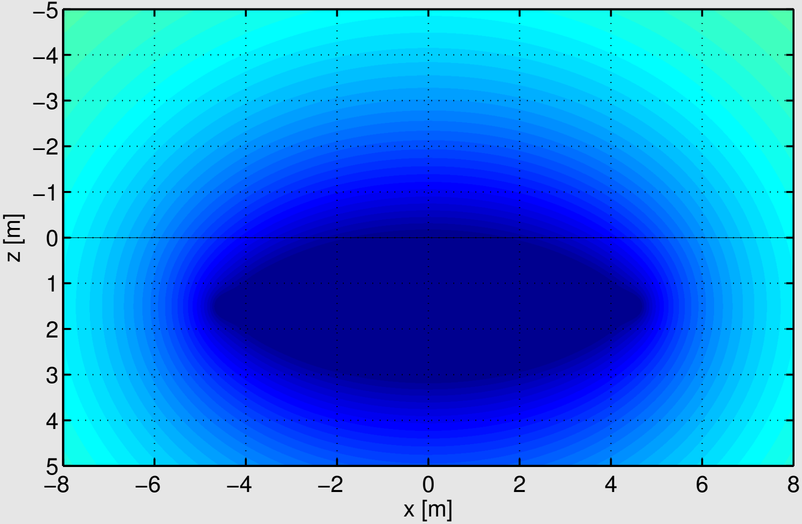
10 Parallel Pipes With 1 m Spacing in 1.5 m Depth After 3 Days



10 Parallel Pipes With 1 m Spacing in 1.5 m Depth After 1 Month



10 Parallel Pipes With 1 m Spacing in 1.5 m Depth After 1 Year



Limitations

- Temperatures of the pipes are not the same.

↓

Same P_l for each pipe is not correct.

↓

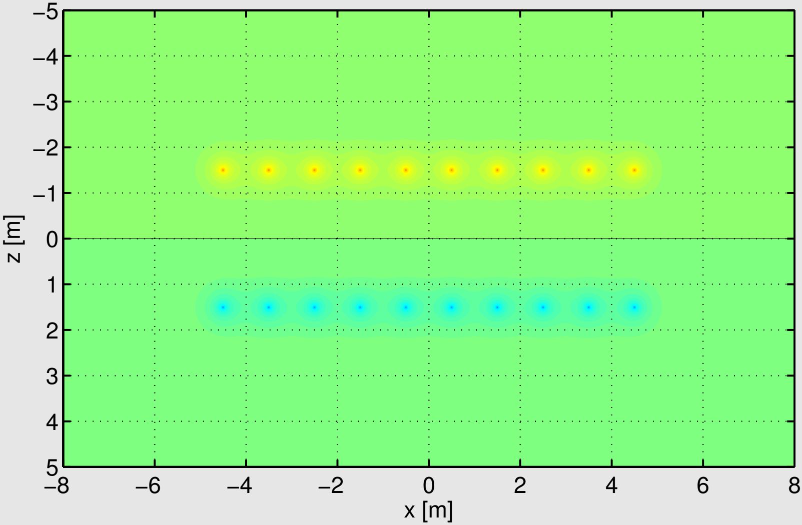
Not a big problem, use mean temperature.

- Surface temperature is affected by the heat collector as if there was no surface, while solar radiation and rapid heat transport in the atmosphere keep the temperature more constant in reality.

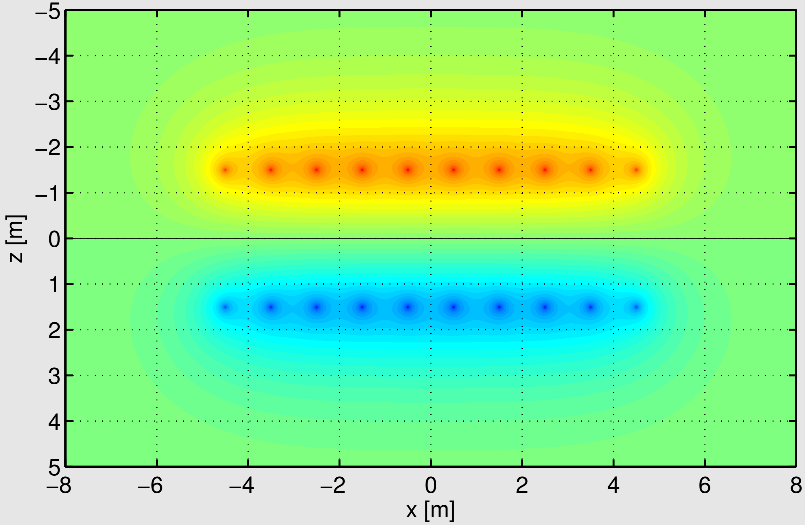
↓

Simplest model: keep surface temperature constant by introducing virtual pipes with $-P_l$ (supplying energy) above the surface.

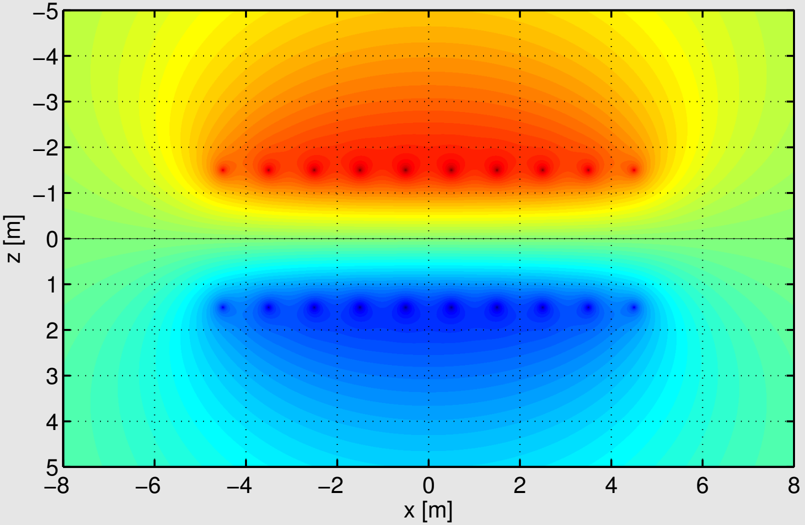
10 Parallel Pipes With 1 m Spacing in 1.5 m Depth After 3 Days



10 Parallel Pipes With 1 m Spacing in 1.5 m Depth After 1 Month



10 Parallel Pipes With 1 m Spacing in 1.5 m Depth After 1 Year



Computing the Average Temperature of Parallel Pipes

Assume n parallel pipes in a depth d below the surface and a horizontal spacing s.

Dist. r	Sign	Mean num.
r_a	$+$	1
s	$+$	$2\left(1-\frac{1}{n}\right)$
$2s$	$+$	$2\left(1-\frac{2}{n}\right)$
$3s$	$+$	$2\left(1-\frac{3}{n}\right)$
...	...	...

Distance r	Sign	Mean num.
$2d$	$-$	1
$\sqrt{s^2+(2d)^2}$	$-$	$2\left(1-\frac{1}{n}\right)$
$\sqrt{(2s)^2+(2d)^2}$	$-$	$2\left(1-\frac{2}{n}\right)$
$\sqrt{(3s)^2+(2d)^2}$	$-$	$2\left(1-\frac{3}{n}\right)$
...	...	...