Near-surface Geophysics

Seismology and Seismics

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Seismology

- Comprises all about earthquakes and the propagation of seismic waves in the Earth.
- One of the main fields of solid-earth geophysics.
- Has provided the majority of our knowledge on Earth's interior.

Seismics

- Exploration of the deep and shallow subsurface with the help of artificial seismic waves.
- The perhaps most important field of applied geophysics.

The First "Seismometer" (132 a.D.)

History of Seismology

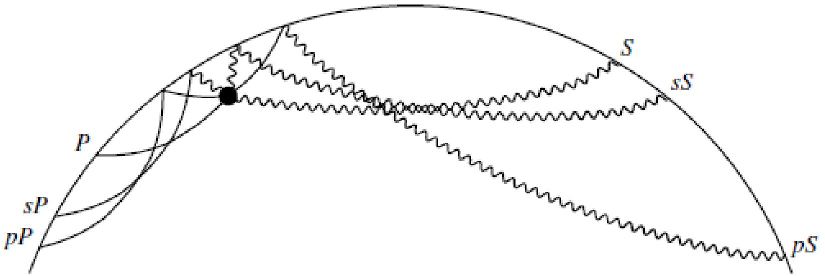
1660	basic law of elasticity	R. Hooke
1821–22	differential equations of elasticity	C. Navier A. L. Cauchy
1830	theory of two fundamental types of elastic waves (P- and S-wave)	S. D. Poisson
1875	First "serious" seismometer	F. Gecchi
1887	theory of the first type of surface waves	J. W. Strutt (3. Lord Rayleigh)
1889	first recording of a distant earthquake	
1892	first compact seismometer, used at about 40 stations	J. Milne
1894	statistics of aftershocks	F. Omori
1903	12 degree scale for the intensity of earthquakes based on the damage	G. Mercalli

History of Seismology

1906–1913	detection of the liquid core of the earth and determination of its size	R. D. Oldham, B. Gutenberg
1909	detection of the crust-mantle discontinuity	A. Mohorovičić
1911	theory of a second type of surface waves	A. E. H. Love
1935	local magnitude as an "objective" measure of earthquake intensity	C. F. Richter
1936	detection of the inner, solid core	I. Lehmann
1954	frequency-magnitude relation of earthquakes	B. Gutenberg, C. F. Richter
1975	first successful short-term prediction of a strong earthquake	
1977	moment magnitude as a measure of earthquake source strength	H. Kanamori

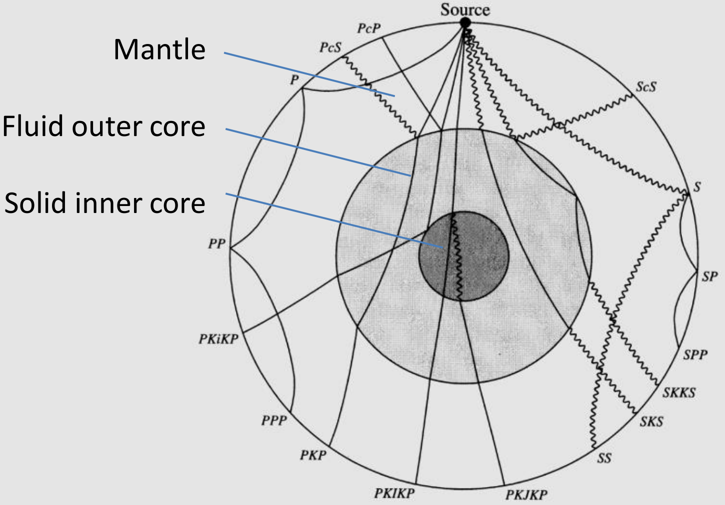
Global Wave Propagation in the Earth´s Interior

- Capital letters indicate the rays of depth phases, going downwards from the source, i.e. vertical angle lower than 90° (P,S)
- Lower case letters indicate the rays of depth phases, going upwards from the source, i.e. vertical angle greater than 90°. (p,s)
- Hypocenter near reflexions at the surface (pP,pS,sP,sS)
- Conversion waves travelling upwards (Ps,Sp)



Source: Shearer 2010, Introduction to Seismology

Global Wave Propagation in the Earth's Interior



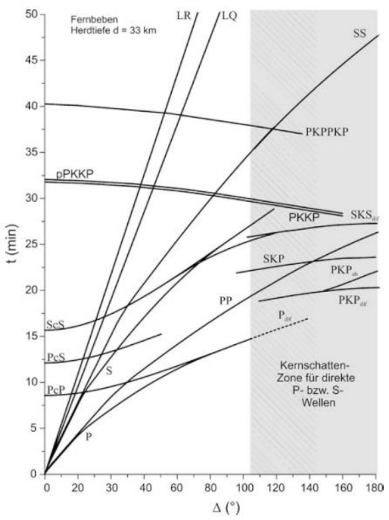
Source: Shearer, Introduction to Seismology

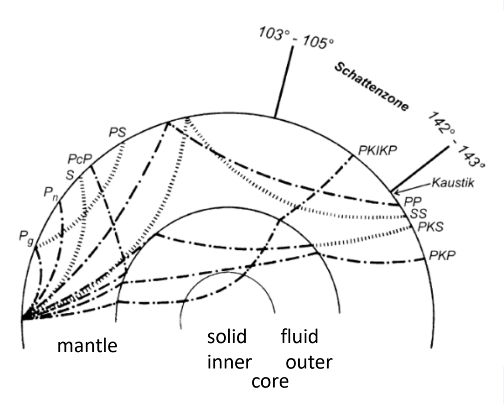
Global Wave Propagation in the Earth´s Interior

- P: P wave in the crust/mantle
- K: P wave in the outer core
- I: P wave in the inner core
- S: S wave in the crust/mantle
- J: S wave in the inner core
- c: reflection off the core–mantle boundary (CMB)
- i: reflection off the inner-core boundary (ICB)

- Combination of different letters indicate the seismic phases along the ray path travelling through the Earth.
- Repeating letters indicate reflection at the surface (PP, SS)

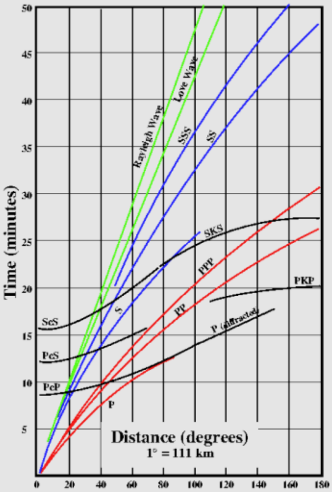
Localization of Earthquakes – Travel Time curves





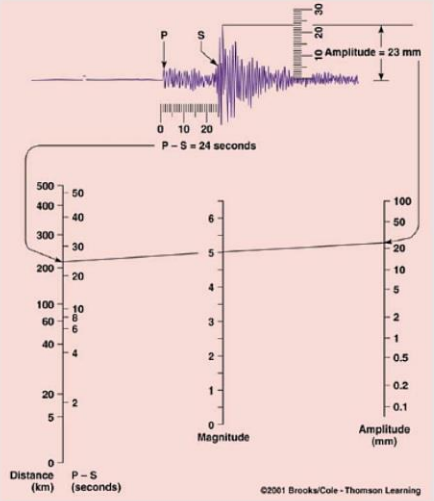
Source: Clauser, Einführung in die Geophysik, figs. 3.15 and 3.16

Localization of Earthquakes – Travel Time Curves



Source: Southern Arizona Seismic Observatory

Localization of Earthquakes



Seismic Waves

Waves in Elastic Media

- Propagating elastic deformation
- The same as sound waves in solids
- More complicated than sound waves in liquids and gases

Theory of Wave Propagation

Navier-Cauchy equations for the displacement $\vec{u}(\vec{x}, t)$ in an elastic medium (http://hergarten.at/extra/continuummechanics.pdf):

$$\rho \frac{\partial^2}{\partial t^2} \vec{u} \;=\; \nabla \left(\lambda \, \mathrm{div}(\vec{u}) \right) + \mathrm{div} \left(\mu \left(\nabla \vec{u} + (\nabla \vec{u})^T \right) \right)$$

with

$$\begin{aligned} \rho(\vec{x}) \;&=\; \text{density} \\ \lambda(\vec{x}),\; \mu(\vec{x}) \;&=\; \text{Lamé's parameters of the medium} \end{aligned}$$

No general analytical solution for an inhomogeneous medium

If λ and μ are constant:

$$\rho \frac{\partial^2}{\partial t^2} \vec{u} \;=\; (\lambda + \mu) \nabla \mathrm{div}(\vec{u}) + \mu \, \Delta \vec{u}$$

Navier-Cauchy equations in an elastic medium

- Linear elasticity is the mathematical study of how solid objects deform and become internally stressed due to prescribed loading conditions.

- The fundamental assumptions of linear elasticity are: infinitesimal strains or small deformations and linear relationships between the components of stress and strain.

- In continuum mechanics, the Lamé parameters are two material-dependent quantities denoted by λ and μ that arise in strain-stress relationships.

  λ: Lamé's first parameter

  μ: shear modulus (ratio of shear stress to the shear strain)

Elastic modulus

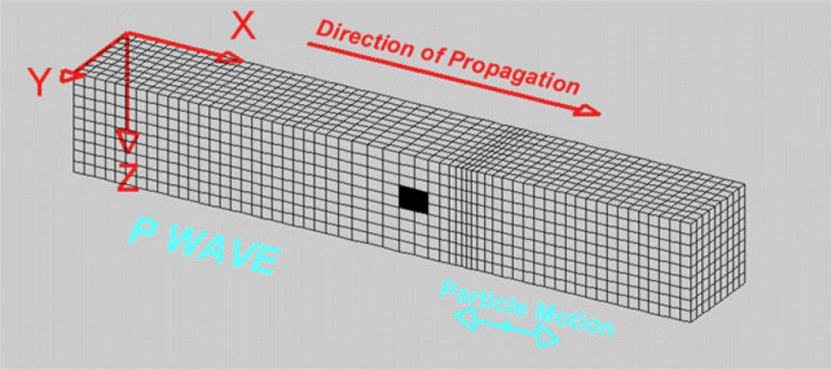
- a number that measures an object or substance's resistance to being deformed elastically
- The elastic modulus of an object is defined as the slope of its stress–strain curve in the elastic deformation region: $\lambda = \frac{stress}{strain}$

- *stress* is the force causing the deformation divided by the area to which the force is applied
- *strain* is the ratio of the change in some length parameter caused by the deformation to the original value of the length parameter

Basic Types of Body Waves

Two types of independent plane waves in an infinite, homogeneous elastic medium:

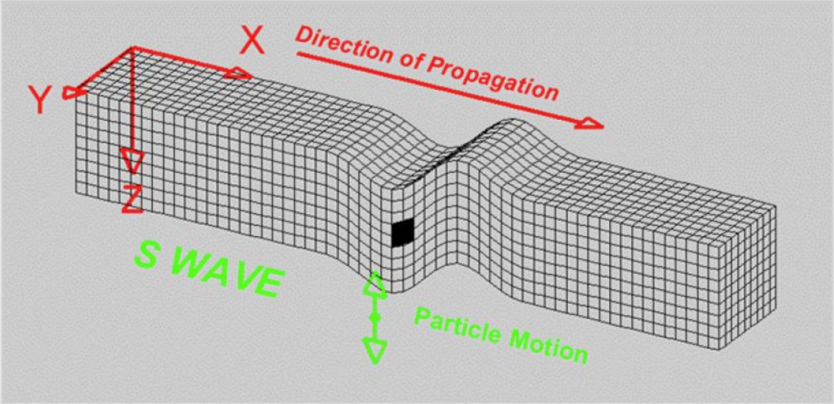
- Compressional wave (longitudinal wave, primary wave)



Source: L. Braile, Purdue University

Basic Types of Body Waves

- Shear wave (transverse wave, secondary wave)



Source: L. Braile, Purdue University

Seismic Waves

Comparison with Sound Waves in Liquids and Gases

The compressional wave is similar to sound waves in liquids and gases, while the shear wave has no counterpart in liquids and gases.

Seismic Velocities

Medium	Compressional wave [$\frac{km}{s}$]	Shear wave [$\frac{km}{s}$]
air	0.34	–
water	1.45	–
wood	about 3	about 1.8
Earth	5.8–13.7	3.2–7.3

Seismic Waves

Seismic Velocities

Velocity v_p of the compressional wave is always higher than the velocity v_s of the shear wave.

$$\Downarrow$$

Compressional wave always arrives prior to the shear wave.

$$\Downarrow$$

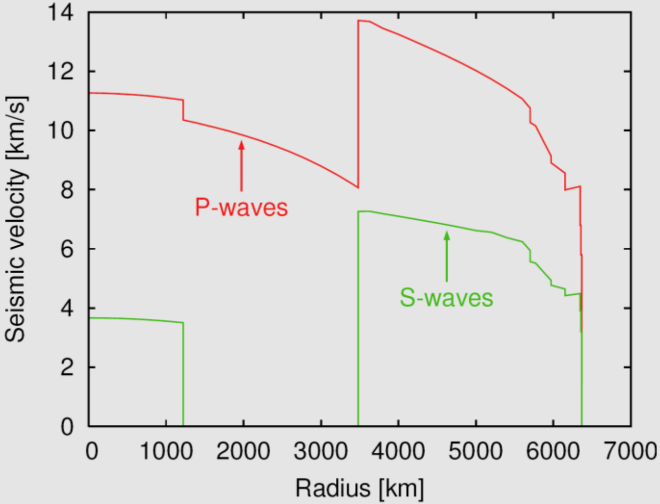
compressional wave = primary wave (P-wave)
shear wave = secondary wave (S-wave)

Rules of Thumb

Solid rock: $v_s \approx 0.5\, v_p - 0.6\, v_p$

Soil / unconsolidated rock: $v_s \approx 0.4\, v_p$

Seismic Velocities according to the Preliminary Reference Earth Model



Seismic Velocities and Elastic Properties

$\vec{s}$ is the "slowness vector" of the wave ($|\vec{s}| = \frac{1}{v}$), so that

$$
\begin{aligned}
v_p &= \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{M}{\rho}} = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}} \\
v_s &= \sqrt{\frac{\mu}{\rho}}
\end{aligned}
$$

with

$$
\begin{aligned}
M &= \text{linear elastic modulus without lateral contraction} \\
K &= \text{bulk modulus} \\
\mu &= \text{shear modulus}
\end{aligned}
$$

v_p und v_s are determined by the density and on the elastic modulus of the respective type of deformation.

Seismic Velocities and Elastic Properties

- Slowness (s) is a quantity introduced in Seismology which is the reciprocal of velocity. Thus travel time of a wave is the distance that the wave travels times the slowness of the medium.

Seismic Waves

Typical P-wave Velocities in the Shallow Subsurface

Medium	v_p $[\frac{km}{s}]$
weathering zone	0.1–0.5
dry sand	0.3–0.6
water-saturated sand	1.3–1.8
sandstone	1.8–4
pit coal	1.6–1.9

Medium	v_p $[\frac{km}{s}]$
clay	1.2–2.8
claystone	2.2–4.2
limestone	3–6
halite	4.5–6.5
granite	5–6.5

Propagation in Inhomogeneous Media

Two different approximations in analogy to optics:

- Computation of wavefronts (Huygens' principle, eikonal equation)
- Computation of ray paths normal to the wavefronts (ray optics)

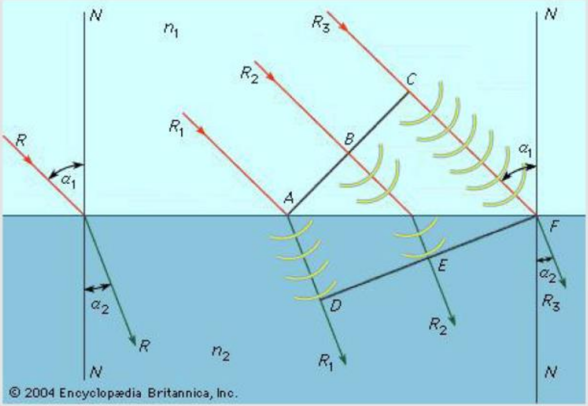
Ray Optics

Ray optics is rather simple in two limiting cases:

- Almost planar interface between two homogeneous media.
- Medium is almost homogeneous on the scale of the wavelength.

Reflection and Refraction

Simplest case: two homogeneous, isotropic halfspaces with different properties (λ, μ, ρ) and plane waves in each of them.

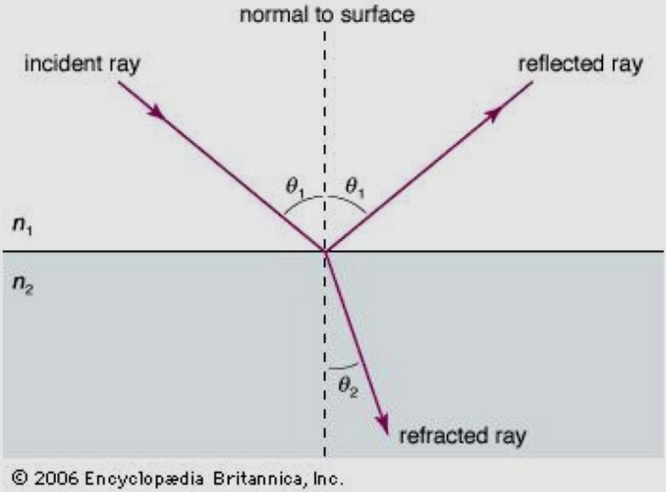


Source: Encyclopaedia Britannica

Snell's law:

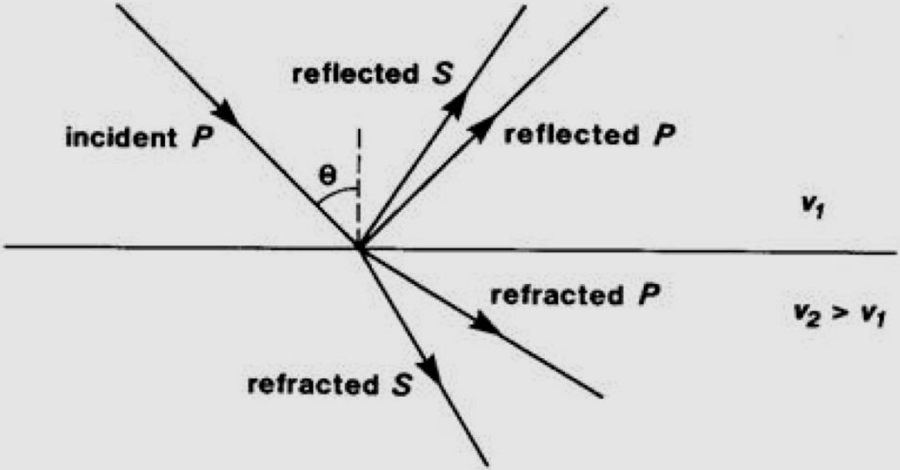
$$\frac{\sin \alpha_1}{\sin \alpha_2} = \frac{v_1}{v_2}$$

Reflection and Refraction



Source: Encyclopaedia Britannica

Reflection and Refraction



Source: University College London

Reflected and Refracted Waves

P- and S-waves are merged when reflected or refracted.

$$\Downarrow$$

Incoming P- or S-wave induces up to 4 reflected and refracted waves.

Snell's law holds for all involved pairs of waves

$$\Downarrow$$

Ray parameter

$$p = \frac{\sin \alpha}{v}$$

(also called horizontal slowness) is the same for all involved waves.

Reflected and Refracted Waves

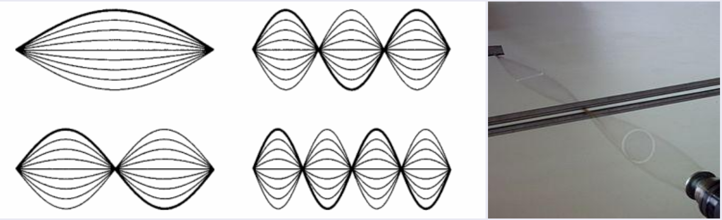
General form of Snell's law:

Horizontal slowness remains constant in reflection and refraction.

- Horizontal velocity is not constant!
- Conservation of horizontal slowness is the main reason why slowness is preferred to veloctiy in seismology.

Polarization

- is a parameter applying to transverse waves that specifies the geometrical orientation of the oscillations
- A simple example of a polarized transverse wave is vibrations traveling along a string. The vibrations can be at any angle perpendicular to the string.



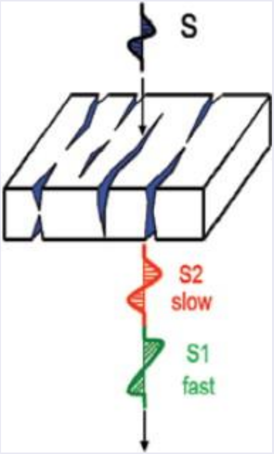
Modes of vibration of a string between fixed endpoints, from Shearer 2010, Introduction to Seismology

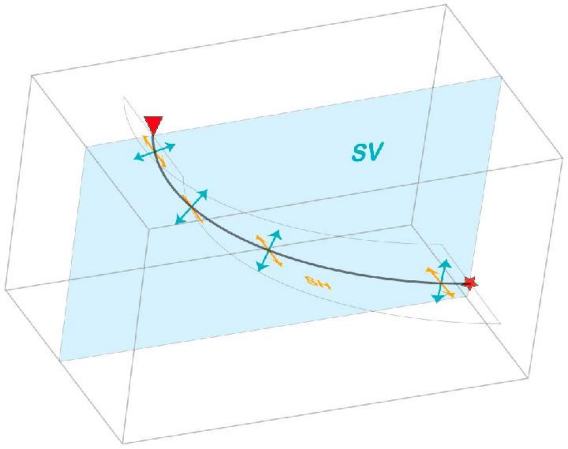
- Sound waves in solid materials exhibit polarization. Differential propagation of the three polarizations through the earth is a crucial in the field of seismology. Horizontally and vertically polarized seismic waves (shear waves) are termed SH and SV, while waves with longitudinal polarization are termed P-waves.

Polarization

Dipankar et al. 2010, Seismic anisotropy beneath the Indian continent from splitting of direct S-waves.

Shear-wave splitting after Long and Becker 2010,
Mantle dynamics and seismic anisotropy





- SH is parallel to the ground surface, while SV is oriented in a vertical plane coinciding with the propagation direction of the wave front

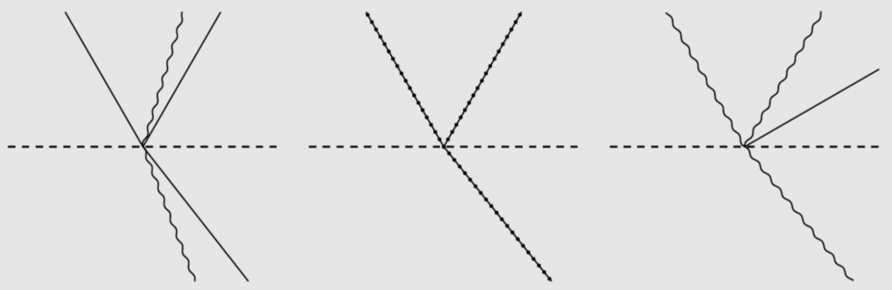
Conversion of Waves in Reflection and Refraction

Conversion of waves depends on the polarization of the S-waves.

Vertically polarized S-wave (SV) merges with P-waves.

Horizontally polarized S-wave (SH) is independent of P-waves and SV-waves.

Incident P-wave | Incident SH-wave | Incident SV-wave



Seismic Waves

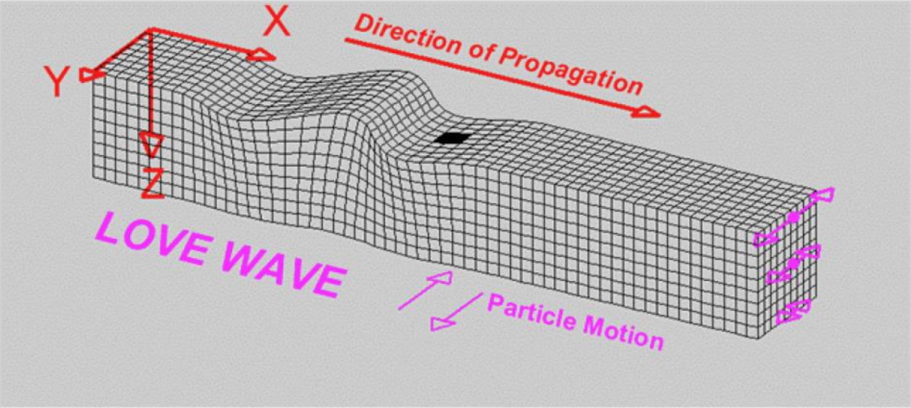
Surface Waves

Infinite domain in 3D → body waves (P- and S-wave)
Semi-infinite halfspace → surface waves (Love wave and Rayleigh wave)

The Love Wave

- Discovered quite late (1911), named after A. E. H. Love.
- In principle a S-wave with horizontal polarization.
- Amplitude decreases exponentially with depth.
- Exists only in inhomogeneous media where the seismic velocities increase with depth.

The Love Wave

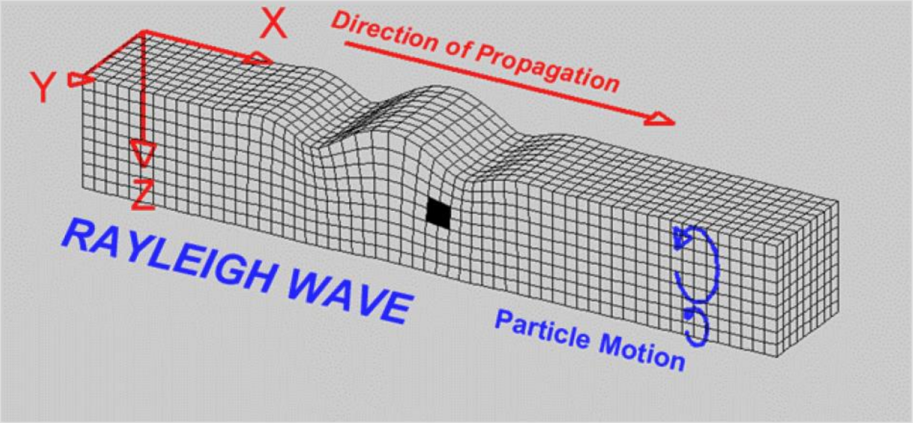


Source: L. Braile, Purdue University

The Rayleigh Wave

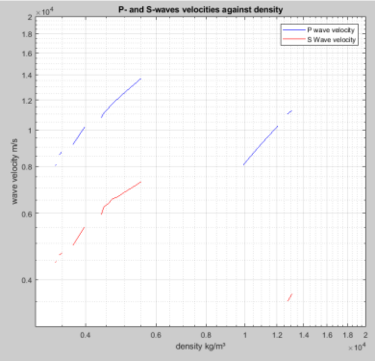
- Named after J. W. Strutt (later 3. Lord Rayleigh).
- In principle a mixture of P-wave and S-wave with vertical polarization.
- Particles rotate backwards on elliptic traces (ground roll).
- Causes most of the damage of earthquakes.
- Amplitude decreases exponentially with depth.
- Also exists in homogeneous media.
- Slightly slower than the S-wave.

The Rayleigh Wave

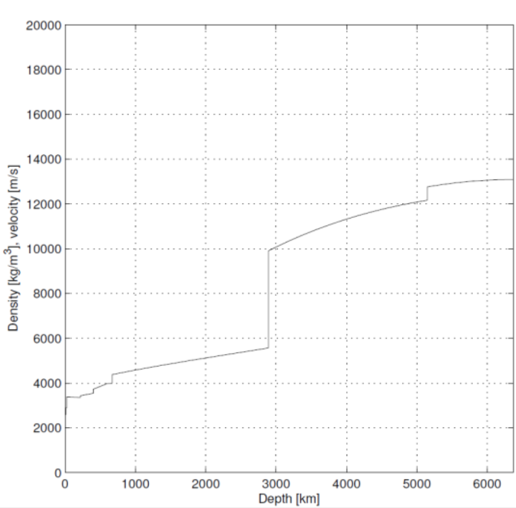


Source: L. Braile, Purdue University

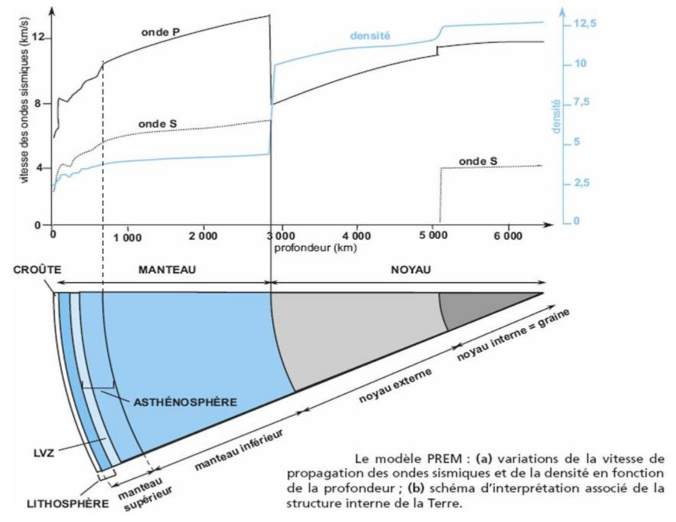
```
% Assignment 1 - Jakob Wilk - MNo: 0815

% Loading data.

data=csvread ('prem.csv');

radius=data(:,1); % Earth radius.
density=data(:,2); %Density.
pwave=data(:,3); % P-wave velocity.
swave=data(:,4); % S-wave velocity.


% Adding a value of NaN for discontinuities


for i=2:numel(radius)
    A=[radius(i-1) radius(i)]
    if A(1)==A(2)
        density(i)=NaN;
        pwave(i)=NaN;
        swave(i)=NaN;
    end
end

% Plotting data with NaN values.

figure
loglog(density,pwave,'b-')
hold on
loglog(density,swave,'r-')

% Labeling figure and axis.

legend('P wave velocity','S Wave velocity')
title('P- and S-waves velocities against density')
xlabel ('density kg/m³')
ylabel ('wave velocity m/s')
grid on
set(gcf, 'color', [0.8 0.8 0.8])
axis ([3*10^3 2*10^4 3*10^3 2*10^4])
```



NSG – Exercise 1



NSG - Exercise 1



Le modèle PREM : (a) variations de la vitesse de propagation des ondes sismiques et de la densité en fonction de la profondeur ; (b) schéma d'interprétation associé de la structure interne de la Terre.

Near-surface Geophysics

Reflection and refraction seismics

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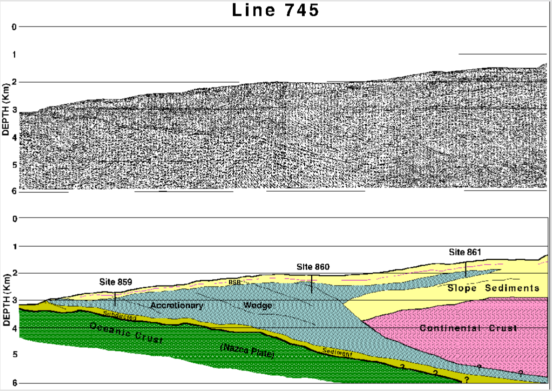
Main Fields of Application

- Prospection and exploration of natural resources (oil, gas, ores, . . .).
- Exploration of the geological structure, e. g., for tunnel construction.
- Mapping of the interface between soil or unconsolidated rock and bedrock (ground investigation, slope instability, . . .).
- Mapping of aquifers.
- Mapping of residual waste sites.

Main Fields of Application

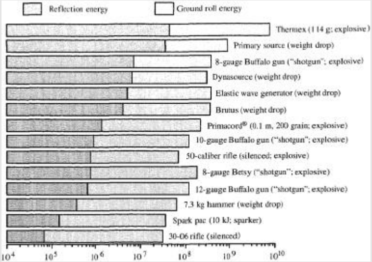


(source: wikipedia)

Sources of Seismic Waves

While seismology uses the waves radiated by natural earthquakes, seismic uses artificial sources, e. g.,

- Hammer stroke
- Weight dropping
- Explosives
- Seismic vibrators ("Vibroseis")
- Airguns (in marine seismics)



The different seismic sources differ concerning their energy and thus concerning their penetration depth.

Hammer Stroke

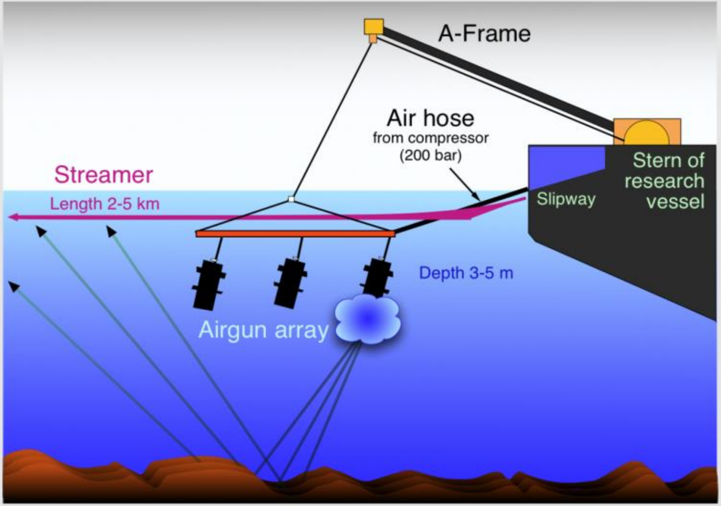
Explosives

Source: Teaching material R. Scholger

Seismic Vibrators ("Vibroseis")

Source: Wikipedia

Airguns (in Marine Seismics)



Source: Wikipedia

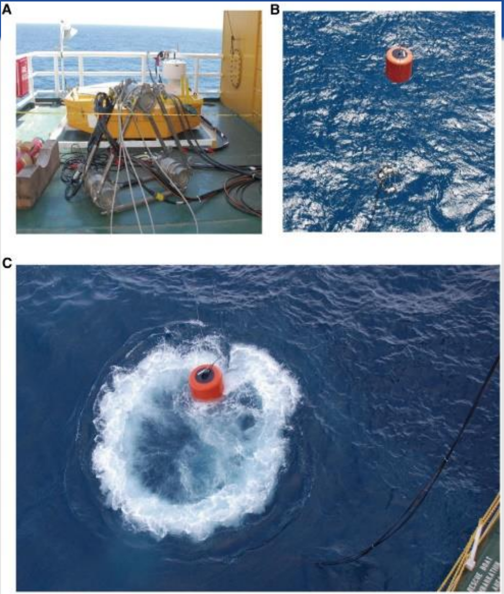
Seismics

Airguns (in Marine Seismics)

A - VSP air gun

B - Air gun array

(source: IODP 2018)



Seismic Recording

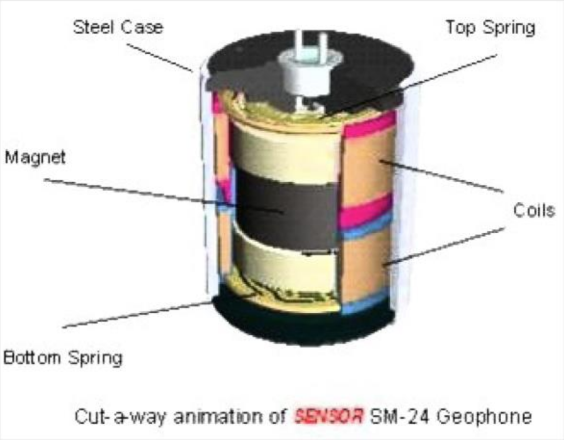
Waves are recorded by either seismometers of geophones. Standard equipment:

Seismic source, e. g., hammer and plate.

Geophones
- In principle a simplified seismometer for small amplitudes.
- Similar to a microphone.
- Consists of a permanent magnet and a coil; one directly connected to the casing, while the other can move. the induced voltage is proportional to the relative velocity.

Seismograph unit records the signals from 6–48 geophones (channels) and the seismic source.

A Simple Geophone

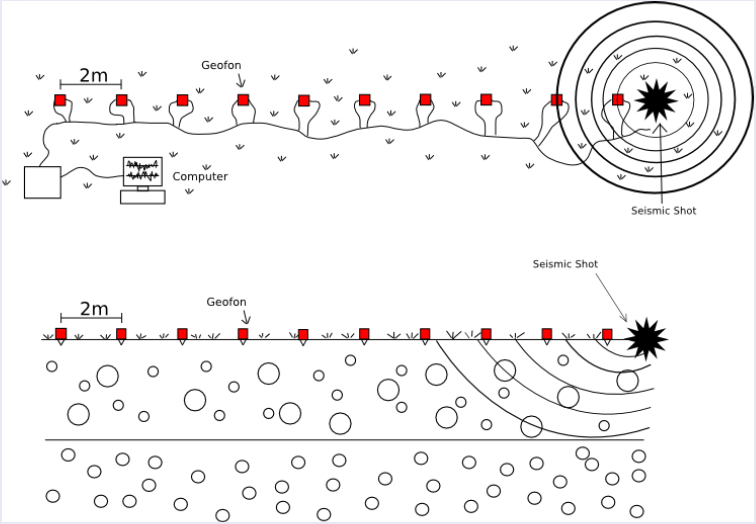


Seismics

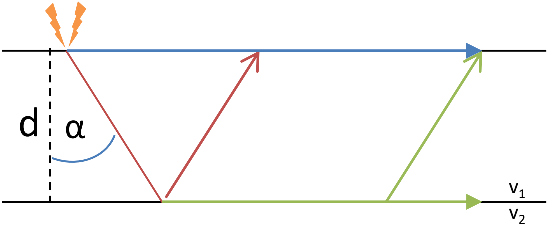
- device that converts ground movement into voltage and the deviation of this measured voltage from the base line is called the seismic response
- Construction
    - passive analog devices that typically comprise a spring-mounted magnetic mass moving within a wire coil to generate an electrical signal
    - microelectromechanical systems (MEMS) which generates an electrical response to ground motion through an active feedback circuit to maintain the position of a small piece of silicon
- waves passing through the earth have a three-dimensional nature, geophones are normally constrained to respond to single dimension - usually the vertical
- Analog geophones are very sensitive devices. Small signals can be drowned by larger signals from local sources.
    - to recover the small signals that are caused by large but distant events the signals from several geophones deployed in an array are correlated. It can be assumed that small signals that register uniformly at all geophones in an array can be attributed to a distant and therefore significant event.

Seismics

Geophone



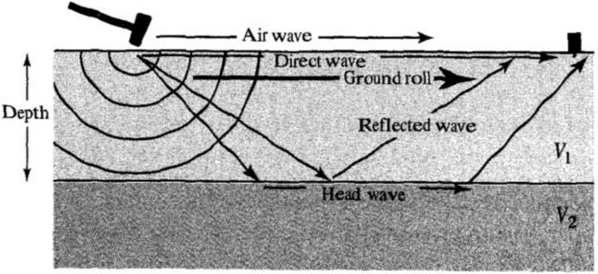
Typical Setup of Hammer Stroke Seismics



Refracted wave

Reflected wave

Direct wave

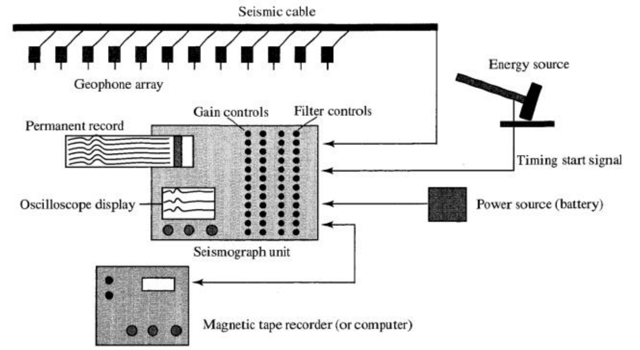


$V_1 = 1500$ m/s

$V_2 = 4000$ m/s

Source: Burger et al., Introduction to Applied Geophysics

Typical Setup of Hammer Stroke Seismics



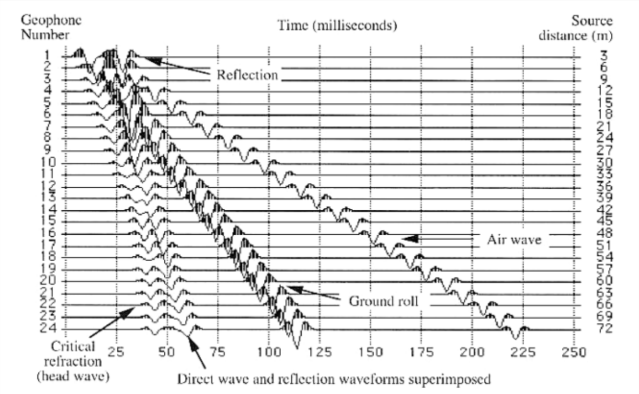
De-Multiplexed Order

	Channel		
Time	1	2	3
0.001	D001	E001	F001
0.002	D002	E002	F002
0.004	D003	E003	F003

Source: Burger et al., Introduction to Applied Geophysics

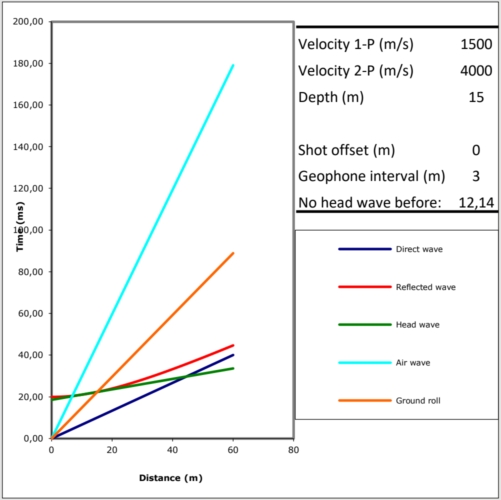
Seismic Picking

In most cases, only the first arrival times of the different waves at the geophones are analyzed.



Source: Burger et al., Introduction to Applied Geophysics

Seismic Picking



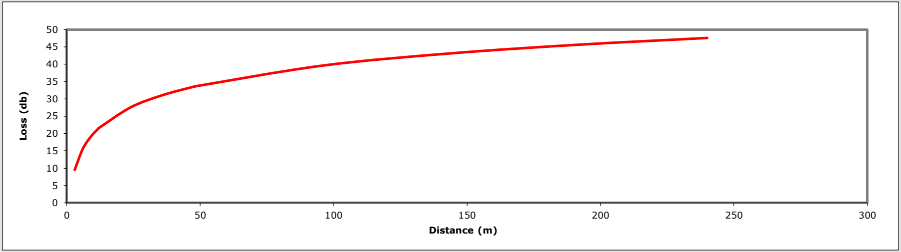
Velocity 1-P (m/s)	1500
Velocity 2-P (m/s)	4000
Depth (m)	15
Shot offset (m)	0
Geophone interval (m)	3
No head wave before:	12,14

Distance from shot (m)	Arrival Times (ms): Direct wave	Reflected wave	Head wave	Air wave	Ground roll
0	0,00	20,00	18,54	0,00	0,00
3	2,00	20,10	19,29	8,96	4,44
6	4,00	20,40	20,04	17,91	8,89
9	6,00	20,88	20,79	26,87	13,33
12	8,00	21,54	21,54	35,82	17,78
15	10,00	22,36	22,29	44,78	22,22
18	12,00	23,32	23,04	53,73	26,67
21	14,00	24,41	23,79	62,69	31,11
24	16,00	25,61	24,54	71,64	35,56
27	18,00	26,91	25,29	80,60	40,00
30	20,00	28,28	26,04	89,55	44,44
33	22,00	29,73	26,79	98,51	48,89
36	24,00	31,24	27,54	107,46	53,33
39	26,00	32,80	28,29	116,42	57,78
42	28,00	34,41	29,04	125,37	62,22
45	30,00	36,06	29,79	134,33	66,67
48	32,00	37,74	30,54	143,28	71,11
51	34,00	39,45	31,29	152,24	75,56
54	36,00	41,18	32,04	161,19	80,00
57	38,00	42,94	32,79	170,15	84,44
60	40,00	44,72	33,54	179,10	88,89

Data from: Burger er al., Introduction to Applied Geophysics

Signal attenuation / absorption

Distance from shot (m)	3	6	9	12	12	24	36	48	48	96	144	192	240
Spherical spreading los (db)	9,5	15,6	19,1	21,6	21,6	27,6	31,1	33,6	33,6	39,6	43,2	45,7	47,6
Loss from previous point (db)		6,02	3,52	2,50		6,02	3,52	2,50		6,02	3,52	2,50	1,94



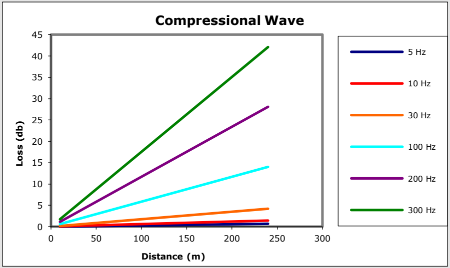
Signal attenuation / absorption

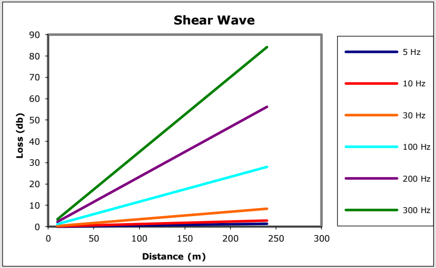
Frequency (Hz)	Compressional Wave Distance (m) 10	60	120	240	Shear Wave Distance (m) 10	60	120	240
5	0,03	0,176	0,35	0,7	0,06	0,35	0,7	1,4
10	0,06	0,351	0,7	1,4	0,12	0,7	1,4	2,81
30	0,18	1,053	2,11	4,21	0,35	2,11	4,21	8,42
100	0,59	3,51	7,02	14	1,17	7,02	14	28,1
200	1,17	7,02	14	28,1	2,34	14	28,1	56,2
300	1,76	10,53	21,1	42,1	3,51	21,1	42,1	84,2

P-wave velocity (m/s): 4000

S-wave velocity (m/s): 2000

Absorption coefficient = 0.55 db/l





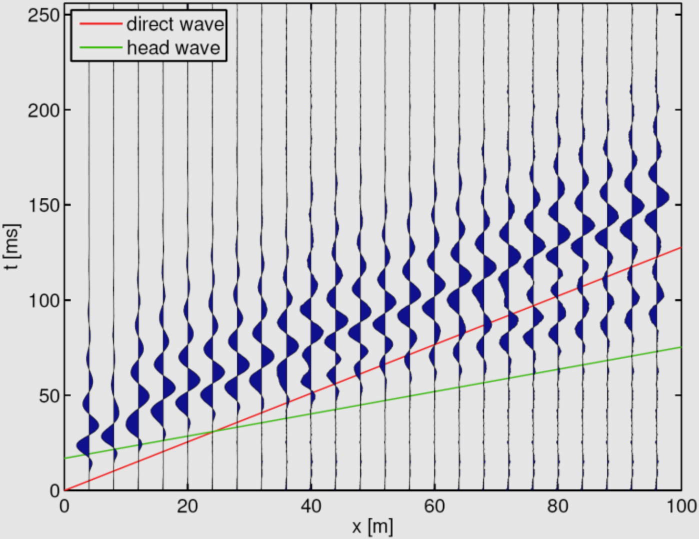
Seismic Picking

The first arrival times are mostly considered to be

- the time when the signal significantly rises above the noise,
- the time of the first maximum minus a quarter of the period, or
- the time of the zero after the first maximum minus a half of the period.

Picking is often supported by software.

Seismic Picking



Seismic Methods

Reflection seismics: Evaluation of the waves reflected at discontinuities.

Refraction seismics: Evaluation of the head wave, i. e., the refracted wave at the limit to total internal refraction.

Seismic tomography: Inverting the signals of many sources and receivers. Mainly applied in large-scale seismology.

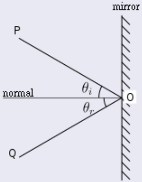
- Both reflection and refraction seismics address discontinuities where the seismic velocities change.
- Im most cases, only the first arrival times of the P-waves are evaluated.

Reflection Seismics

- is the change in direction of a wavefront at an interface between two different media so that the wavefront returns into the medium from which it originated.

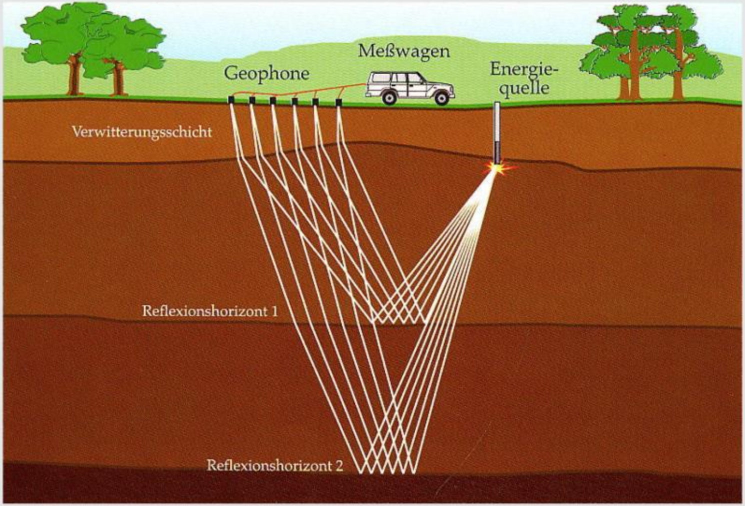
The laws of reflection are as follows:

- The incident ray, the reflected ray and the normal to the reflection surface at the point of the incidence lie in the same plane.
- The angle which the incident ray makes with the normal is equal to the angle which the reflected ray makes to the same normal.
- The reflected ray and the incident ray are on the opposite sides of the normal.



- These three laws can all be derived from the Fresnel equations.

Basic Setup



Meßwagen
Geophone
Energie-
quelle
Verwitterungsschicht
Reflexionshorizont 1
Reflexionshorizont 2

Reflection at the First, Horizontal Discontinuity

Travel time of the reflected wave to a geophone at distance x from the source (geophone offset):

$$t = \frac{\sqrt{4d_1^2 + x^2}}{v_1}$$

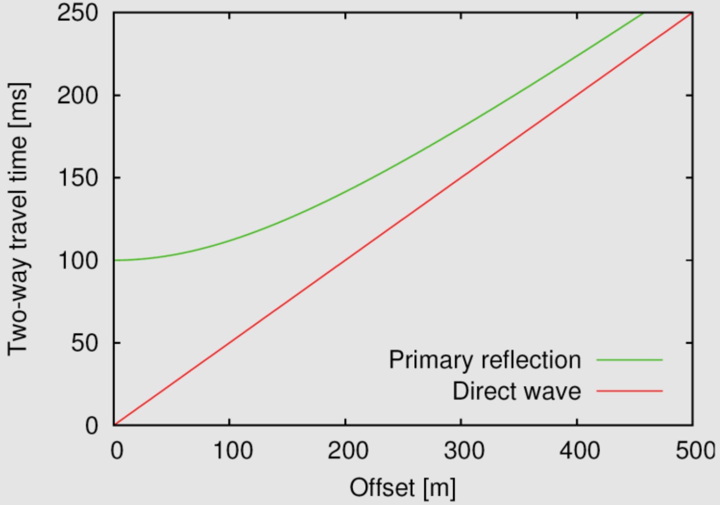
with

$$\begin{aligned} d_1 &= \text{thickness of the upper layer} \\ v_1 &= \text{P-wave velocity of the upper layer} \end{aligned}$$

This equation describes a hyperbola:

$$\frac{t^2}{(2d_1/v_1)^2} - \frac{x^2}{(2d_1)^2} = 1$$

The Reflection Hyperbola



Evaluating the Reflection Hyperbola

- v_1 can be determined from the travel time curve of the direct wave.
- d_1 can be determined from v_1 and the hyperbola.

Question

What are v_1 and d_1 in the example on the previous page?

Evaluating the Reflection Hyperbola

v_1 can also be determined without the direct wave, either

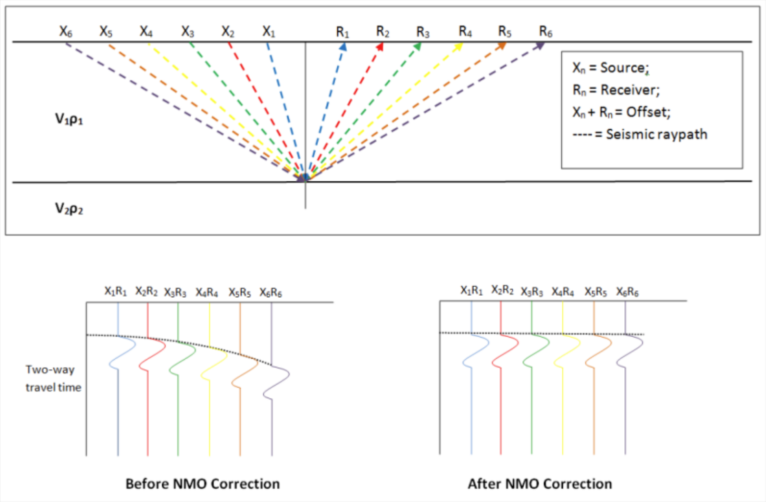
- from the asymptotic slope of the hyperbola for $x \to \infty$ (requires large offsets), or
- from the curvature of the hyperbola.

Normal Moveout Correction

The increase of the travel time with the offset of called Normal Moveout (NMO).

The correction to zero offset is called NMO correction or Common Midpoint (CMP) method. It is applied before stacking the seismic signals radiated by several sources or recorded by several geophones in order to improve the quality of the signal.

Normal Moveout Correction



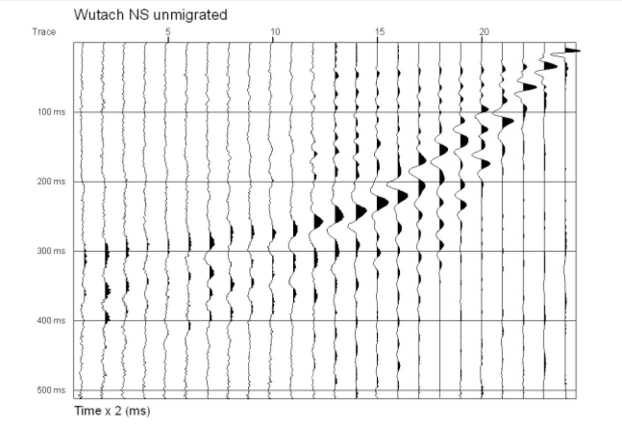
Source: Wikipedia

Normal Moveout Correction

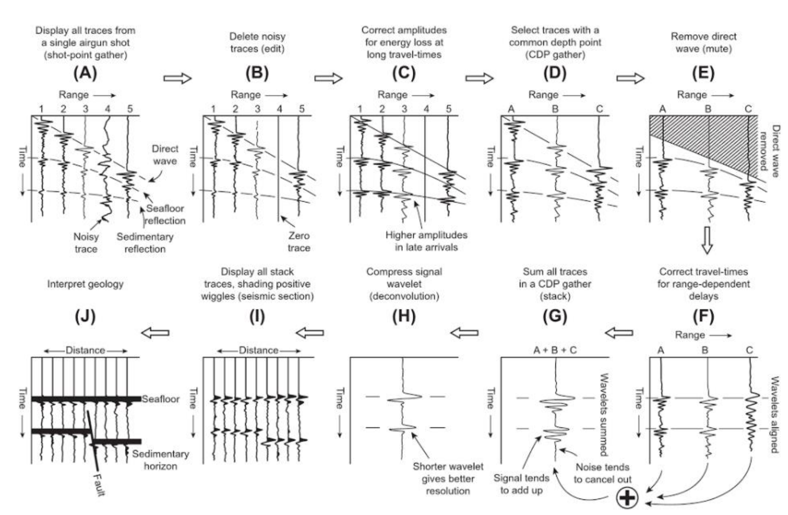
Noise:

- Air wave
- Rayleigh wave
- Ground roll
- Surface wave
- Refraction
- Head wave
- Multiple reflection
- Cultural noise

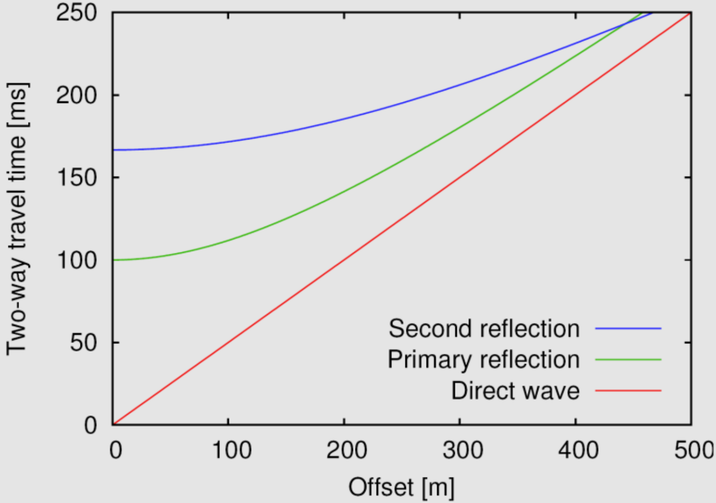
application of a high pass filter (above 380 MHz):



Seismic processing



Multiple Reflections



Multiple Reflections

- Even in horizontally layered media, secondary and later reflections are only approximately hyperbola.
- v_2, v_3, ... can in principle be derived from the hyperbolas, but this is more complicated than for the primary reflection.
- The uncertainty increases from layer to layer.

Question

What is the depth of the second discontinuity in the example on the previous page?

Reflection at Tilted or Curved Interfaces

Transfer of travel times to depths (depth migration) can be done, e. g., by constructing spherical wave fronts.

Intensity of the Reflected Waves

The intensity of the reflected waves depends on the contrast in the acoustic impedance

$$Z = \rho v.$$

Reflection coefficient R = amplitude ratio of reflected and incident wave

- In general, R increases with the contrast in Z.
- For normal incidence, P- and S-waves do not merge, and

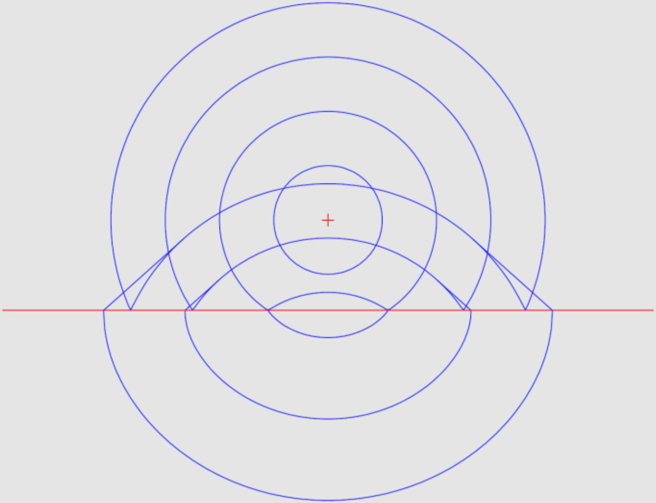
$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{\rho_2 v_2 - \rho_1 v_1}{\rho_2 v_2 + \rho_1 v_1}.$$

- Multiple reflexion costs much energy.

$$\Downarrow$$

Multiple-layer reflection seismics requires strong seismic sources.

Reflection and Refraction of a Spherical Wave at a Planar Interface



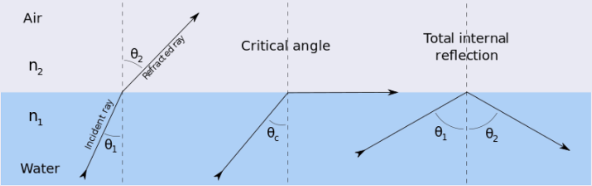
Refraction

- When a seismic wave travelling through the Earth encounters an interface between two materials with different acoustic impedances, some of the wave energy will reflect off the interface and some will refract through the interface.

- At its most basic, the seismic reflection technique consists of generating seismic waves and measuring the time taken for the waves to travel from the source, reflect off an interface and be detected by a spread of receivers (or geophones) at the surface

Refraction

- is the change in direction of wave propagation due to a change in its transmission medium.
- explained by the conservation of energy and the conservation of momentum. Due to the change of medium, the phase velocity of the wave is changed but its frequency remains constant.
- Refraction is described by Snell's law



Source: Wikipedia

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

$$v_{refl} = l - (2\cos\theta_1)n \qquad v_{refr} = \left(\frac{n_1}{n_2}\right) l + \left(\frac{n_1}{n_2}\cos\theta_1 \pm \cos\theta_2\right) n$$

Refraction

Headwave

- Is the wave that strikes at a critical angle on a boundary surface from a seismic medium to another medium with a higher seismic velocity and is refracted at a right angle. It runs along the boundary surface and continuously radiates wave energy below the critical angle.

- Parallel to the boundary with the velocity of the layer beneath it, with:

$$\sin(i) = \frac{v_0}{v_1}$$

Refraction at the Critical Angle

If the angle of incidence on a planar interface from a medium with v_1 to a medium with $v_2 > v_1$ achieves the critical value α_c with

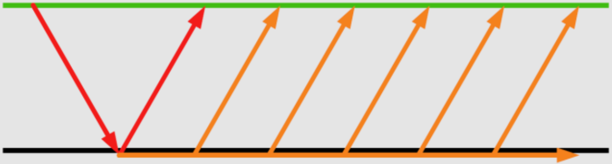
$$\sin \alpha_c = \frac{v_1}{v_2}$$

the refracted waves propagates along the interface with the velocity v_2.

The Seismic Head Wave

Wave refracted at the critical angle = head wave = Mintrop wave (according to L. Mintrop, 1880–1956).

- Continuously radiates back into the first medium at the angle α_c.

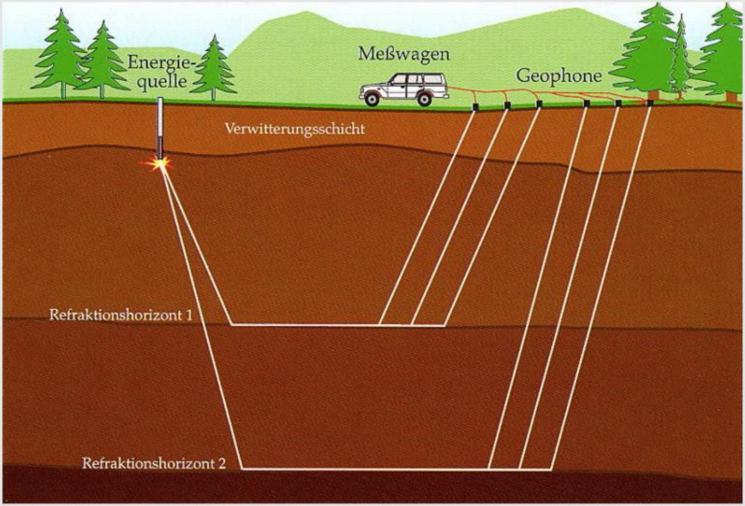


- Propagates along the surface with $v_2 > v_1$.

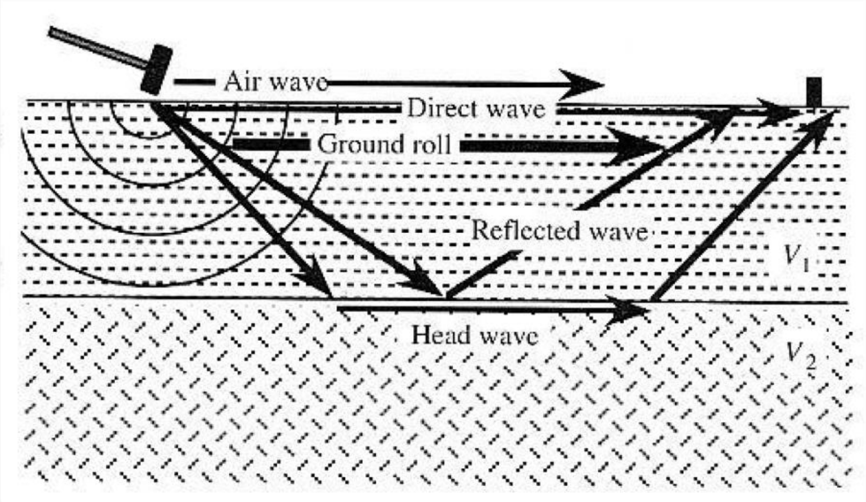
$\Downarrow$

Head wave finally outruns the direct wave (v_1)

Basic Setup


Energie-
quelle
Meßwagen
Geophone
Verwitterungsschicht
Refraktionshorizont 1
Refraktionshorizont 2

Types of Waves



Source: Burger et al., Introduction to Applied Geophysics

Travel Time of the Head Wave at a Horizontal Interface

The wave reflected at the critical angle α_c needs the time

$$t = \frac{2d}{v_1 \cos \alpha_c}$$

(d = layer's thickness) and arrives at the surface again at

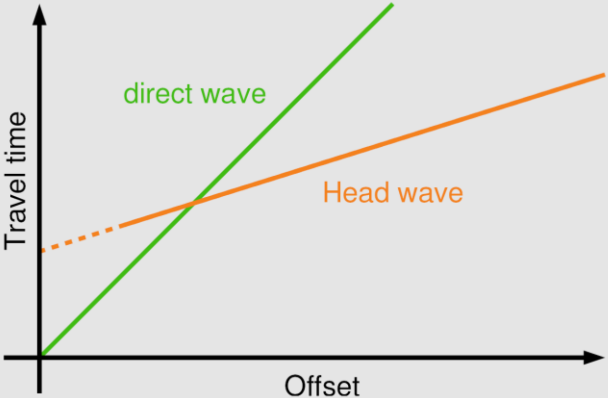
$$x = 2d \tan \alpha_c .$$

$$\Downarrow$$

The head wave reaches a receiver with offset x after

$$t = \frac{2d}{v_1 \cos \alpha_c} + \frac{x - 2d \tan \alpha_c}{v_2} = \frac{2\, d \cos \alpha_c}{v_1} + \frac{x}{v_2}$$

Travel-Time Diagram for a Horizontal Interface



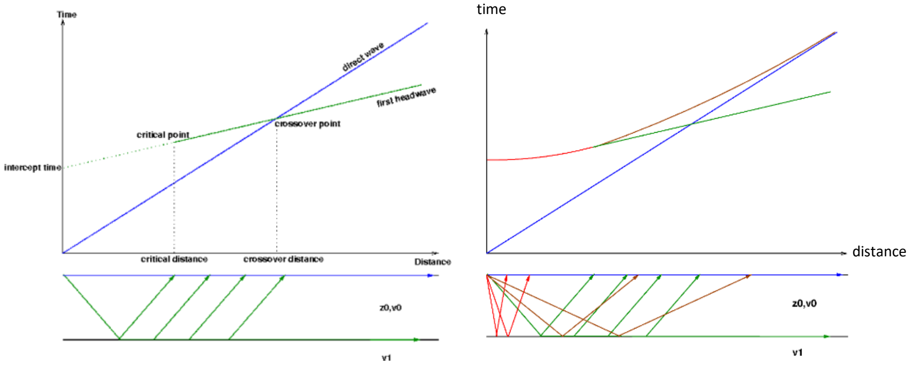
Question

What would the travel time curve for the reflected wave look like?

Travel time curves

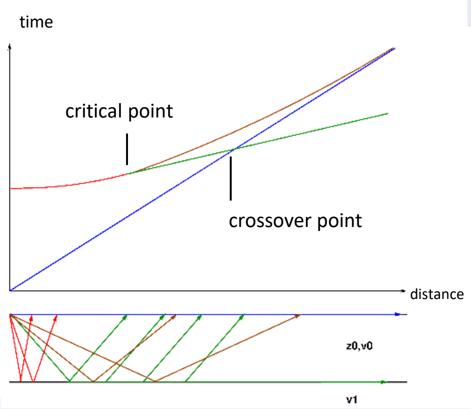
Seismic Refraction Outlines

- Propagating seismic waves (bottom) and related travel time diagram (top) of the direct (blue) and the first refracted phase (green)



Travel time curves

- blue: direct wave
  - *Slope reflects invert of velocity* $1/v_0$
- green: Head wave
- red: reflected wave, under critical
- brown: reflected wave, over critical



Travel Time Diagram

- Mostly, the waves radiated at one source (shot point) are recorded with several geophones located on a straight line..
- Direct and refracted waves are straight lines in the diagram, while reflected waves are (approximately) hyperbola.
- Often only the earliest arriving wave is picked.

Quantitative Analysis in Case of a Horizontal Interface

1. Slope of the travel time curve of the direct wave $S_d = \frac{1}{v_1}$
2. Slope of the travel time curve of the refracted wave $S_r = \frac{1}{v_2}$
3. Intercept time = extrapolated time where the travel time curve of the refracted wave crosses the time axis

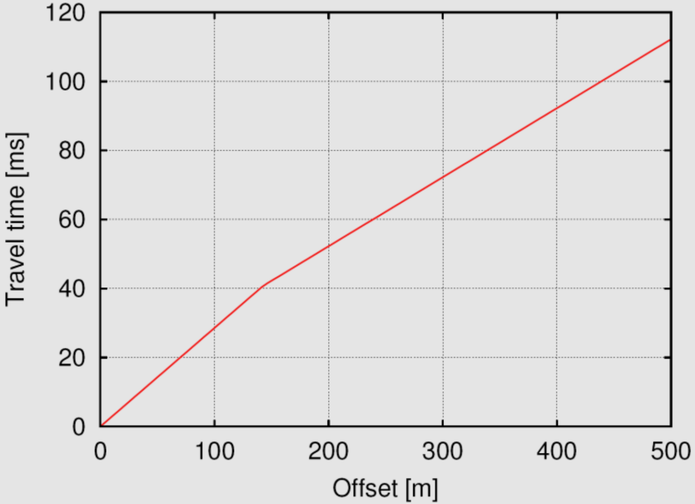
$$t_0 = \frac{2\,d\,\cos\alpha_c}{v_1} = 2\,d\sqrt{\frac{1}{v_1^2} - \frac{1}{v_2^2}}$$

so that

$$d = \frac{t_0\,v_1}{2\cos\alpha_c} = \frac{t_0}{2\sqrt{\frac{1}{v_1^2} - \frac{1}{v_2^2}}}$$

Refraction Seismics

Example



What is a Useful Offset?

The travel time curves of the direct wave and the head wave intersect at

$$\frac{x}{v_1} = \frac{2\,d\,\cos\alpha_c}{v_1} + \frac{x}{v_2}$$

$$\Downarrow$$

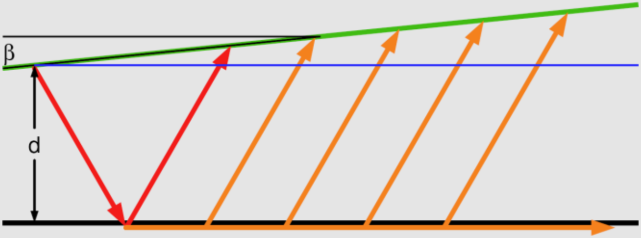
$$x = \frac{2\,d\,\cos\alpha_c\,v_2}{v_2 - v_1} = 2\,d\sqrt{\frac{v_2 + v_1}{v_2 - v_1}}$$

The total offset (shot point to last geophone) should be a least two times x to recognize the travel time curves of both waves.

Question

What would be a reasonable offset for locating the ground water level in an aquifer of sand an gravel in about 12 m depth?

Travel Time Curve for a Sloping Interface



Intersection of the arriving rays with the blue line:

$$x_b = x \cos\beta - x \sin\beta \tan\alpha_c$$

Additional travel time from the blue line to the surface:

$$t - t_b = \frac{x \sin\beta}{v_1 \cos\alpha_c}$$

Travel Time Curve for a Sloping Interface

Head wave arrives at x_b on the blue line at

$$t_b = \frac{2\,d\,\cos\alpha_c}{v_1} + \frac{x_b}{v_2}$$

so that the total travel time is

$$\begin{aligned} t &= \frac{2\,d\,\cos\alpha_c}{v_1} + \frac{x\,\cos\beta - x\,\sin\beta\,\tan\alpha_c}{v_2} + \frac{x\,\sin\beta}{v_1\,\cos\alpha_c} \\ &= \frac{2\,d\,\cos\alpha_c}{v_1} + \left(\frac{\cos\beta}{v_2} + \frac{\sin\beta\,\cos\alpha_c}{v_1}\right)x \end{aligned}$$

Travel Time Curve for a Sloping Interface

Slope of the travel time curve:

$$S_r = \frac{\cos\beta}{v_2} + \frac{\sin\beta \, \cos\alpha_c}{v_1} = \frac{\sin(\alpha_c + \beta)}{v_1}$$

$$\Downarrow$$

If the interface dips in shot direction (downdip), the head wave is apparently slower than for a horizontal interface and vice versa.

Intercept time

$$t_0 = \frac{2\,d\,\cos\alpha_c}{v_1}$$

is the same as for a horizontal interface if the depth is taken perpendicularly to the interface (not to the surface).

Shot and Reverse Shot

For a sloping interface the travel time curve is not sufficient to determine v_2, d, and β.

Measure a second travel time curve opposite to the original one (reverse shot).

Shot and Reverse Shot

Steps of analysis:

1. Slope of the travel time curve of the direct wave $S_d = \frac{1}{v_1}$ (same as for a horizontal interface).
2. Use either
   (a) mean value of the absolute slopes of the travel time curves of both head waves:
   $$\overline{S}_r = \frac{\cos\beta}{v_2} \approx \frac{1}{v_2}$$
   if β is not too large or
   (b) absolute slopes of the travel time curves of both head waves:
   $$S_r = \frac{\sin\left(\alpha_c \pm \beta\right)}{v_1}.$$

Shot and Reverse Shot

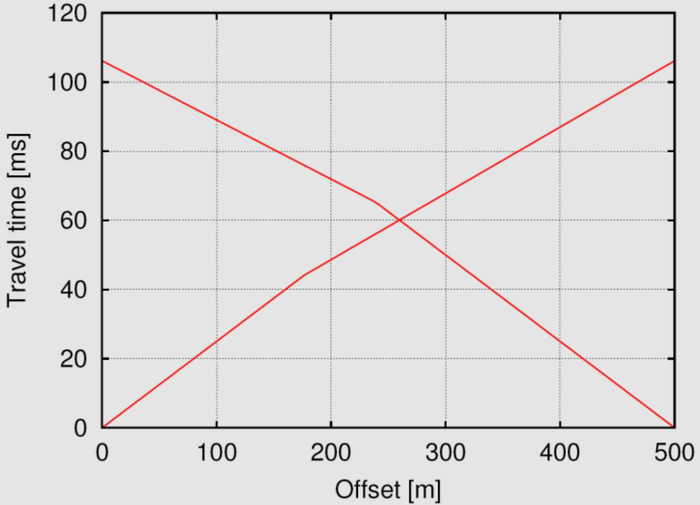
3. Depth of the interface is similar to the horizontal case:

$$d = \frac{t_0 \, v_1}{2 \cos \alpha_c} = \frac{t_0}{2 \sqrt{\frac{1}{v_1^2} - \frac{1}{v_2^2}}}$$

where the intercept times of shot and reverse shot yield the depth below the respective shot point (normal to the interface).

If β is not small, the (vertical) depths can be obtained by $\frac{d}{\cos \beta}$.

Example



120
100
80
60
40
20
0
Travel time [ms]
0
100
200
300
400
500
Offset [m]

Multiple Refraction

- Several head waves.
- Evaluation becomes more complicated, but without principal problems.
- Important limitation: Only interfaces where the velocity increases towards to lower layer can be detected (also applies to the case of two layers).

Refraction Seismics vs. Reflection Seismics

Advantages of Reflection Seismics

- High spatial resolution
- Complex geological structures (non-planar interfaces) can in principle be resolved.
- Layers with lower velocities can also be detected.

Advantages of Refraction Seismics

- Relatively simple evaluation.
- Moderate requirements on seismic energy.