Geothermics and Geothermal Energy
Closed Geothermal Systems

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Principle

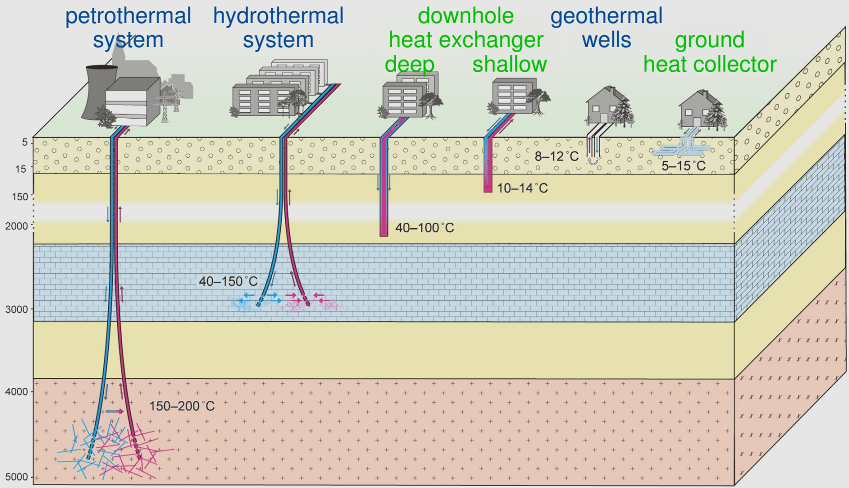
- Fluid circulates in a closed heat exchanger.
- Heat is transported to the fluid by heat conduction.
- Heat transport in the surrounding rock or soil by heat conduction; in some cases also by advection (groundwater).

Fluids

- Water or alcohol-water mixtures.
- Water has the best properties (heat capacity, thermal conductivity, viscosity) as long as $T > 0°\mathrm{C}$.

Closed Geothermal Systems

Types of Geothermal Systems



Source: Ingolstädter Kommunalbetriebe (modified)

Closed Geothermal Systems

Limitation

Heat is transported to the heat exchanger by conduction.

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Requires a temperature gradient towards the exchanger.

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Temperature in the exchanger is lower than the undisturbed subsurface temperature; temperature drop depends on

- extracted power
- thermal properties of the subsurface (mainly the thermal conductivity)
- properties of the exchanger (size, shape, material)

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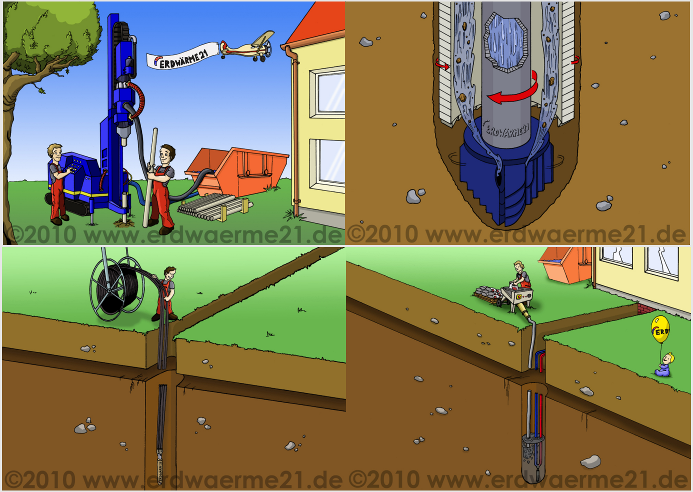
Production of electricity is economically not reasonable (so far?).

Downhole Heat Exchangers (Borehole Heat Exchangers)



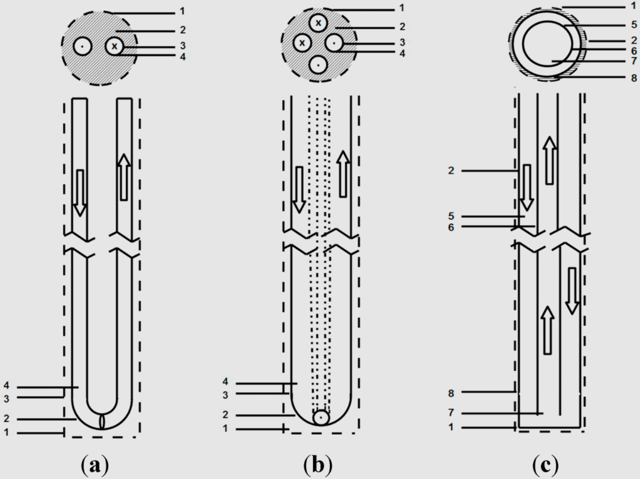
Source: Erdwärme21

Downhole Heat Exchangers (Borehole Heat Exchangers)

Source: Baublog: Villa Lugana in Teltow

Geothermal Heat Exchangers

Downhole Heat Exchangers (Borehole Heat Exchangers)



(a) single U-tube
(b) double U-tube
(c) coaxial
1 borehole wall
2 grout
3 interior of U-tube
4 U-tube wall
5 annulus space
6 inner column (pipe)
7 interior of inner column (pipe)
8 external column

Source: Sliwa & Rosen (2015), *Sustainability*, 7(10), 13104, doi:10.3390/su71013104

Application of Downhole Heat Exchangers

Shallow heat exchangers ($d \lessapprox 400\,\mathrm{m}$, $T \lessapprox 25^\circ\mathrm{C}$): heating of buildings with the help of heat pumps

Deep heat exchangers ($d \gtrapprox 1000\,\mathrm{m}$, $T \gtrapprox 40^\circ\mathrm{C}$): direct heating

- down to depths of about 3000 m so far (coaxial type only)
- mostly reuse or deepen abandoned hydrocarbon boreholes
- economically still questionable

Ground Heat Collectors

Source: www.bauweise.net

Ground Heat Collectors

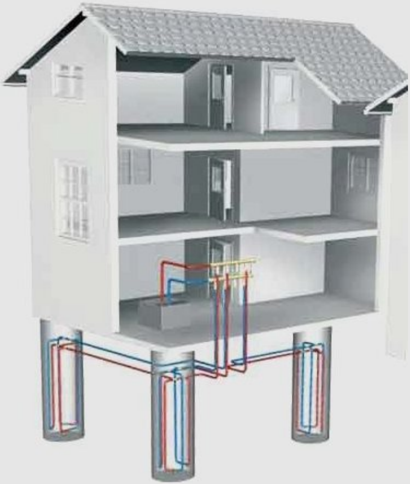
Source: Rehau AG & Co

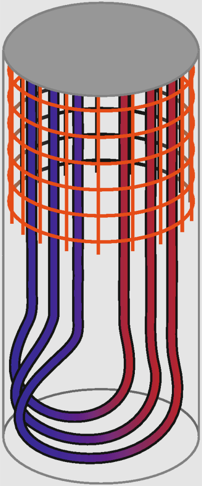
Geothermal Baskets

Source: Heizungsjournal

Geothermal Heat Exchangers

Geothermal Energy Piles





Sources: Energy Systems Research Unit, University of Strathclyde; Stober & Bucher, Geothermie

Superposition of Solutions

The heat conduction equation is linear.

$$\Downarrow$$

Can be solved by superposing individual components:

$$T(\vec{x},t) \;=\; T_m(\vec{x}) + T_y(\vec{x},t) + T_1(\vec{x},t) + T_2(\vec{x},t) + \ldots \qquad (1)$$

with

$$\begin{aligned} T_m(\vec{x}) &= \text{steady-state geotherm} \\ T_y(\vec{x},t) &= \text{natural seasonal variation} \\ T_i(\vec{x},t) &= \text{temperature drop caused by the } i^{\text{th}} \text{ heat exchanger} \end{aligned}$$

We use T instead of T_i for the rest of the chapter.

Analytical Approximations

All three components can be approximated by analytical solutions of the heat conduction equation reasonably well in most cases.

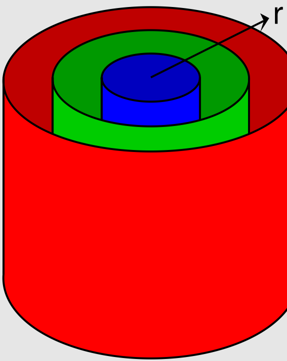
$\Downarrow$

No need for numerical simulations and / or specific software

Analytical solutions use

- symmetries for reducing the spatial dimension (mainly from 3 to 1) and
- scaling properties (length vs. time).

Cylindrical Symmetry



$T(x, y, z, t)$ only depends on $r = \sqrt{x^2 + y^2}$ and t.

Limitations:

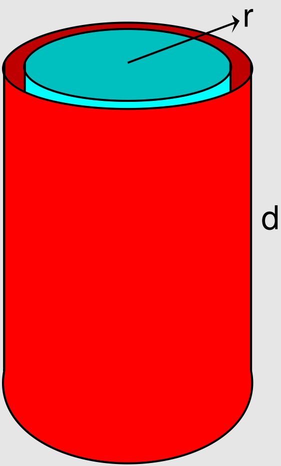
- Fluid temperature in the heat exchanger must increase with the geothermal gradient.

  ⬇

  Only applicable to shallow boreholes and to deep coaxial heat exchangers under specific conditions.
- Seasonal temperature variation in the upper region cannot be taken into account.
- Borehole length l must be much larger than $L(t) = \sqrt{\kappa t}$.

The Heat Conduction Equation for Cylindrical Symmetry

Energy balance for a cylindrical shell:



Change in thermal energy E contained in the shell:

$$\begin{aligned}\frac{\partial E}{\partial t} &= \left(\pi r^2 d - \pi r^2 d\right)\rho c \frac{\partial T}{\partial t} \\ &= 2\pi r d\, q - 2\pi r d\, q \end{aligned} \quad (2)$$

where q is the heat flux density in radial direction.

↓

$$\rho c \frac{\partial}{\partial t} T(r,t) = -\frac{1}{r}\frac{\partial}{\partial r}\left(r q(r,t)\right) \quad (3)$$

for $r - r \to 0$

The Heat Conduction Equation for Cylindrical Symmetry

With $q = -\lambda \frac{\partial T}{\partial r}$:

$$\rho c \frac{\partial}{\partial t} T(r,t) = \frac{1}{r} \frac{\partial}{\partial r} \left(r \lambda \frac{\partial}{\partial r} T(r,t) \right) \tag{4}$$

If λ is constant:

$$\frac{\partial}{\partial t} T(r,t) = \kappa \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} T(r,t) \right) \tag{5}$$

with $\kappa = \frac{\lambda}{\rho c}$

Solution of the Heat Conduction Equation for Cylindrical Symmetry

Look for solutions where the shape of the temperature profile remains constant, while only the spatial scale changes.

↓

Look for solutions $T(r, t)$ which depend on

$$u = \frac{r}{2L(t)} = \frac{r}{2\sqrt{\kappa t}} \tag{6}$$

instead of r and t only. The factor 2 is only for convenience.

Solution of the Heat Conduction Equation for Cylindrical Symmetry

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial u}\frac{\partial u}{\partial t} = \frac{\partial T}{\partial u}\frac{r}{2\sqrt{\kappa}}\frac{-1}{2t^{\frac{3}{2}}} = \frac{\partial T}{\partial u}\frac{-u}{2t} \quad (7)$$

$$r\frac{\partial T}{\partial r} = r\frac{\partial T}{\partial u}\frac{\partial u}{\partial r} = \frac{\partial T}{\partial u}\frac{r}{2\sqrt{\kappa t}} = u\frac{\partial T}{\partial u} \quad (8)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial u}\left(u\frac{\partial T}{\partial u}\right)\frac{\partial u}{\partial r} = \frac{1}{u}\frac{\partial}{\partial u}\left(u\frac{\partial T}{\partial u}\right)\frac{1}{4\kappa t} \quad (9)$$

⇓

$$\frac{\partial T}{\partial u}\frac{-u}{2t} = \kappa\frac{1}{u}\frac{\partial}{\partial u}\left(u\frac{\partial T}{\partial u}\right)\frac{1}{4\kappa t} \quad (10)$$

⇓

$$\frac{\partial}{\partial u}\left(u\frac{\partial T}{\partial u}\right) = -2u\left(u\frac{\partial T}{\partial u}\right) \quad (11)$$

Solution of the Heat Conduction Equation for Cylindrical Symmetry

Solution:

$$u \frac{\partial T}{\partial u} = a\, e^{-u^2} \tag{12}$$

with an arbitrary constant a.

Extracted power per borehole length l:

$$\begin{aligned} P_l &= \frac{P}{l} = -\lim_{r \to 0} \left(2\pi r\, q(r,t)\right) = 2\pi\lambda \lim_{r \to 0} r \frac{\partial T}{\partial r} \\ &= 2\pi\lambda \lim_{u \to 0} u \frac{\partial T}{\partial u} = 2\pi\lambda \lim_{u \to 0} a\, e^{-u^2} = 2\pi\lambda a \end{aligned} \tag{13}$$

⬇

$$\frac{\partial T}{\partial u} = \frac{P_l}{2\pi\lambda} \frac{e^{-u^2}}{u} \tag{14}$$

Solution of the Heat Conduction Equation for Cylindrical Symmetry

Solution for $T(u)$ with the condition $T(u) \to 0$ for $u \to \infty$:

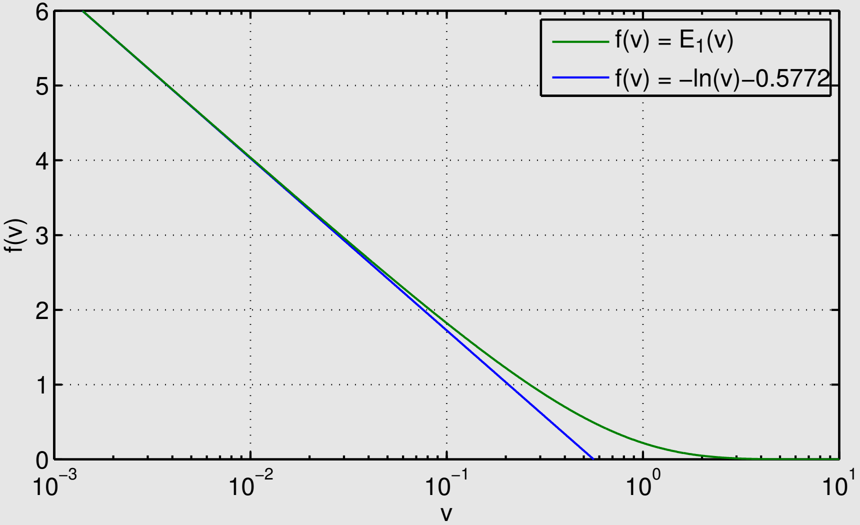
$$\begin{aligned} T(u) &= -\frac{P_l}{2\pi\lambda}\int\limits_{u}^{\infty}\frac{e^{-\xi^2}}{\xi}d\xi = -\frac{P_l}{4\pi\lambda}\int\limits_{u^2}^{\infty}\frac{e^{-x}}{x}dx \\ &= -\frac{P_l}{4\pi\lambda}E_1(u^2) \end{aligned} \tag{15}$$

with the function

$$E_1(v) = \int\limits_{v}^{\infty}\frac{e^{-x}}{x}dx \tag{16}$$

which is called exponential integral.

Solution of the Heat Conduction Equation for Cylindrical Symmetry



Solution of the Heat Conduction Equation for Cylindrical Symmetry

- The exponential integral can be approximated by

$$E_1(v) \approx -\ln(v) - 0.5772 \tag{17}$$

  for $v \ll 1$.
- $E_1(v) \to 0$ for $u \to \infty$

$$\Downarrow$$

$$E_1(v) \approx \max\{-\ln(v) - 0.5772, 0\} \tag{18}$$

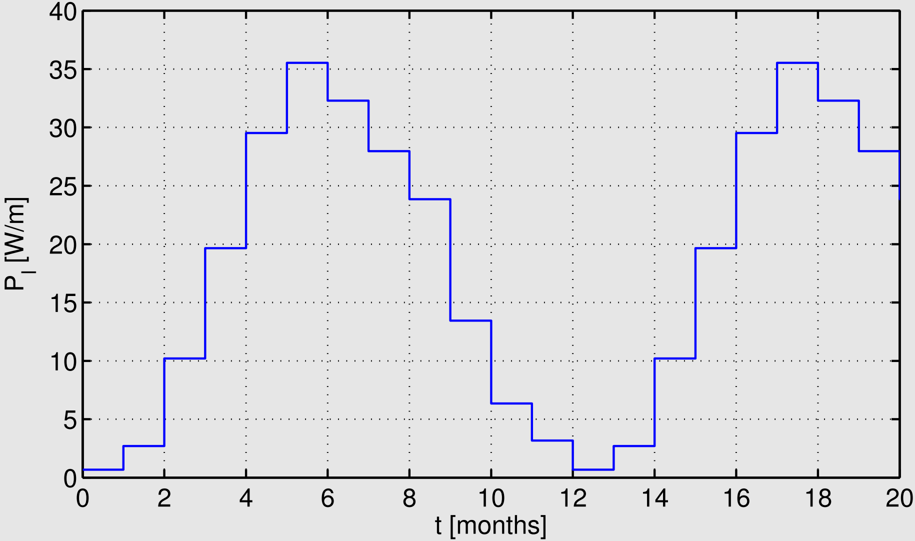
is a reasonable approximation if E_1 is not available.

Solution of the Heat Conduction Equation for Cylindrical Symmetry

Solution written in terms of r and t:

$$T(r,t) = -\frac{P_l}{4\pi\lambda} E_1\left(\frac{r^2}{4L(t)^2}\right) = -\frac{P_l}{4\pi\lambda} E_1\left(\frac{r^2}{4\kappa t}\right) \qquad (19)$$

Solution for Step-Like Heat Extraction



Solution for Step-Like Heat Extraction

Superposition of several heat exchangers switched on at different times:

Month #	P_l $[\frac{W}{m}]$
1	0.7
2	2.7
3	10.2
4	19.7
5	29.5
6	35.5
7	32.3
8	28.0
9	23.8
10	13.4
11	6.3
12	3.2
...	...

Exchanger #	Starting time [mon]	P_l $[\frac{W}{m}]$
1	0	0.7
2	1	2.0
3	2	7.5
4	3	9.5
5	4	9.8
6	5	6.0
7	6	−3.2
8	7	−4.3
9	8	−4.2
10	9	−10.4
11	10	−7.1
12	11	−3.1
...	...	...

Solution for Step-Like Heat Extraction

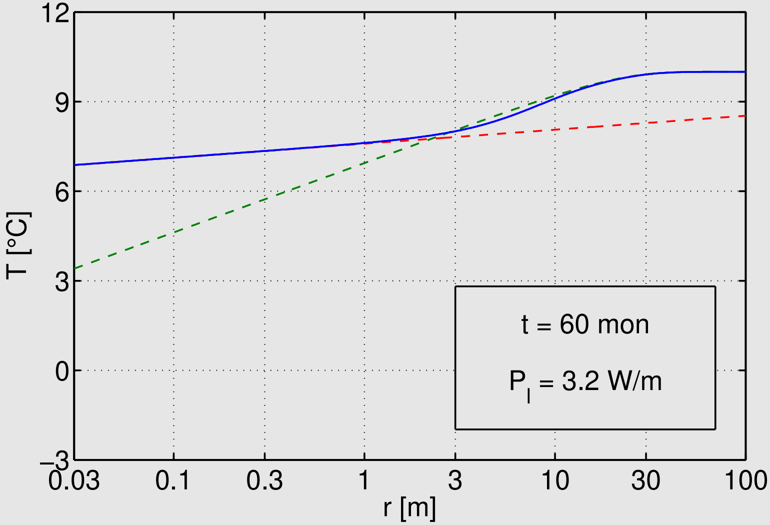
Formally:

$$P_l(t) = \begin{cases} 0 & \text{for} \quad t < t_0 \\ P_{l,i} & \phantom{\text{for}} \quad t_{i-1} \leq t < t_i \end{cases} \tag{20}$$

$$\Downarrow$$

$$\begin{aligned} T(r,t) &= \frac{P_{l,1}}{4\pi\lambda} E_1\left(\frac{r^2}{4\kappa(t-t_0)}\right) \\ &+ \frac{(P_{l,2}-P_{l,1})}{4\pi\lambda} E_1\left(\frac{r^2}{4\kappa(t-t_1)}\right) \\ &+ \ldots \\ &+ \frac{(P_{l,n+1}-P_{l,n})}{4\pi\lambda} E_1\left(\frac{r^2}{4\kappa(t-t_n)}\right) \end{aligned} \tag{21}$$

Solution for Step-Like Heat Extraction



Solution for Step-Like Heat Extraction

- Behavior for $r \gg L(t)$ is determined by the long-term mean heat extraction $\overline{P}_l$:

$$T(r,t) \approx -\frac{\overline{P}_l}{4\pi\lambda} E_1\left(\frac{r^2}{4L(t)^2}\right) \tag{22}$$

- Behavior for $r < L(\delta t)$ is determined by the actual heat extraction P_l if P_l has been constant for a time interval δt:

$$\begin{aligned} T(r,t) &\approx -\frac{P_l}{4\pi\lambda} E_1\left(\frac{r^2}{4L(\delta t)^2}\right) + f(t) && (23) \\ &\approx \frac{P_l}{4\pi\lambda} \ln\left(\frac{r^2}{4L(\delta t)^2}\right) + 0.5772 + f(t) && (24) \end{aligned}$$

where the function $f(t)$ depends on the history of P_l.

The Thermal Resistance

Consider two boreholes of different radii r_1 and r_2.

$\Downarrow$

$$T(r_1, t) - T(r_2, t) = \frac{P_l}{4\pi\lambda}\left(\ln\left(\frac{r_1^2}{4L(\delta t)^2}\right) - \ln\left(\frac{r_2^2}{4L(\delta t)^2}\right)\right) \quad (25)$$
$$= \frac{P_l}{2\pi\lambda}\ln\left(\frac{r_1}{r_2}\right) \quad (26)$$

for $r_1 \ll L(\delta t)$ and $r_2 \ll L(\delta t)$.

$\Downarrow$

Temperature difference is proportional to the actual P_l.

The Thermal Resistance

Basically the same result for the effect of the borehole's filling and the walls of the heat exchanger:

$$T_f(t) = T(r_b, t) - R\,P_l \tag{27}$$

where

$$\begin{aligned} T_f(t) &= \text{temperature of the fluid in the heat exchanger} \\ r_b &= \text{radius of the borehole} \\ R &= \text{resistance of the borehole / heat exchanger} \end{aligned}$$

- Borehole resistance depends on the geometry (single U-tube, double U-tube, coaxial) and on the material used for filling.
- Typical values for double U-tube heat exchangers:
  $R_b \approx 0.1\,\frac{\mathrm{m\,K}}{\mathrm{W}}$ (standard filling)
  $R_b \approx 0.08\,\frac{\mathrm{m\,K}}{\mathrm{W}}$ (thermally improved filling)

Including the Thermal Resistance in the Calculation

Define an apparent borehole radius r_a in such a way that

$$T_f(t) = T(r_a, t) \quad (28)$$

↓

$$T(r_b, t) - T_f(t) = T(r_b, t) - T(r_a, t) = \frac{P_l}{2\pi\lambda} \ln\left(\frac{r_b}{r_a}\right) = R\, P_l \quad (29)$$

with

$$R = \frac{1}{2\pi\lambda} \ln\left(\frac{r_b}{r_a}\right) \quad (30)$$

↓

$$r_a = r_b\, e^{-2\pi\lambda R} \quad (31)$$

The Thermal Resistance of a Single Tube

Assume a single tube of outer radius r_b, wall thickness d and thermal conductivity λ_t (in general smaller than λ of the surrounding rock or soil).

$\Downarrow$

$$R = \frac{1}{2\pi\lambda_t} \ln\left(\frac{r}{r-d}\right) \tag{32}$$

or in terms of an apparent radius r_a:

$$r_a = r_b\, e^{-2\pi\lambda R} = r_b\, e^{-\frac{\lambda}{\lambda_t} \ln\left(\frac{r_b}{r_b-d}\right)} = r_b \left(\frac{r_b-d}{r_b}\right)^{\frac{\lambda}{\lambda_t}} \tag{33}$$

Modeling Approaches

- Infinite horizontal plane
- Set of parallel pipes

Too small

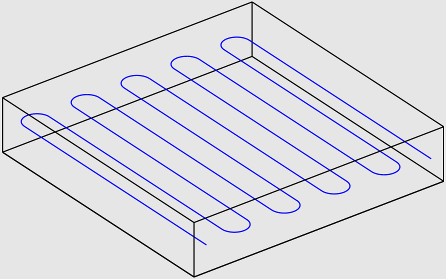
Source: www.bauweise.net

Large enough

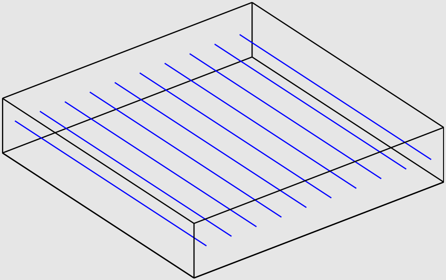
Source: Rehau AG & Co

Modeling as a Set of Parallel Pipes

Typical configuration



Simplified model



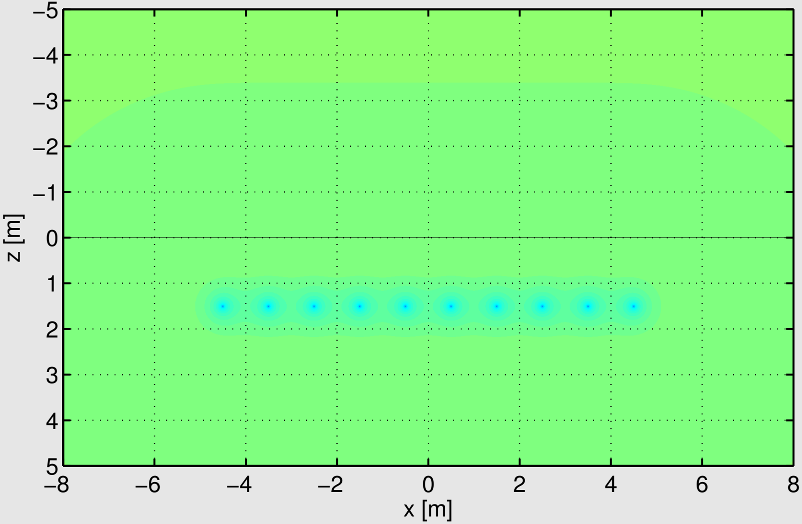
- Parallel horizontal pipes of infinite length

↓

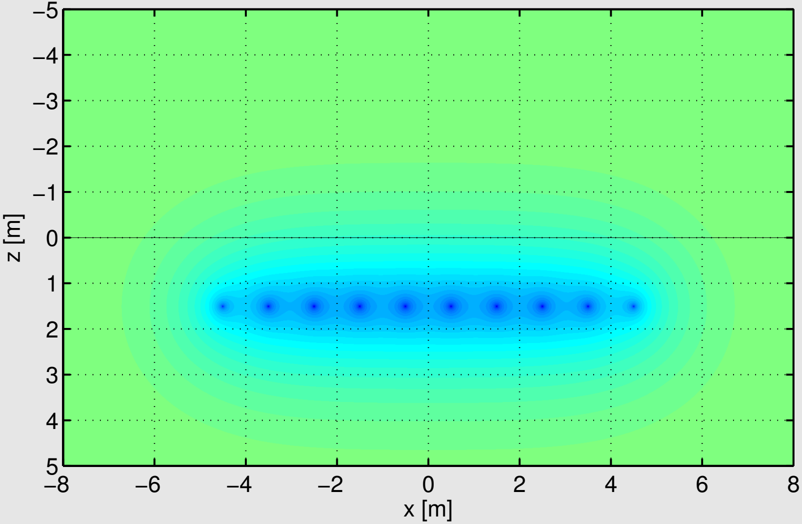
Theory of shallow downhole heat exchangers can be applied.

- Assume the same power per length for all pipes.

10 Parallel Pipes With 1 m Spacing in 1.5 m Depth After 3 Days

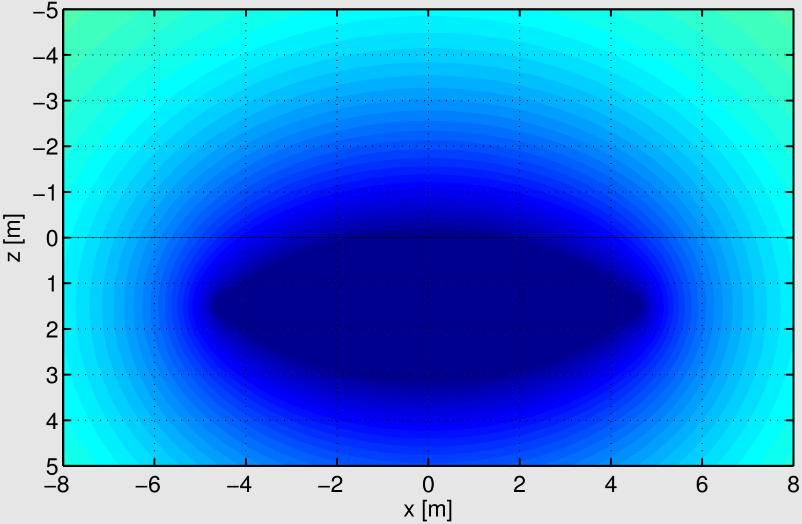


10 Parallel Pipes With 1 m Spacing in 1.5 m Depth After 1 Month



Ground Heat Collectors

10 Parallel Pipes With 1 m Spacing in 1.5 m Depth After 1 Year



Limitations

- Temperatures of the pipes are not the same.

↓

Same P_l for each pipe is not correct.

↓

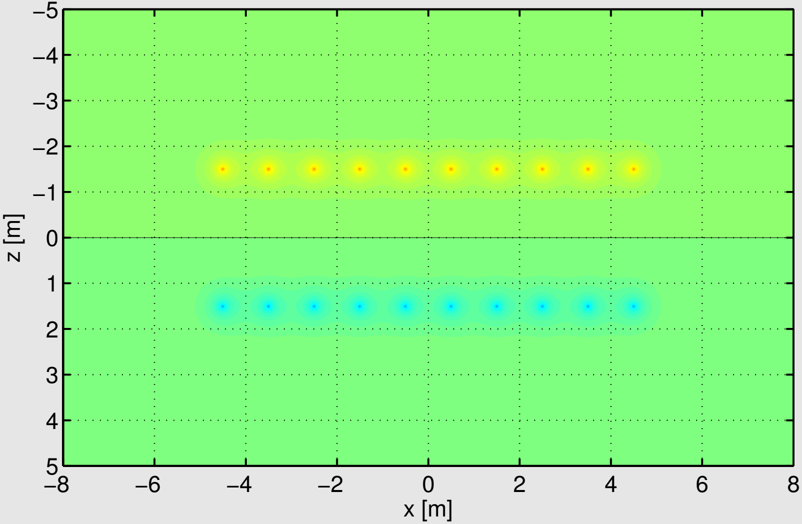
Not a big problem, use mean temperature.

- Surface temperature is affected by the heat collector as if there was no surface, while solar radiation and rapid heat transport in the atmosphere keep the temperature more constant in reality.

↓

Simplest model: keep surface temperature constant by introducing virtual pipes with $-P_l$ (supplying energy) above the surface.

10 Parallel Pipes With 1 m Spacing in 1.5 m Depth After 3 Days



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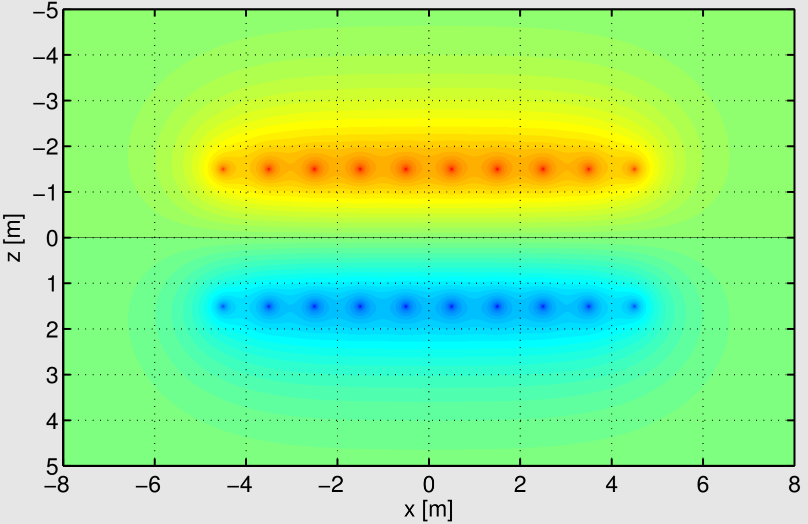

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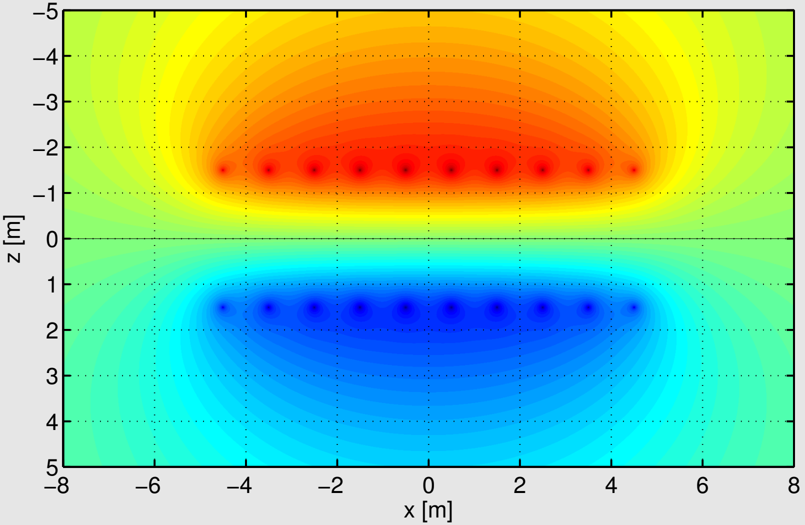


Computing the Average Temperature of Parallel Pipes

Assume n parallel pipes in a depth d below the surface and a horizontal spacing s.

Dist. r	Sign	Mean num.
r_a	$+$	1
s	$+$	$2\left(1-\frac{1}{n}\right)$
$2s$	$+$	$2\left(1-\frac{2}{n}\right)$
$3s$	$+$	$2\left(1-\frac{3}{n}\right)$
...	...	...

Distance r	Sign	Mean num.
$2d$	$-$	1
$\sqrt{s^2+(2d)^2}$	$-$	$2\left(1-\frac{1}{n}\right)$
$\sqrt{(2s)^2+(2d)^2}$	$-$	$2\left(1-\frac{2}{n}\right)$
$\sqrt{(3s)^2+(2d)^2}$	$-$	$2\left(1-\frac{3}{n}\right)$
...	...	...

Main Difference towards Shallow Heat Exchangers

Significant variation in temperature along the borehole

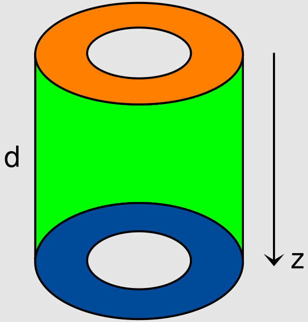
$$\Downarrow$$

Only coaxial heat exchangers can be used.

Main Field of Application

Direct (district) heating without heat pumps.

Energy Balance for the Fluid in the Heat Exchanger



Steady-state energy balance for the fluid:

$$\rho_f c_f Q T_f - \rho_f c_f Q T_f + P_l d = 0 \quad (34)$$

with

$$\rho_f = \text{density of the fluid}$$
$$c_f = \text{specific heat capacity of the fluid}$$
$$Q = \text{flow rate } [\tfrac{m^3}{s}]$$
$$T_f = \text{fluid temperature}$$

Energy Balance for the Fluid in the Heat Exchanger

$$\downarrow$$

$$\rho_f c_f Q \frac{T_f - T_f}{d} = P_l \tag{35}$$

$$\downarrow \text{ for } d \to 0$$

$$\rho_f c_f Q \frac{\partial T_f}{\partial z} = P_l \tag{36}$$

Simplest Situation: Perfectly Adjusted Flow Rate

Assume that $T_m(z)$ is the undisturbed temperature, and $\frac{T_m}{\partial z} = \text{const}$.

Adjust $Q(t)$ to $P_l(t)$ according to

$$Q = \frac{P_l}{\rho_f \, c_f \, \frac{\partial T_m}{\partial z}} \tag{37}$$

↓

$$\frac{\partial T_f}{\partial z} = \frac{\partial T_m}{\partial z} \tag{38}$$

↓

$$T_m - T_f = \text{const} \quad \text{and} \quad P_l = \text{const} \quad \text{for all } z \tag{39}$$

↓

Theory for shallow downhole heat exchangers can be applied.

General Case

- Solution with adjusted flow rate yields too low power in general; not useful for application.
- General case where with $P_l(z, t)$ cannot be solved analytically.

Possible solutions / approximations:

- Numerical simulation in 2D $\longrightarrow$ $T(r, t)$
- Sequence of coupled heat exchangers ... later
- Steady-state approximation

Steady-State Approximation for an Arbitrary Flow Rate

Assume that P_l depends on z, but not on t and

$$T_m(z) - T_f(z) \;=\; R_{\text{eff}}\, P_l(z) \tag{40}$$

with an effective thermal resistance R_{eff} ($> R$ of the heat exchanger) taking into account the seasonal variation in heating power (assignment 9).

Introduce asymptotic P_l^∞ according to Eq. 37:

$$P_l^\infty \;=\; \rho_f\, c_f\, Q\, \frac{\partial T_m}{\partial z} \tag{41}$$

Steady-State Solution for an Arbitrary Flow Rate

Combine Eqs. 36, 40 and 41

↓

$$P_l(z) - P_l^{\infty} = \rho_f c_f \, Q \, \frac{\partial T_f}{\partial z} - \rho_f c_f \, Q \, \frac{\partial T_m}{\partial z} \quad (42)$$

$$= -\rho_f c_f \, Q \, R_{\text{eff}} \, \frac{\partial}{\partial z} P_l(z) = -a \, \frac{\partial}{\partial z} P_l(z) \quad (43)$$

with $a = R_{\text{eff}} \, \rho_f \, c_f \, Q$

↓

$$\frac{\partial}{\partial z} \left(P_l(z) - P_l^{\infty} \right) = -\frac{1}{a} \left(P_l(z) - P_l^{\infty} \right) \quad (44)$$

if $P_l^{\infty} = \text{const}$ ($\frac{\partial T_m}{\partial z} = \text{const}$)

Steady-State Solution for an Arbitrary Flow Rate

Solution:

$$P_l(z) - P_l^{\infty} \;=\; \left(P_l(0) - P_l^{\infty}\right) e^{-\frac{z}{a}} \tag{45}$$

$$\Downarrow$$

- $P_l(z) \to P_l^{\infty}$ for $z \to \infty$
- a is the depth where the difference between $P_l(z)$ and P_l^{∞} has decreased to $\frac{1}{e}$ of the difference at the surface.

Steady-State Solution for an Arbitrary Flow Rate

Solution in terms of temperatures:

$$T_m(z) - T_f(z) - a\frac{\partial T_m}{\partial z} = \left(T_m(0) - T_f(0) - a\frac{\partial T_m}{\partial z} \right) e^{-\frac{z}{a}} \quad (46)$$

$$\Downarrow$$

$$T_f(z) = T_m(z) - a\frac{\partial T_m}{\partial z} + \left(T_f(0) - \left(T_m(0) - a\frac{\partial T_m}{\partial z} \right) \right) e^{-\frac{z}{a}} \quad (47)$$

Time-Dependent Solution Using Coupled Heat Exchangers

Consider fluid temperature $T_f(z,t)$ and power per length $P_l(z,t)$ depending on depth and time and assume that the flow rate $Q(t)$ is given.

Two conditions must be met:

- Change of fluid temperature with depth according to Eq. 36
- Fluid temperature as a function of time:

$$T_f(z,t) \;=\; T_m(z) + T(z,t) \tag{48}$$

with $T(z,t)$ depending on $P_l(z,t)$ in the past according to Eq. 21.

Time-Dependent Solution Using Coupled Heat Exchangers

Insert

$$\frac{\partial}{\partial z} T_f(z,t) \;=\; \frac{T_f(z,t) - T_f(z-\delta z,t)}{\delta z} \tag{49}$$

into Eq. 36 and $P_l(z,t)$ for $P_{l,n+1}$ in Eq. 21.

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$P_l(z,t)$ as a function of $P_l(z,t)$ in the past and $T_f(z-\delta z,t)$

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$T_f(z,t)$ from Eq. 21