Geothermics and Geothermal Energy
Deep Open Geothermal Systems

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Converting Geothermal Energy to Electricity

Steps of Conversion

Thermal energy → mechanical work: turbine; rather low efficiency due to thermodynamic limitation

Mechanical work → electricity: generator; high efficiency

The Thermodynamic Limitation (Carnot Cycle)

$$\delta S \;=\; \frac{\delta Q_h}{T_h} + \frac{\delta Q_c}{T_c} \;\geq\; 0 \tag{1}$$

where

$$\begin{aligned}
\delta Q_h &= \text{thermal energy supplied to to the hot system} \\
&< 0 \text{ (from the geothermal reservoir into the turbine)} \\
T_h &= \text{temperature of the hot system} \\
\delta Q_c &= \text{thermal energy supplied to the cold system} \\
&> 0 \text{ (out of the turbine)} \\
T_c &= \text{temperature of the cold system}
\end{aligned}$$

Converting Geothermal Energy to Electricity

The Thermodynamic Limitation (Carnot Cycle)

Mechanical work yielded by one cycle (conservation of energy):

$$\delta W \;=\; -\,(\delta Q_h + \delta Q_c) \;\leq\; -\,\delta Q_h \left(1 - \frac{T_c}{T_h}\right) \;=\; -\,\eta_{\max}\,\delta Q_h \qquad (2)$$

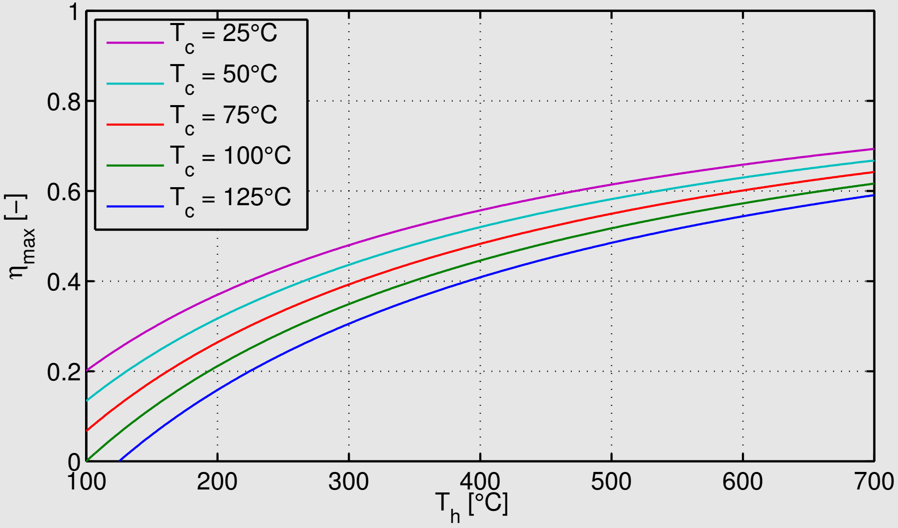
with the maximum efficiency

$$\eta_{\max} \;=\; \frac{T_h - T_c}{T_h} \qquad (3)$$

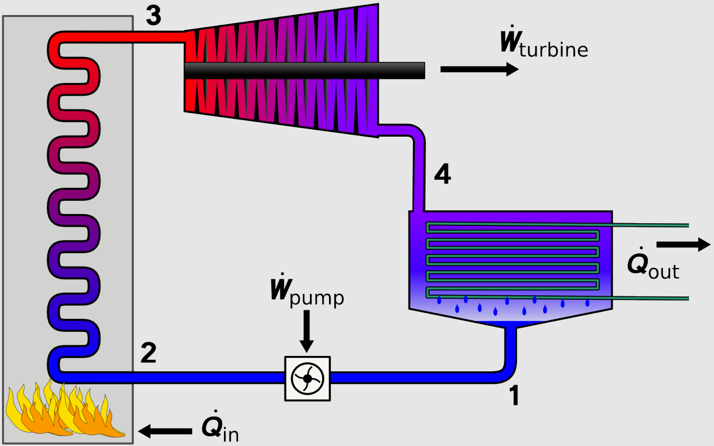
Consequence for the electrical, mechanical and thermal power of a geothermal power plant:

$$P_{\mathrm{el}} \;<\; P_{\mathrm{me}} \;<\; \eta_{\max}\,P_{\mathrm{th}} \qquad (4)$$

Maximum Efficiency of Converting Thermal Energy



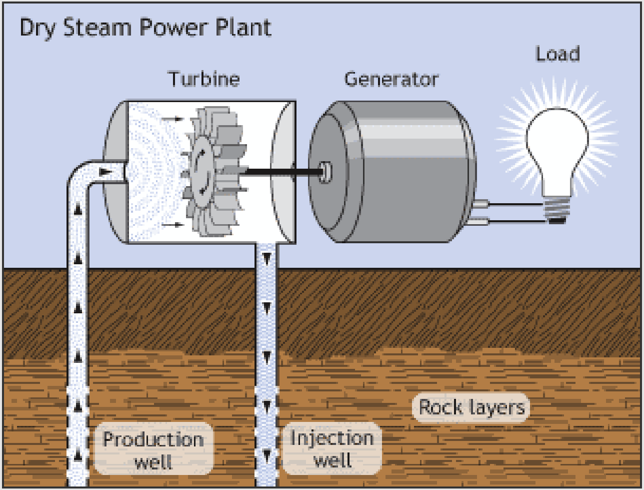
General Principle: Clausius-Rankine Cycle



Source: Wikipedia, © A. Ainsworth

Main Types of Geothermal Power Plants

Dry Steam Power Plants



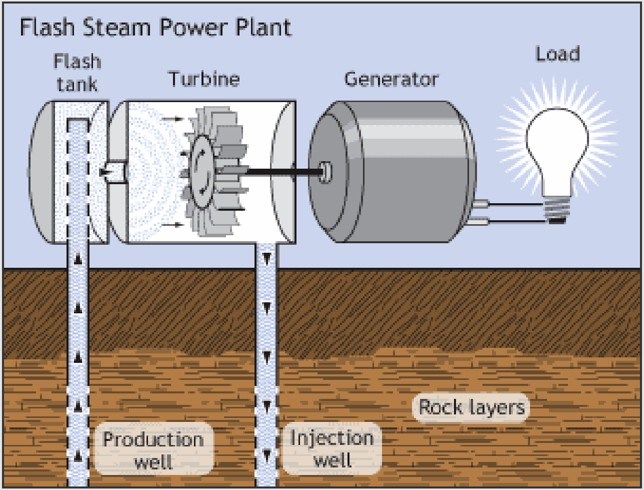
Source: Office of Energy Efficiency and Renewable Energy

Dry Steam Power Plants

- Rather simple technology
- First geothermal production of electricity: Larderello 1904
- Biggest geothermal power plant on Earth: “The Geysers”, California, USA, 750 MW_{el}
- Limited to few locations on Earth

Main Types of Geothermal Power Plants

Flash Steam Power Plants



Source: Office of Energy Efficiency and Renewable Energy

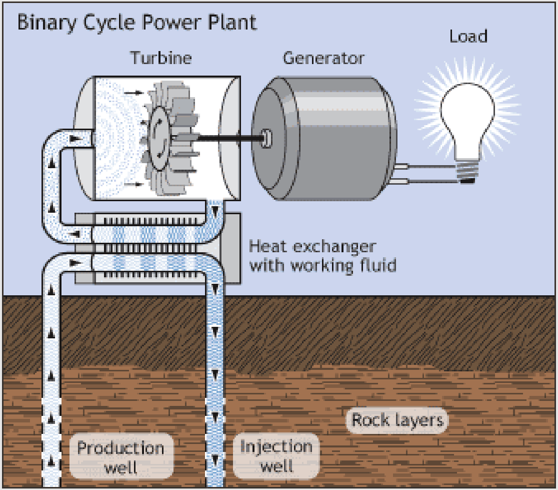
Main Types of Geothermal Power Plants

Flash Steam Power Plants

- Most common type of geothermal power generation plants in operation today
- Reasonable efficiency only for high-enthalpy resources ($T > 200°C$)

Main Types of Geothermal Power Plants

Binary Cycle Power Plants



Source: Office of Energy Efficiency and Renewable Energy

Binary Cycle Power Plants

- Heat transfer to a fluid with a boiling point below 100°C by a heat exchanger
- Applicable to low-enthalpy resources ($T < 200$°C)
- Expensive technology
- Types:

  Organic Rankine Cycle (ORC): Transfer heat to an organic fluid with a low boiling point and operate the turbine with this fluid, e. g., n-perfluorpentane (C_5F_{12}, boiling point 30°C, $T_c \approx 75$°C)

  Kalina cycle: Ammonia solution where the concentration of ammonia varies during the cycle; power plants in Germany at Unterhaching and Bruchsal

ORC Power Plants in Germany

Location	Depth [m]	T [°C]	Q [$\frac{l}{s}$]	P_{th} [MW]	P_{el} [MW]
Dürrnhaar	4114	130	135		5.5
Grünwald	4083	130	150	50	4.5
Insheim	3650	165	85		4.8
Kirchstockach	3750	139	145		5.5
Landau	3340	155	70		3.8
Neustadt-Glewe	2455	99	35	5.5	0.23
Sauerlach	4480	143	110		4.0
Simbach/Braunau	1942	80	74	7	0.2
Traunreut	4500	118	130		5.0 (?)

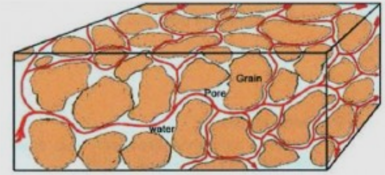
Kalina Cycle Power Plants in Germany

Location	Depth [m]	T [°C]	Q [$\frac{l}{s}$]	P_{th} [MW]	P_{el} [MW]
Unterhaching	3350	122	150	38	3.36*
Bruchsal	2542	120	24	5.5	0.55

* until 2017

Principle

- Hot water is extracted at one or more wells.
- Cold water is (re)injected at another well; in most cases at the same rate as total extraction.
- Flow of water through a porous rock.



Source: Borehole Wireline

Major Problem

Maintaining the fluid circulation in the rock consumes a considerable part of the produced energy.

Porosity of Rocks

Total porosity: $\phi = \frac{\text{void volume}}{\text{total volume}}$; $0 \leq \phi < 1$; often measured in percent

Effective porosity: only accessible pores and volume of water that can be extracted

Typical porosity values:

	ϕ_{tot} [%]	ϕ_{eff} [%]
equally sized spheres	26–48	26–48
soil	55	40
clay	50	2
sand	25	22
limestone	20	18
sandstone (semiconsolidated)	11	6
granite	0.1	0.09

Source: GlobalSecurity.org

Darcy's Law

- Empirically found by Henry Darcy (1856).
- Describes the average flow through a porous medium on macroscopic scales.
- Simplest form (without gravity):

$$\vec{v}(\vec{x},t) \;=\; -\frac{k}{\eta}\nabla p(\vec{x},t) \qquad (5)$$

where

$$\begin{aligned}
\vec{v} &= \text{volumetric flow rate (Darcy velocity) } [\tfrac{\text{m}}{\text{s}}] \\
p &= \text{fluid pressure [Pa]} \\
k &= \text{hydraulic permeability } [\text{m}^2] \\
\eta &= \text{dynamic viscosity of the fluid [Pa s]}
\end{aligned}$$

- Basically the same as Fourier's law of heat conduction.

The Hydraulic Permeability

- Units:
  SI unit: m^2
  Widely used unit: Darcy (D)

$$1\,\mathrm{D} \;=\; 9.869 \times 10^{-13}\,\mathrm{m}^2 \;\approx\; 10^{-12}\,\mathrm{m}^2 \;=\; 1\,\mu\mathrm{m}^2$$

- $k = 1\,\mathrm{D}$ results in a flow rate of $1\,\frac{\mathrm{cm}}{\mathrm{s}}$ at a pressure drop of $1\,\frac{\mathrm{atm}}{\mathrm{cm}}$ in water at 20°C ($\eta = 10^{-3}\,\mathrm{Pas}$).
- Typical values:

Medium	k [D]
gravel	$10 - 1000$
sand	$0.01 - 10$
silt	$10^{-3} - 0.1$

Medium	k [D]
limestone	$10^{-6} - 100$
fractured igneous rocks	$10^{-6} - 10$
unfractured igneous rocks	$10^{-9} - 10^{-6}$

The Darcy Equation

Balance equation for the mass of water per bulk volume

$$\frac{\partial \chi}{\partial t} \;=\; -\,\mathrm{div}\,(\rho_f \vec{v}) \;=\; \mathrm{div}\left(\rho_f \frac{k}{\eta} \nabla p\right) \tag{6}$$

where

$$\begin{aligned} \chi &= \text{mass of fluid per bulk volume } [\tfrac{\mathrm{kg}}{\mathrm{m}^3}] \\ \rho_f &= \text{fluid density } [\tfrac{\mathrm{kg}}{\mathrm{m}^3}] \end{aligned}$$

$$\Downarrow$$

$$S \frac{\partial p}{\partial t} \;=\; -\,\mathrm{div}\,(\rho_f \vec{v}) \;=\; \mathrm{div}\left(\rho_f \frac{k}{\eta} \nabla p\right) \quad \text{with} \quad S \;=\; \frac{\partial \chi}{\partial p} \tag{7}$$

The Darcy Equationi Compared to the Heat Conduction Equation

Basically the same equation as the heat conduction equation with a different meaning of the parameters.

heat conduction	T	λ	ρc	$\kappa = \frac{\lambda}{\rho c}$
Darcy flow	p	$\rho_f \frac{k}{\eta}$	S	$\tilde{\kappa} = \frac{\rho_f k}{\eta S}$

If all parameters are constant:

$$\frac{\partial T}{\partial t} = \kappa \Delta T, \qquad \vec{q} = -\lambda \nabla T \tag{8}$$

$$\frac{\partial p}{\partial t} = \tilde{\kappa} \Delta p, \qquad \vec{v} = -\frac{k}{\eta} \nabla p \tag{9}$$

Superposition of Solutions

The simplest form of Darcy's equation is linear.

$\Downarrow$

Solutions can be superposed:

$$p(\vec{x}, t) = p_0(\vec{x}) + p_1(\vec{x}, t) + p_2(\vec{x}, t) + \ldots \quad (10)$$

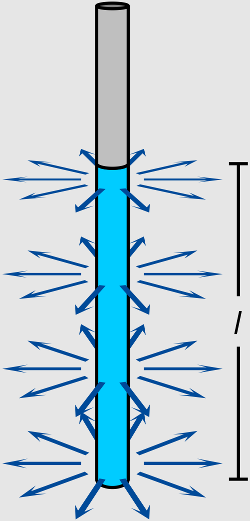
$$\vec{v}(\vec{x}, t) = \vec{v}_0(\vec{x}) + \vec{v}_1(\vec{x}, t) + \vec{v}_2(\vec{x}, t) + \ldots \quad (11)$$

where

$p_0, \vec{v}_0$ = natural pressure and Darcy velocity without wells

$p_i, \vec{v}_i$ = additional pressure and Darcy velocity caused by well $\# i$

The Simplest Model for a Hydrothermal Well



Vertical borehole in an aquifer of a thickness l

Simplifications:

- All parameters (k, S, ρ_f, η) constant
- Only horizontal flow in radial direction

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Basically the same solution as for the temperature drop of a downhole heat exchanger

Use variables p and $\vec{v}$ for the additional pressure and Darcy velocity instead instead of p_i and $\vec{v}_i$.

The Simplest Model for a Hydrothermal Well

Downhole heat exchanger:

$$T(r,t) \;=\; -\frac{P_l}{4\pi\lambda} E_1\left(\frac{r^2}{4\kappa t}\right) \tag{12}$$

Hydrothermal well:

$$p(r,t) \;=\; \frac{\frac{\rho_f Q}{l}}{4\pi\rho_f \frac{k}{\eta}} E_1\left(\frac{r^2}{4\tilde{\kappa} t}\right) \;=\; \frac{\eta Q}{4\pi k l} E_1\left(\frac{r^2}{4\tilde{\kappa} t}\right) \tag{13}$$

where

$$Q \;=\; \text{rate of injection } [\tfrac{\mathrm{m}^3}{\mathrm{s}}],\; Q < 0 \text{ for extraction}$$

Well Doublets

$\tilde{\kappa} \gtrapprox 1\,\frac{\mathrm{m}^2}{\mathrm{s}}$ for highly permeable rocks ($k \gtrapprox 0.01\,\mathrm{D}$) required for hydrothermal systems if the rock is fully saturated with water.

$\Downarrow$

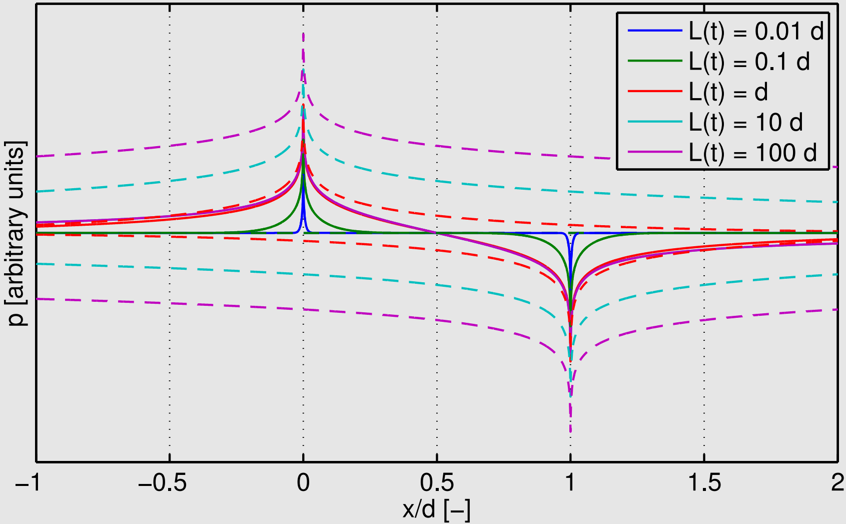
Pressure rapidly decreases around an extraction well.

Solution: Use a well doublet consisting of an injection well and an extraction well working at the same flow rate.

$$p(x, y, t) = \frac{\eta Q}{4\pi k l}\left(E_1\left(\frac{r_i^2}{4\tilde{\kappa} t}\right) - E_1\left(\frac{r_e^2}{4\tilde{\kappa} t}\right)\right) \qquad (14)$$

where $r_{i/e}$ is the distance of the considered point from the injection / extraction well.

Pressure Distribution of a Well Doublet Compared to a Single Well



Well Doublets

Use the approximation

$$E_1(v) \approx -\ln(v) - 0.5772 \quad \text{for} \quad v \ll 1 \tag{15}$$

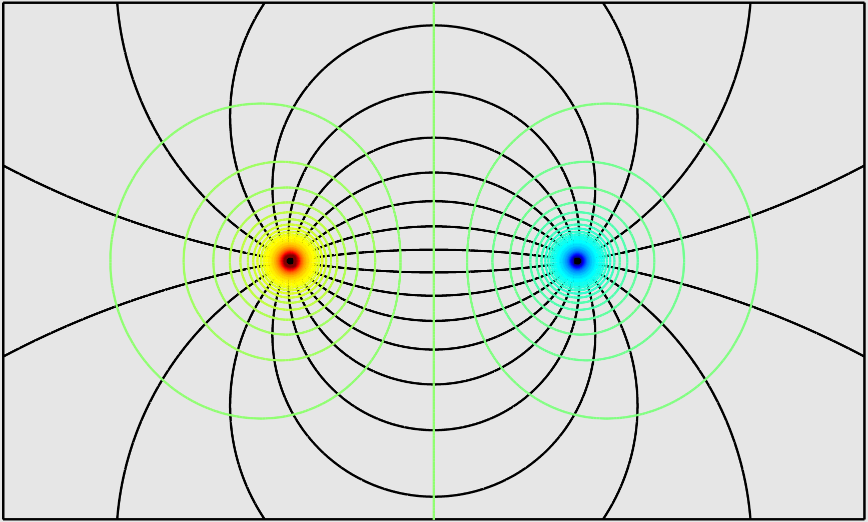
$$\Downarrow$$

$$p(x,y,t) \approx \frac{\eta Q}{4\pi kl}\left(-\ln\left(\frac{r_i^2}{4\tilde{\kappa}t}\right) + \ln\left(\frac{r_e^2}{4\tilde{\kappa}t}\right)\right) \tag{16}$$

$$= \frac{\eta Q}{2\pi kl}\ln\frac{r_e}{r_i} \tag{17}$$

is independent of t (steady-state flow conditions).

Pressure and Flow Lines of a Simple Well Doublet



The Simplest Model for a Well Doublet

Limitation: In principle only valid

- for confined aquifers or
- if the horizontal distance of the wells is much smaller than the open borehole length l

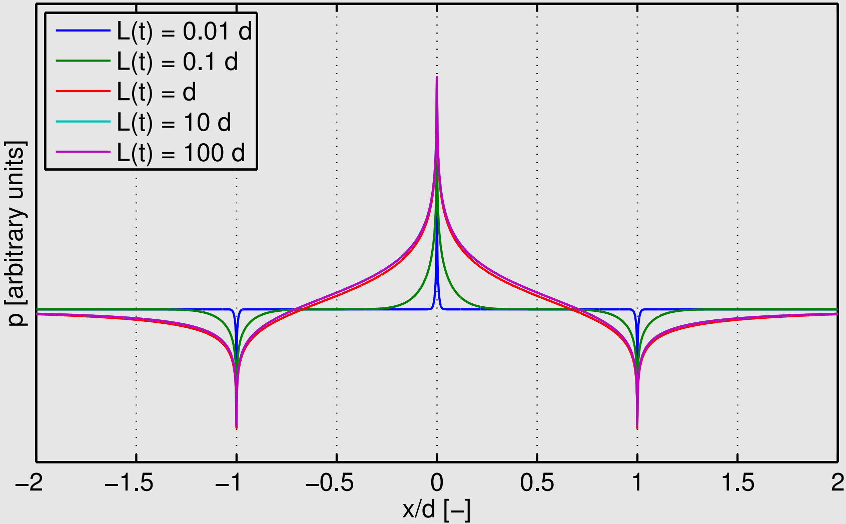
Mechanical power required for maintaining the flow:

$$P = (p_i - p_e)\,Q \tag{18}$$

where

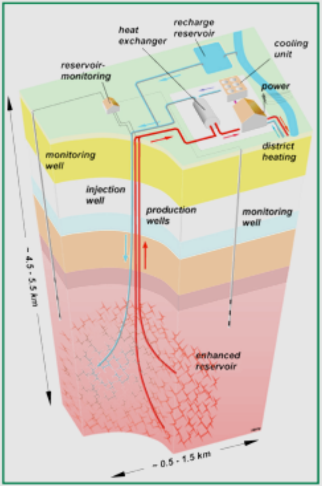
$$p_i = \text{pressure at the walls of the injection well}$$
$$p_e = \text{pressure at the walls of the extraction well}$$

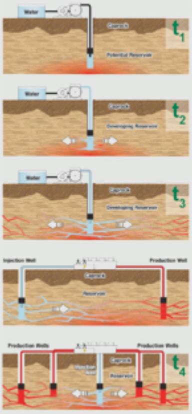
Well Triplet



Enhanced Geothermal Systems

Hydraulic Fracturing for Increasing the Permeability





Drill a well to explore

Inject water to cause slip on faults (high water pressure pushes fractures open)

Injection extends a network of connected fractures

Inject water to sweep heat to a production well

Maximize production rate and lifetime

Source: NewEnergyNews

Environmental Issues related to Hydraulic Fracturing

- Large amounts of contaminated water if fracturing is supported by additional chemicals
- Fluid-induced seismicity

Heat Transport in Geothermal Systems

Mechanisms of Heat Transport in Porous Media

Solid matrix: conduction

Fluid: conduction and advection

Heat Exchange Between Fluid and Matrix

Length scale of heat conduction:

$$L(t) = \sqrt{\kappa t} \tag{19}$$

Water: $\kappa = 1.4 \times 10^{-7} \, \frac{\mathrm{m}^2}{\mathrm{s}}$

Rocks: $\kappa \approx 10^{-6} \, \frac{\mathrm{m}^2}{\mathrm{s}}$

$\Downarrow$

Fluid and matrix rapidly adjust to the same temperature locally.

The Heat Equation for a Fluid

Heat flux density for a fluid moving at a velocity $\vec{v}$:

$$\vec{q} = \underset{\text{conduction}}{\underset{\uparrow}{-\lambda \nabla T}} + \underset{\text{advection}}{\underset{\uparrow}{\rho c T \vec{v}}} \tag{20}$$

$$\Downarrow$$

$$\rho c \frac{\partial T}{\partial t} = -\mathrm{div}(\vec{q}) \tag{21}$$

$$= \mathrm{div}\left(\lambda \nabla T - \rho c T \vec{v}\right) \tag{22}$$

The Heat Equation for a Porous Medium

$$(\rho_m c_m + \phi \rho_f c_f) \frac{\partial T}{\partial t} = \operatorname{div}\left((\lambda_m + \phi \lambda_f) \nabla T - \rho_f c_f T \vec{v}\right) \qquad (23)$$

where

$$\begin{aligned} \rho_f, c_f, \lambda_f &= \text{parameters of the fluid} \\ \rho_m, c_m, \lambda_m &= \text{parameters of the dry matrix (not the solid!)} \\ \phi &= \text{porosity} \\ \vec{v} &= \text{Darcy velocity} \end{aligned}$$

⬇

Effective velocity of heat advection:

$$\vec{v}_a = \frac{\rho_f c_f}{\rho_m c_m + \phi \rho_f c_f} \vec{v} \approx 1.5 \vec{v} \qquad (24)$$

Velocities of Fluid Flow and Heat Transport

Mean interstitial velocity of the water particles

$$\vec{v}_p = \frac{\vec{v}}{\phi} \tag{25}$$

is significantly higher than the flow rate (Darcy velocity) $\vec{v}$.

Effective velocity of heat advection

$$\vec{v}_a = \frac{\rho_f c_f}{\rho_m c_m + \phi \rho_f c_f} \vec{v} \approx 1.5\, \vec{v} \tag{26}$$

is also higher than the flow rate $\vec{v}$, but lower than the mean interstitial velocity $\vec{v}_p$.

Velocities of Fluid Flow and Heat Transport

$$\vec{v}_a = \frac{\phi \rho_f c_f}{\rho_m c_m + \phi \rho_f c_f} \vec{v}_p = \frac{\vec{v}_p}{R} \tag{27}$$

where

$$R = \frac{\rho_m c_m + \phi \rho_f c_f}{\phi \rho_f c_f} = 1 + \frac{\rho_m c_m}{\phi \rho_f c_f} \tag{28}$$

is the coefficient of retardation.

↓

Water circulates R times between the wells until the cold temperature front breaks through (if heat conduction is neglected).