Introduction

Basic Idea

Measure the natural magnetic field of the Earth and its amplification or attenuation by inhomogeneities in the subsurface.

Main Fields of Application

- Exploration of regional and local geological structures
- Prospection and exploration of natural resources, in particular ores; first published study on applied geophysics (1879)
- Detection of residual waste and other "non-natural" objects in the subsurface

Basics

The Maxwell Equations

Electric and magnetic fields and their interaction are described by the Maxwell equations:

- Electric charges are sources of the electric field.
- There is nothing like a charge for the magnetic field; magnetic fields are solenoidal vector fields (closed field lines, no sources).
- Temporal changes in a magnetic field induce a solenoidal electric field.
- Electric currents (i. e, moving charges) and temporal changes in an electric field induce a magnetic field.

Basics

The Magnetic Flux Density

The magnetic flux density $\vec{B}$ is defined by the (Lorentz) force $\vec{F}$ acting on a moving charged particle or on a current:

$$\vec{F} \;=\; q\,\vec{v} \times \vec{B} \;=\; I\,\vec{s} \times \vec{B}$$

where I is the current and $\vec{s}$ a vector from the origin of the conductor to its end.

- SI unit: $[B] = 1\,\mathrm{T}$ (Tesla) $= 1\,\frac{\mathrm{N}}{\mathrm{Am}} = 1\,\frac{\mathrm{Vs}}{\mathrm{m}^2}$
- Sometimes called magnetic induction or simply magnetic field.

Basics

The Magnetic Flux Density

- 1 T is a rather strong magnetic field: A force of 1 N would act on a wire of 1 m length exposed to a current of 1 A.

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  A horizontal copper wire of 1 mm^2 cross section would levitate at $\mathrm{I} = 0.1\,\mathrm{A}$.
- Earth's magnetic field amounts to about 30–60 μT at the surface.
- Other units:
  $1\,\Gamma$ (Gauß, Gs) $= 10^{-4}\,\mathrm{T}$
  $1\,\gamma = 1\,\mathrm{nT} = 10^{-9}\,\mathrm{T}$

Basics

The Magnetic Field

The magnetic field $\vec{H}$ (also called field strength) describes the generation of magnetic fields by electric currents.

- An infinite wire with a current I generates a magnetic field

  $$H = \frac{I}{2\pi r}$$

  at a distance r.
- Unit: $[H] = 1\,\frac{\mathrm{A}}{\mathrm{m}}$
- In vacuum, $\vec{B}$ and $\vec{H}$ are directly related by

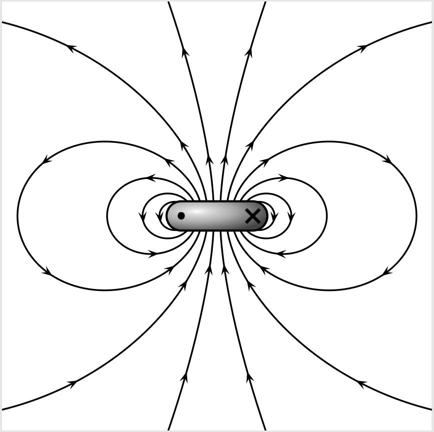
  $$\vec{B} = \mu_0 \vec{H}$$

  with the magnetic constant (also called vacuum permeability) $\mu_0 = 4\pi \times 10^{-7}\,\frac{\mathrm{N}}{\mathrm{A}^2}$.

Basics

Magnetic Dipoles

A small current loop or a rotating charged particle generates a dipole field.



Source: Wikipedia

Basics

Magnetic Fields in Matter

Magnetization: Orbital angular momentums and spins of electrons (in principle of protons, too) generate internal magnetic moments that may be aligned in an external magnetic field, so that

$$\vec{B} \;=\; \mu_0 \left(\vec{H} + \vec{M}\right)$$

$\vec{M}$ is the total magnetic moment per volume and is called magnetization or magnetic polarization.

Magnetic susceptibility: In the simplest case

$$\vec{M} \;=\; \chi \vec{H},$$

where χ is the magnetic susceptibility of the medium.

- Media with $\chi > 0$ amplify external magnetic fields.
- Media with $\chi < 0$ attenuate external magnetic fields.

Basics

Magnetic Fields in Matter

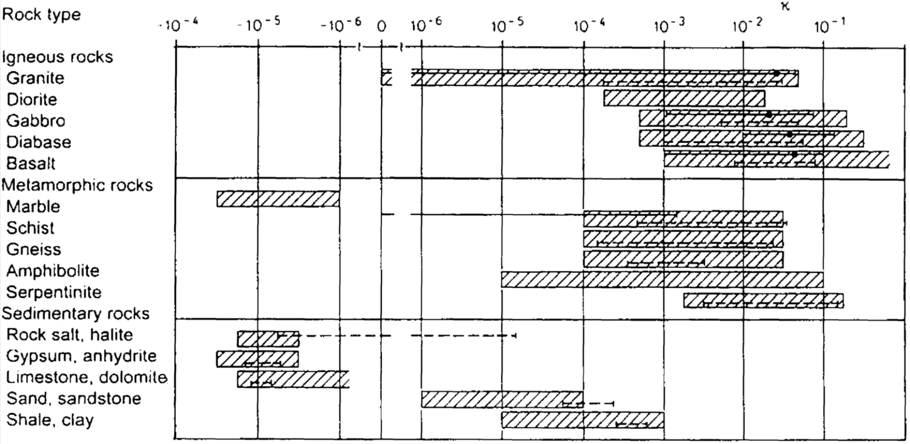
Magnetic permeability:

$$\vec{B} \;=\; \mu_0 \left(\vec{H} + \vec{M}\right) \;=\; \mu_0 \,(1 + \chi)\, \vec{H} \;=\; \mu_0\, \mu_r\, \vec{H} \;=\; \mu\, \vec{H}$$

with

$$\begin{aligned} \mu_r &= 1 + \chi = \text{relative magnetic permeability of the material} \\ \mu &= \mu_0\, \mu_r = \text{absolute magnetic permeability of the material} \end{aligned}$$

Magnetic Susceptibility of Rocks



Basics

Magnetic Fields in CGS Units

In the CGS (centimeter-gram-second) system, B and H have the same unit, and there is no μ_0.

⇓

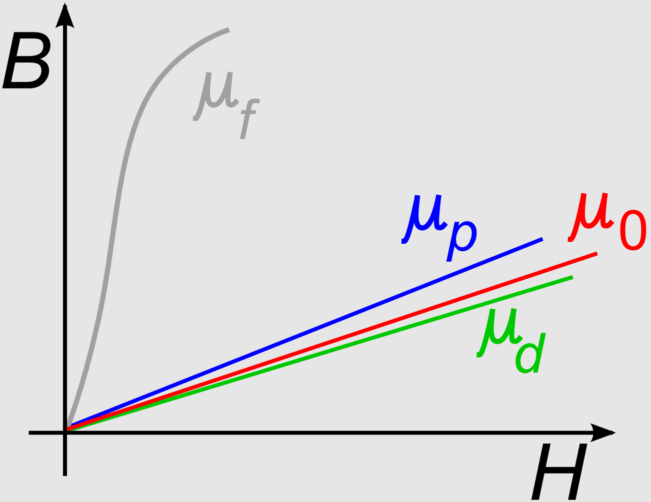
$$\vec{B} \;=\; \vec{H} + 4\pi\,\vec{M} \;=\; (1 + 4\pi\,\chi)\,\vec{H}.$$

⇓

χ values in CGS units are by a factor 4π smaller than in SI units, although nondimensional.

Basics

Types of Magnetic Behavior



Source: Wikipedia

Basics

Diamagnetism

- No alignment of internal dipoles in an external magnetic field.
- Precession of electrons generates a small magnetic field opposite to the external magnetic field:

$$\chi < 0, \quad \mu_r < 1, \quad \mu < \mu_0$$

- Water: $\chi = -9 \times 10^{-6}$; quartz: $\chi = -1.6 \times 10^{-5}$ (SI units)
- Weakest type of magnetization
- Is in principle present in all materials, but often shadowed by stronger effects.
- Linear and reversible; the magnetization is proportional to the external magnetic field and vanishes if the external field is removed.
- Force towards smaller magnetic field strength.

Basics

Paramagnetism

- Alignment of orbital angular momentums and spins of unpaired electrons in an external magnetic field.
- Generates a small magnetic field in direction of the external field:

$$\chi > 0, \quad \mu_r > 1, \quad \mu > \mu_0$$

- Illite: $\chi = 4 \times 10^{-4}$; siderite ($FeCO_3$): $\chi = 4 \times 10^{-3}$ (SI units)
- Linear and reversible
- Force towards higher magnetic field strength.

Basics

Ferromagnetism

- Interaction of the magnetic moments of different electrons

$$\Downarrow$$

Parallel alignment within spatial domains

- Generates strong magnetic field in direction of an external magnetic field that may be much stronger than the external field:

$$\chi \gg 1, \quad \mu_r \gg 1, \quad \mu \gg \mu_0$$

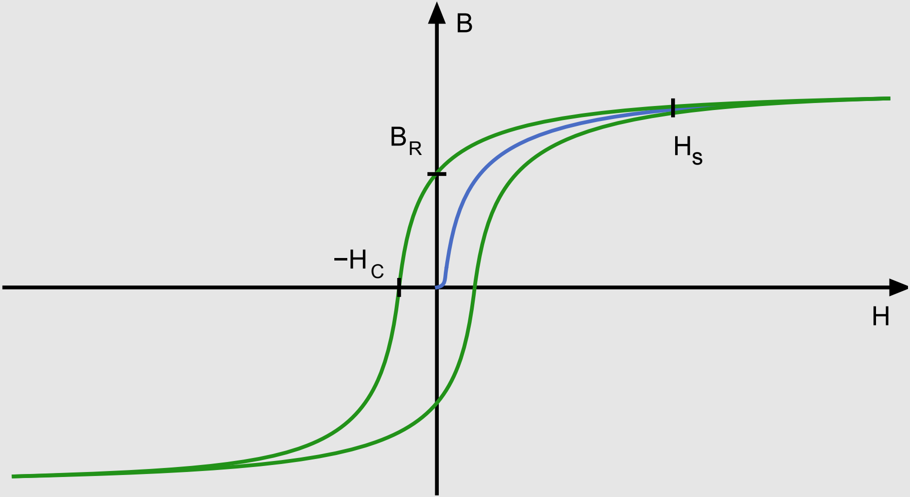
- Iron ($\chi \approx 1000$ at 20°C), nickel, and cobalt are the only ferromagnetic elements at standard temperature.
- Strongly nonlinear with saturation, hysteresis and residual magnetism

$$\Downarrow$$

χ, μ_r, and μ are not constant.

Basics

Typical Hysteresis Loop of a Ferromagnetic Material



Source: Wikipedia

Basics

Ferrimagnetism

- Opposite alignment of neighbored magnetic moments, but no complete compensation
- On large scales similar to ferromagnetism, but with a much lower saturation
- Magnetite (Fe_3O_4, $\chi = 2.5$), titaniferous magnetite, pyrrhotite (FeS ... $Fe_{11}S_{12}$), and hematite (Fe_2O_3, $\chi = 3 \times 10^{-3}$) are the most widespread ferrimagnetic minerals.

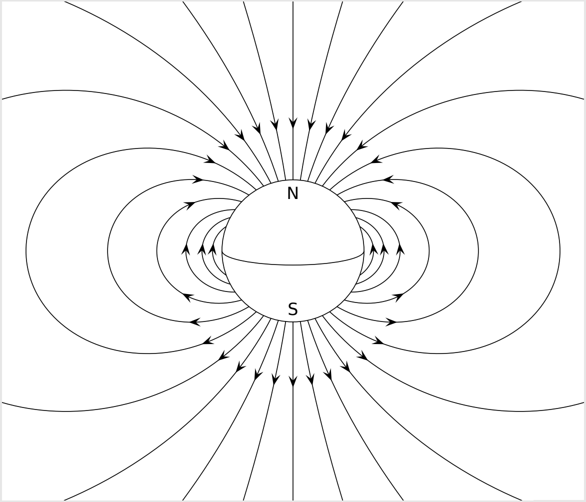
The Magnetic Field of the Earth

The Most Important Properties

- Amounts to about 30–60 μT (0.3–0.6 Γ) at the surface.
- Mainly a dipole field outside the Earth.
- At the surface the contribution of the dipole component is about 90 %; less in the Earth's interior.
- Mainly generated by electric currents in the Earth's core.
- Ionosphere and magnetosphere contribute about 2 %.
- Magnetic matter in the upper crust (down to about 20 km) also contributes about 2 % in total, but with an inhomogeneous spatial distribution.
- The Earth's magnetic field shows significant temporal variations at all scales.
- Dipole axis (magnetic poles) neither intersects the geographic poles nor the center of the Earth.

The Magnetic Field of the Earth

The Earth's Dipole Field



Source: Wikipedia

The Magnetic Field of the Earth

The Earth's Dipole Field

Flux density

$$\vec{B} \;=\; \frac{\mu_0}{4\pi}\,\frac{3(\vec{m}\cdot\vec{x})\vec{x} - |\vec{x}|^2\vec{m}}{|\vec{x}|^5}$$

with $m \approx 7.75 \times 10^{22}\,\mathrm{Am}^2$ (2010)

Total intensity

$$F \;=\; |\vec{B}| \;=\; \frac{\mu_0}{4\pi}\,m\,\frac{\sqrt{1+3\sin^2\phi}}{r^3}$$

where ϕ is the magnetic latitude.

Horizontal component

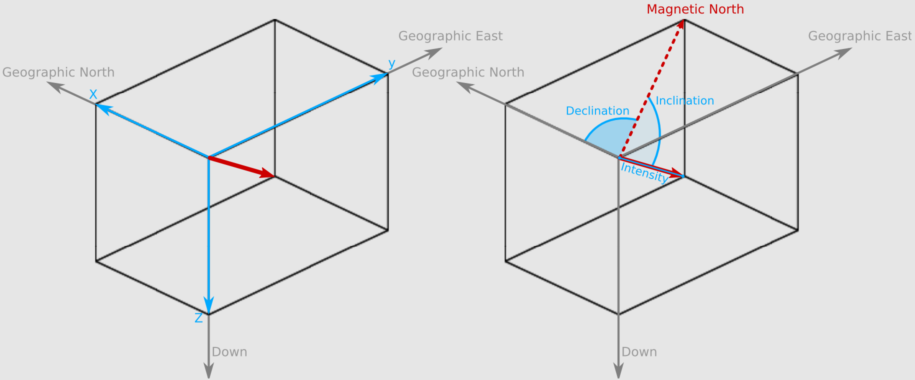
$$H \;=\; \frac{\mu_0}{4\pi}\,m\,\frac{\cos\phi}{r^3}$$

Vertical component

$$Z \;=\; \frac{\mu_0}{4\pi}\,m\,\frac{2\sin\phi}{r^3}$$

The Magnetic Field of the Earth

Components of the Earth’s Magnetic Field



Source: Wikipedia

The Magnetic Field of the Earth

Components of the Earth's Magnetic Field

Declination $D =$ deviation (angle) between magnetic and geographic north direction (clockwise).

Inclination (magnetic dip) $I =$ dip angle of the magnetic field with respect to the Earth's surface. Important for the analysis of geomagnetic measurements. For a dipole field:

$$\tan I \;=\; 2 \tan \phi$$

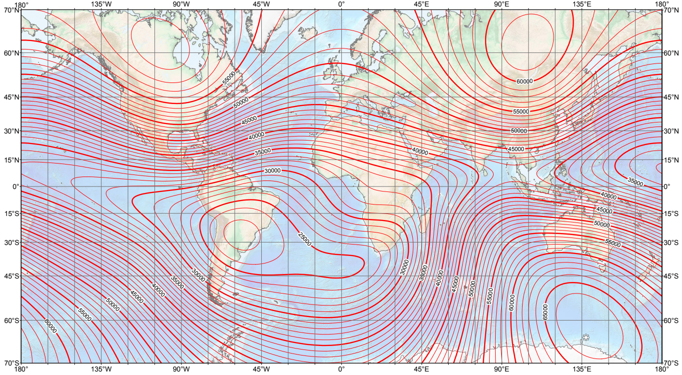
The International Geomagnetic Reference Field

The International Geomagnetic Reference Field (IGRF) provides reference values for the magnetic field including the long-term temporal variation. Data are available from an online tool at http://www.ngdc.noaa.gov/geomag-web/#igrfwmm

The Magnetic Field of the Earth

Total Intensity

US/UK World Magnetic Model -- Epoch 2010.0
Main Field Total Intensity (F)



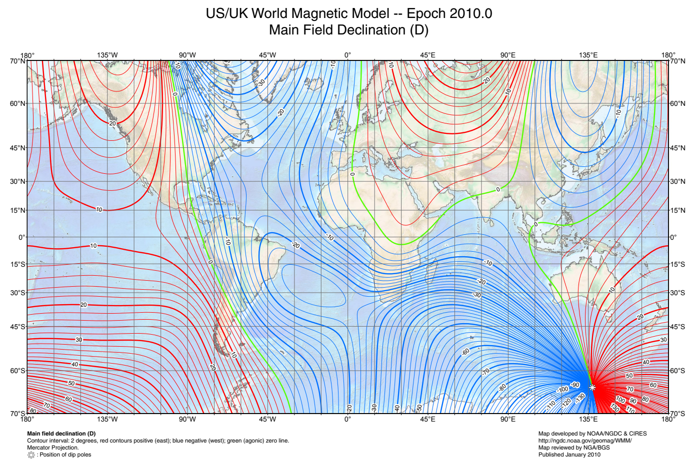
Main Field Total Intensity (F)
Contour interval: 1000 nT.
Mercator Projection.
✺ : Position of dip poles

Map developed by NOAA/NGDC & CIRES
http://ngdc.noaa.gov/geomag/WMM/
Map reviewed by NGA/BGS
Published January 2010

Source: National Geophysical Data Center at NOAA

The Magnetic Field of the Earth

Declination



US/UK World Magnetic Model -- Epoch 2010.0
Main Field Declination (D)

Main field declination (D)
Contour interval: 2 degrees, red contours positive (east); blue negative (west); green (agonic) zero line.
Mercator Projection.
✳ : Position of dip poles

Map developed by NOAA/NGDC & CIRES
http://ngdc.noaa.gov/geomag/WMM/
Map reviewed by NGA/BGS
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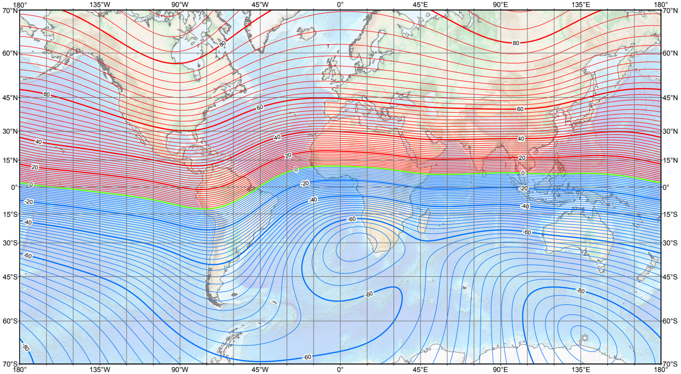
Source: National Geophysical Data Center at NOAA

The Magnetic Field of the Earth

Inclination



US/UK World Magnetic Model -- Epoch 2010.0
Main Field Inclination (I)

Main field inclination (I)
Contour interval: 2 degrees, red contours positive (down); blue negative (up); green zero line.
Mercator Projection.
✲ : Position of dip poles

Map developed by NOAA/NGDC & CIRES
http://ngdc.noaa.gov/geomag/WMM/
Map reviewed by NGA/BGS
Published January 2010

Source: National Geophysical Data Center at NOAA

The Magnetic Field of the Earth

Long-Term Variations

Polar motion (secular variation) can be directly measured on the scale of years.

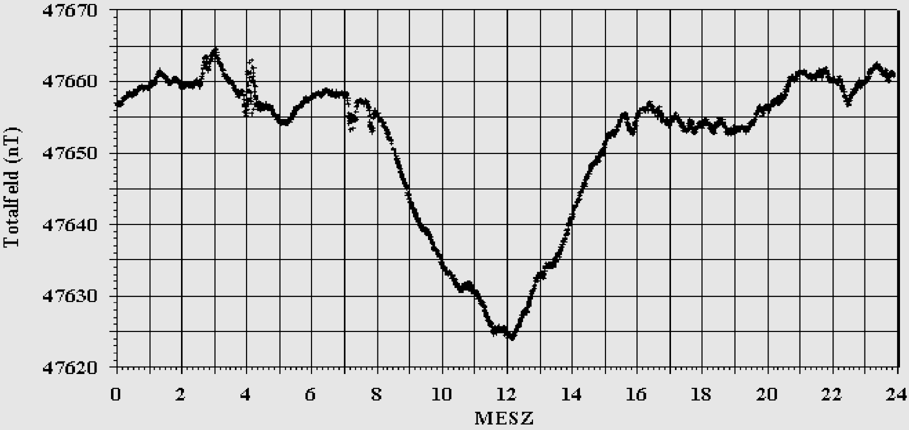
Field reversals occur at intervals of about 0.1 Ma to 50 Ma.

Variations in intensity up to about $10\,\mu$T, even stronger during field reversals.

The Magnetic Field of the Earth

Short-Term Variations

Daily variations of some 10 nT due to solar wind

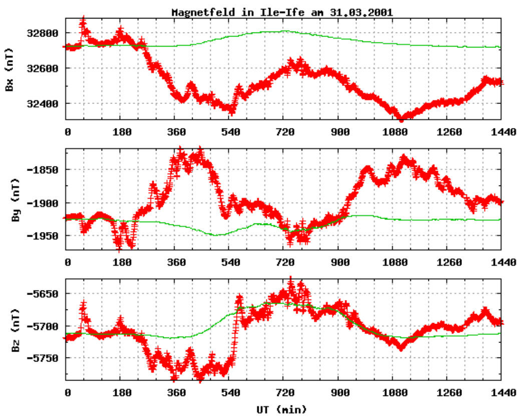


Source: Teaching material R. Scholger

The Magnetic Field of the Earth

Short-Term Variations

Magnetic storms: some 100 nT



Source: Wikipedia

Magnetic Field Measurements

Fluxgate Magnetometer

Principle:

- An easily magnetizable core is driven to saturation by a strong artificial, alternating magnetic field.
- Resulting magnetic field is recorded by two coils with opposite orientation.
- External magnetic field causes an asymmetric signal; asymmetry is proportional to the respective component of the external field.

Measured property: One component of the field; manual orientation of the instrument. Alternatively instruments with three sensors.

Accuracy: Some nT

Type of measurement: Relative measurement; instrument must be calibrated.

Sampling frequency: Continuous recording

Magnetic Field Measurements

Proton Precession Magnetometer

Principle:

- Spins of protons in a liquid containing much hydrogen are aligned in a strong artificial magnetic field.
- After removing the magnetic field, the spins perform a precession in the magnetic field of the Earth.
- The frequency of the precession (Larmor frequency) only depends on the total intensity F and on atomic constants:

$$f = \frac{\gamma}{2\pi} F \quad \text{with} \quad \gamma = 0.2675222005(63)\,\frac{\mathrm{Hz}}{\mathrm{nT}}$$

Measured property: Total intensity

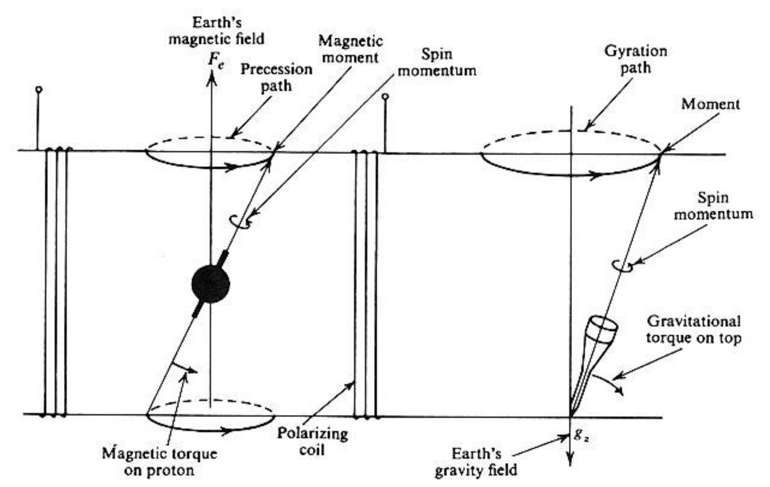
Accuracy: Up to 1 nT

Type of measurement: Absolute measurement; no calibration

Sampling frequency: One measurement per 2–3 s

Magnetic Field Measurements

Proton Precession Magnetometer



Source: Telford et al., Applied Geophysics

Interpretation of Magnetic Field Measurements

Measurement of the Total Intensity

The additional flux density $\delta\vec{B}$ of magnetized bodies in the subsurface is much smaller than the original flux density $\vec{B}_0$.

Measured total intensity:

$$F \;=\; |\vec{B}| \;=\; |\vec{B}_0 + \delta\vec{B}| \;\approx\; |\vec{B}_0| + \frac{\vec{B}_0 \cdot \delta\vec{B}}{|\vec{B}_0|}$$

$$\Downarrow$$

$$\delta F \;=\; |\vec{B}| - |\vec{B}_0| \;\approx\; \delta B_{||}$$

where $\delta B_{||}$ is the component of $\delta\vec{B}$ parallel to $\vec{B}_0$.

$$\Downarrow$$

The anomaly in the total intensity δF is not $|\delta\vec{B}|$, but the component of $\delta\vec{B}$ parallel to $\vec{B}_0$.

Interpretation of Magnetic Field Measurements

The Magnetic Anomaly of a Uniformly Magnetized Sphere

Magnetization (magnetic moment per volume) inside the sphere:

$$\vec{M} \;=\; \chi\,\vec{H}_0 \;=\; \frac{\chi}{\mu_0}\,\vec{B}_0$$

Uniformly magnetized sphere with a volume V generates the same field (outside the sphere) as a point-like dipole with

$$\vec{m} \;=\; V\,\vec{M}.$$

$$\Downarrow$$

$$\delta\vec{B} \;=\; \frac{\mu_0}{4\pi}\,\frac{3(\vec{m}\cdot\vec{x})\vec{x} - |\vec{x}|^2\vec{m}}{|\vec{x}|^5} \;=\; \frac{\chi V}{4\pi}\,\frac{3(\vec{B}_0\cdot\vec{x})\vec{x} - |\vec{x}|^2\vec{B}_0}{|\vec{x}|^5}$$

if the sphere is centered at the origin.

Interpretation of Magnetic Field Measurements

The Magnetic Anomaly of a Uniformly Magnetized Sphere

$$\Downarrow$$

$$\delta F \;=\; F_0\,\frac{\chi V}{4\pi}\,\frac{3\cos^2\alpha - 1}{|\vec{x}|^3}$$

where α is the angle between $\vec{B}_0$ and $\vec{x}$.

$$\Downarrow$$

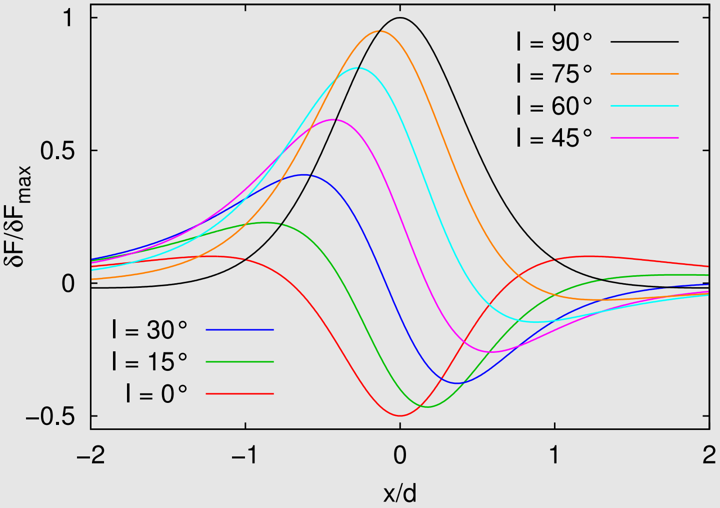
Positive anomaly for $\chi > 0$ if $\alpha < 55^\circ$ or $\alpha > 125^\circ$, negative anomaly else.

Sphere in a depth d, x in magnetic dip direction:

$$\delta F \;=\; F_0\,\frac{\chi V}{4\pi}\,\frac{3\left(x\cos I - d\sin I\right)^2 - \left(x^2+y^2+d^2\right)}{(x^2+y^2+d^2)^{\frac{5}{2}}}$$

Interpretation of Magnetic Field Measurements

The Magnetic Anomaly of a Uniformly Magnetized Sphere



Interpretation of Magnetic Field Measurements

The Magnetic Anomaly of a Uniformly Magnetized Sphere

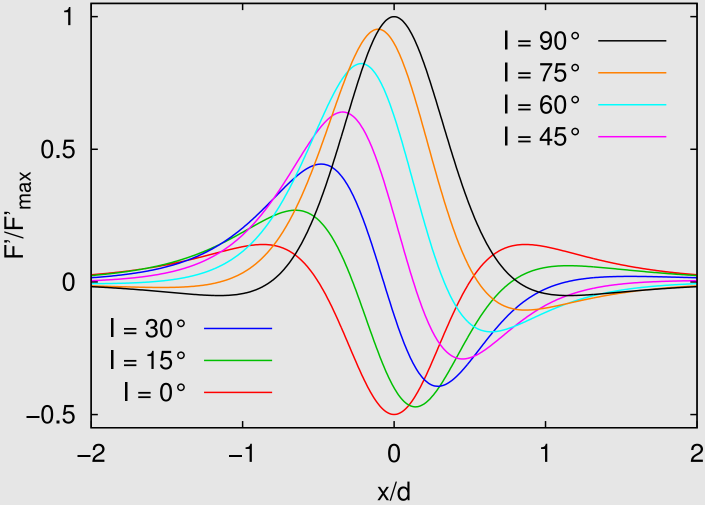
- The maximum anomaly occurs at the magnetic poles ($I = 90^\circ$) vertically above the magnetized sphere ($x = y = 0$) and is

$$\delta F_{\max} \;=\; F_0 \, \frac{\chi V}{2\pi \, d^3}$$

- In case of positive magnetization, the maximum of the anomaly is shifted towards the magnetic equator, but this shift is smaller than one would guess from the inclination.
- The depth of the magnetized body roughly corresponds to the full width at half maximum of the profile (for large inclination) or the distance between minimum and maximum anomaly (for small inclination).

Interpretation of Magnetic Field Measurements

Gradiometer Measurements above a Uniformly Magnetized Sphere



Interpretation of Magnetic Field Measurements

Gradiometer Measurements above a Uniformly Magnetized Sphere

- The profile of $\delta F'$ is qualitatively similar to that of δF, but is narrower.
- The maximum anomaly occurs at the magnetic poles ($I = 90^\circ$) vertically above the magnetized sphere ($x = y = 0$) and is

$$\delta F'_{\mathrm{max}} \; = \; F_0 \, \frac{3\chi V}{2\pi \, d^4}$$

- The depth of the magnetized body roughly corresponds to 1.3 times the full width at half maximum (for large inclination) or 1.3 times the distance between minimum and maximum anomaly (for small inclination).