Geothermics and Geothermal Energy
Deep Open Geothermal Systems

Stefan Hergarten

Institut für Geo- und Umweltnaturwissenschaften
Albert-Ludwigs-Universität Freiburg

Converting Geothermal Energy to Electricity

Steps of Conversion

Thermal energy → mechanical work: turbine; rather low efficiency due to thermodynamic limitation

Mechanical work → electricity: generator; high efficiency

Fundamentals – Thermodynamics

Entropy

Source: Wehrli, Die Kunst aufzuräumen

Entropy

Definition of entropy:

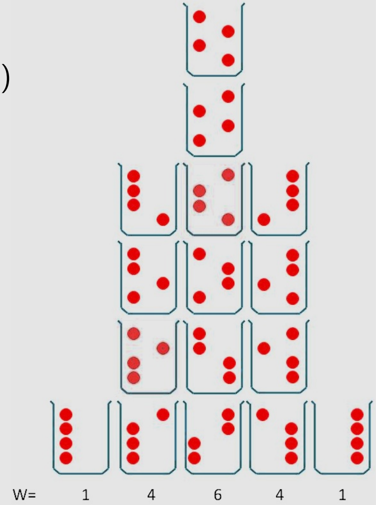
$$S = k \ln N \qquad (1)$$

where

$$k = 1.38 \times 10^{-23} \frac{\mathrm{J}}{\mathrm{K}}$$

= Boltzmann constant

N = number of states that cannot be distinguished



Source: Wikipedia

Entropy

- Second law of thermodynamics: Entropy of a closed system cannot decrease through time.
- Processes where the entropy is constant are reversible.
- Relationship between entropy, thermal energy and temperature in classical thermodynamics: Adding an amount of thermal energy δQ (or extracting if $\delta Q < 0$) at constant temperature T results in a change of entropy

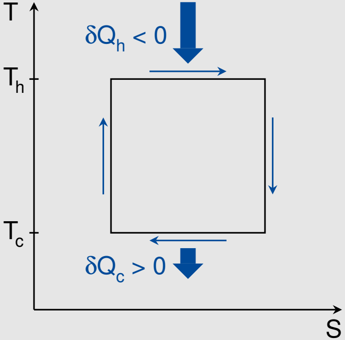
$$\delta S = \frac{\delta Q}{T} \tag{2}$$

or in integral form

$$\delta S = \int \frac{dQ}{T} \tag{3}$$

The Carnot Cycle

- Transfer of thermal energy from a "hot" reservoir of temperature T_h to a "cold" reservoir of temperature T_c yielding the maximum amount of mechanical work.
- Only a theoretical thermodynamic cycle, not a real physical process.
- Assumes two reservoirs of infinite capacity and a hypothetic gas.



Directions:

$\rightarrow$	isothermal expansion (coupled to large reservoir)
$\leftarrow$	isothermal compression (coupled to large reservoir)
$\downarrow$	isentropic cooling (by rapid expansion)
$\uparrow$	isentropic heating (by rapid compression)

The Thermodynamic Limit of the Carnot Cycle

Total change in entropy after one cycle:

$$\delta S \;=\; \delta S_h + \delta S_c \;=\; \frac{\delta Q_h}{T_h} + \frac{\delta Q_c}{T_c} \;\geq\; 0 \tag{4}$$

where

$$\begin{aligned} \delta Q_h &= \text{thermal energy supplied to the hot system } (< 0) \\ T_h &= \text{temperature of the hot system} \\ \delta Q_c &= \text{thermal energy supplied to the cold system } (> 0) \\ T_c &= \text{temperature of the cold system} \end{aligned}$$

⇓

$$\delta Q_c \;\geq\; -\,\delta Q_h \frac{T_c}{T_h} \tag{5}$$

Maximum Efficiency of Converting Thermal Energy

Mechanical work yielded by one Carnot cycle (conservation of energy):

$$\delta W \;=\; -(\delta Q_h + \delta Q_c) \;\leq\; -\delta Q_h \left(1 - \frac{T_c}{T_h}\right) \;=\; \eta_{\mathrm{max}}\,(-\delta Q_h) \tag{6}$$

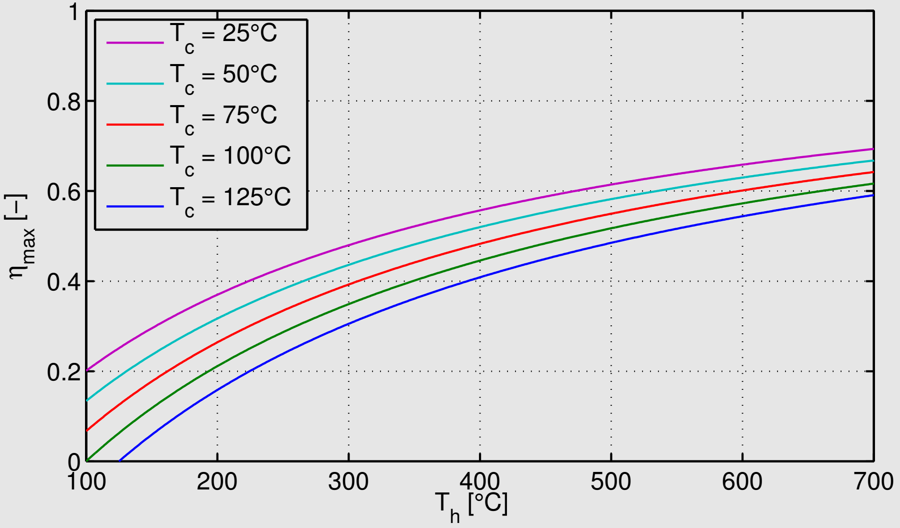
with the maximum efficiency

$$\eta_{\mathrm{max}} \;=\; \frac{T_h - T_c}{T_h} \tag{7}$$

Consequence for the electrical, mechanical and thermal power of a geothermal power plant:

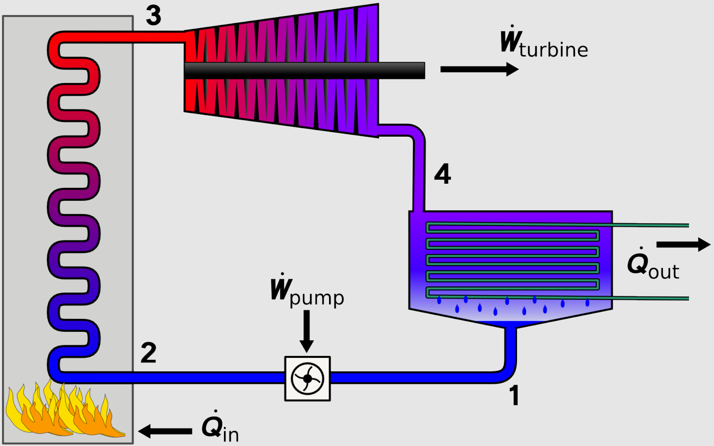
$$P_{\mathrm{el}} \;<\; P_{\mathrm{me}} \;<\; \eta_{\mathrm{max}}\,P_{\mathrm{th}} \tag{8}$$

Maximum Efficiency of Converting Thermal Energy



Main Types of Geothermal Power Plants

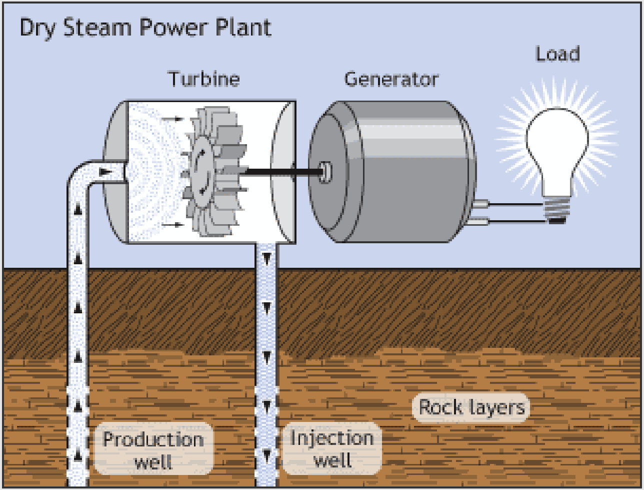
General Principle: Clausius-Rankine Cycle



Source: Wikipedia, © A. Ainsworth

Main Types of Geothermal Power Plants

Dry Steam Power Plants



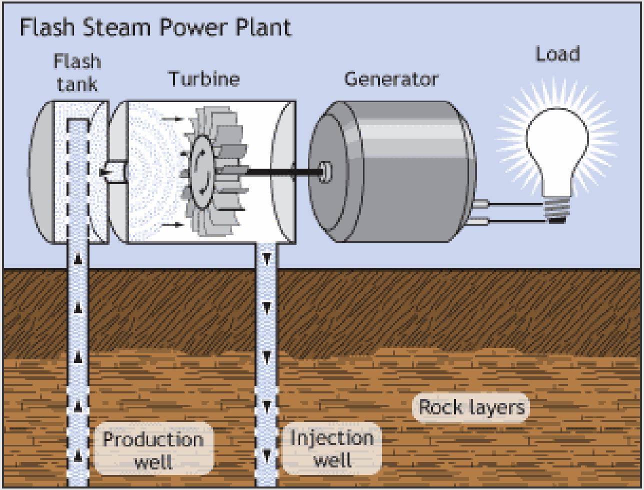
Source: Office of Energy Efficiency and Renewable Energy

Main Types of Geothermal Power Plants

Dry Steam Power Plants

- Rather simple technology
- First geothermal production of electricity: Larderello 1904
- Biggest geothermal power plant on Earth: “The Geysers”, California, USA, 750 MW_{el}
- Limited to few locations on Earth

Flash Steam Power Plants



Source: Office of Energy Efficiency and Renewable Energy

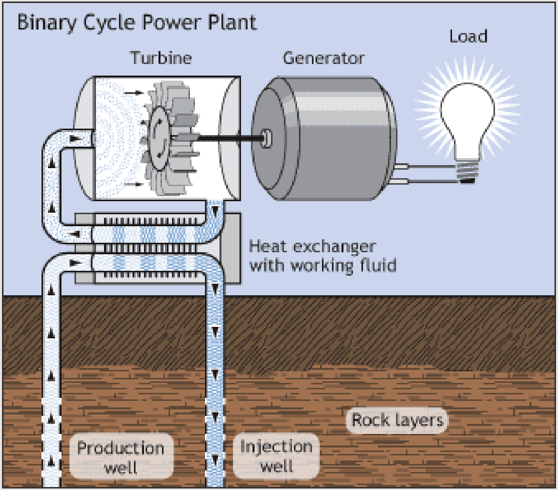
Main Types of Geothermal Power Plants

Flash Steam Power Plants

- Most common type of geothermal power generation plants in operation today
- Reasonable efficiency only for high-enthalpy resources ($T > 200^\circ$C)

Main Types of Geothermal Power Plants

Binary Cycle Power Plants



Source: Office of Energy Efficiency and Renewable Energy

Main Types of Geothermal Power Plants

Binary Cycle Power Plants

- Heat transfer to a fluid with a boiling point below 100°C by a heat exchanger
- Applicable to low-enthalpy resources ($T < 200$°C)
- Expensive technology
- Types: Organic Rankine Cycle (ORC) and, Kalina cycle

The Organic Rankine Cycle

Principle: Transfer heat to an organic fluid with a low boiling point and operate the turbine with the gas.

Fluids: various, e. g., n-perfluorpentane (C_5F_{12}, boiling point 30°C, $T_c \approx 75°C$)

Technical challenges: not many; rather robust

Environmental issues: Some fluids act as greenhouse gases if released.

Installations in Germany: several

Main Types of Geothermal Power Plants

The Kalina Cycle

Development: in the 1970s by Aleksandr Kalina

Principle: Uses an ammonia (NH_3) solution in water; solubility decreases with temperature.

↓

Separate ammonia from the solution at high temperature and operate the turbine with ammonia and then dissolve it at lower temperature.

Advantage: higher efficiency than ORC at low temperatures

Technical challenges: several, in particular corrosion by ammonia and maintainance

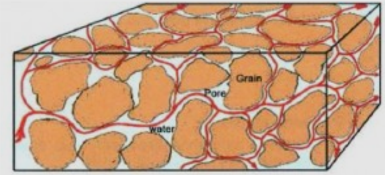
Environmental issues: Ammonia is highly hazardous.

Installations in Germany: Unterhaching (2009–2017), Bruchsal, Taufkirchen

Open Geothermal Systems

Principle

- Hot water is extracted at one or more wells.
- Cold water is (re)injected at another well; in most cases at the same rate as total extraction.
- Flow of water through a porous rock.



Source: Borehole Wireline

Major Problem

Maintaining the fluid circulation in the rock consumes a considerable part of the produced energy.

Porosity of Rocks

Total porosity: $\phi = \frac{\text{void volume}}{\text{total volume}}$; $0 \leq \phi < 1$; often measured in percent

Effective porosity: only accessible pores and volume of water that can be extracted

Typical porosity values:

	ϕ_{tot} [%]	ϕ_{eff} [%]
equally sized spheres	26–48	26–48
soil	55	40
clay	50	2
sand	25	22
limestone	20	18
sandstone (semiconsolidated)	11	6
granite	0.1	0.09

Source: GlobalSecurity.org

Darcy's Law

- Empirically found by Henry Darcy (1856).
- Describes the average flow through a porous medium on macroscopic scales.
- Simplest form (without gravity):

$$\vec{v}(\vec{x}, t) \;=\; -\frac{k}{\eta}\nabla p(\vec{x}, t) \tag{9}$$

where

$$\begin{aligned} \vec{v} &= \text{volumetric flow rate (Darcy velocity) } [\tfrac{\mathrm{m}}{\mathrm{s}}] \\ p &= \text{fluid pressure [Pa]} \\ k &= \text{hydraulic permeability } [\mathrm{m}^2] \\ \eta &= \text{dynamic viscosity of the fluid [Pa s]} \end{aligned}$$

Darcy's Law

- Basically the same as Fourier's law of heat conduction.
- With gravity almost the same, but p is the difference between pressure and hydrostatic pressure then.

The Hydraulic Permeability

- Units:

  SI unit: m^2

  Widely used unit: Darcy (D)

$$1\,\mathrm{D} \;=\; 9.869 \times 10^{-13}\,\mathrm{m}^2 \;\approx\; 10^{-12}\,\mathrm{m}^2 \;=\; 1\,\mu\mathrm{m}^2$$

- $k = 1\,\mathrm{D}$ results in a flow rate of $1\,\frac{\mathrm{cm}}{\mathrm{s}}$ at a pressure drop of $1\,\frac{\mathrm{atm}}{\mathrm{cm}}$ in water at $20°\mathrm{C}$ ($\eta = 10^{-3}\,\mathrm{Pas}$).
- Typical values:

Medium	k [D]
gravel	$10 - 1000$
sand	$0.01 - 10$
silt	$10^{-3} - 0.1$

Medium	k [D]
limestone	$10^{-6} - 100$
fractured igneous rocks	$10^{-6} - 10$
unfractured igneous rocks	$10^{-9} - 10^{-6}$

The Darcy Equation

Balance equation for the mass of water per bulk volume

$$\frac{\partial \chi}{\partial t} = -\operatorname{div}(\rho_f \vec{v}) = \operatorname{div}\left(\rho_f \frac{k}{\eta} \nabla p\right) \qquad (10)$$

where

$$\chi = \text{mass of fluid per bulk volume } [\tfrac{\text{kg}}{\text{m}^3}]$$
$$\rho_f = \text{fluid density } [\tfrac{\text{kg}}{\text{m}^3}]$$

↓

$$S \frac{\partial p}{\partial t} = -\operatorname{div}(\rho_f \vec{v}) = \operatorname{div}\left(\rho_f \frac{k}{\eta} \nabla p\right) \quad \text{with} \quad S = \frac{\partial \chi}{\partial p} \qquad (11)$$

The Darcy Equation Compared to the Heat Conduction Equation

Basically the same equation as the heat conduction equation with a different meaning of the parameters.

heat conduction	T	λ	ρc	$\kappa = \frac{\lambda}{\rho c}$
Darcy flow	p	$\rho_f \frac{k}{\eta}$	S	$\tilde{\kappa} = \frac{\rho_f k}{\eta S}$

If all parameters are constant:

$$\frac{\partial T}{\partial t} = \kappa \, \Delta T, \qquad \vec{q} = -\lambda \nabla T \tag{12}$$

$$\frac{\partial p}{\partial t} = \tilde{\kappa} \, \Delta p, \qquad \vec{v} = -\frac{k}{\eta} \nabla p \tag{13}$$

Superposition of Solutions

The simplest form of Darcy's equation is linear.

$$\Downarrow$$

Solutions can be superposed:

$$p(\vec{x}, t) = p_0(\vec{x}) + p_1(\vec{x}, t) + p_2(\vec{x}, t) + \ldots \quad (14)$$

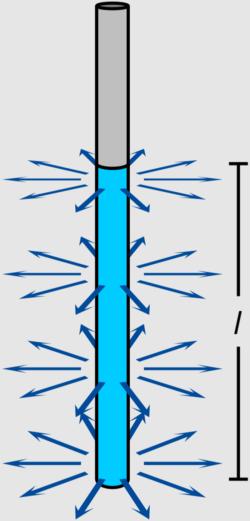
$$\vec{v}(\vec{x}, t) = \vec{v}_0(\vec{x}) + \vec{v}_1(\vec{x}, t) + \vec{v}_2(\vec{x}, t) + \ldots \quad (15)$$

where

$p_0, \vec{v}_0$ = natural pressure and Darcy velocity without wells

$p_i, \vec{v}_i$ = additional pressure and Darcy velocity caused by well $\# i$

The Simplest Model for a Hydrothermal Well



Vertical borehole in an aquifer of a thickness l

Simplifications:

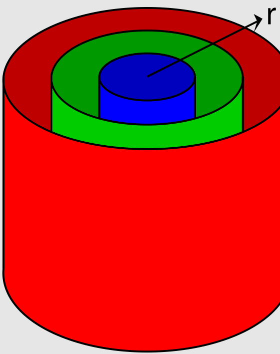
- All parameters (k, S, ρ_f, η) constant
- Only horizontal flow in radial direction

⬇

Widely used in hydrogeology for aquifer tests; introduced by C. V. Theis (1935).

Use variables p and $\vec{v}$ for the additional pressure and Darcy velocity instead instead of p_i and $\vec{v}_i$.

Cylindrical Symmetry



Flow in radial direction requires that $p(x, y, z, t)$ only depends on $r = \sqrt{x^2 + y^2}$ and t; use $p(r, t)$ instead of $p(x, y, z, t)$.

⬇

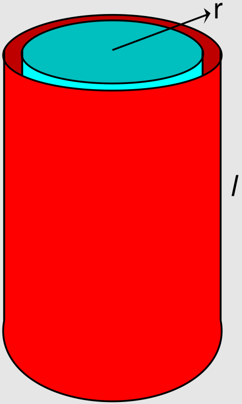
Darcy velocity

$$v(r, t) = -\frac{k}{\eta}\frac{\partial}{\partial r}p(r, t) \tag{16}$$

In other words (a bit sloppy):

$$\nabla = \frac{\partial}{\partial r} \tag{17}$$

Water Balance for a Cylindrical Shell



Change in mass of water m contained in the shell:

$$\begin{aligned}\frac{\partial m}{\partial t} &= \left(\pi r^2 l - \pi r^2 l\right) S \frac{\partial p}{\partial t} \\ &= 2\pi r l\, \rho_f v - 2\pi r l\, \rho_f v \end{aligned} \tag{18}$$

$$S \frac{\partial}{\partial t} p(r,t) = -\frac{1}{r}\frac{\partial}{\partial r}\left(r \rho_f v(r,t)\right) \tag{19}$$

for $r - r \to 0$

In other words (a bit sloppy):

$$\text{div} = \frac{1}{r}\frac{\partial}{\partial r} r \tag{20}$$

Solution of the Darcy Equation for Cylindrical Symmetry

Insert (16) into (19):

$$S\frac{\partial}{\partial t}p(r,t) \quad = \quad \frac{1}{r}\frac{\partial}{\partial r}\left(r\rho_f\frac{k}{\eta}\frac{\partial}{\partial r}p(r,t)\right) \tag{21}$$

If all parameters are constant:

$$\frac{\partial}{\partial t}p(r,t) \quad = \quad \kappa\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}p(r,t)\right) \tag{22}$$

with $\tilde{\kappa} = \frac{\rho_f k}{\eta S}$

Solution of the Darcy Equation for Cylindrical Symmetry

Rescaling approach already used for 1D heat conduction: Look for solutions where the shape of the pressure profile remains constant, while only the spatial scale changes:

$$p(r,t) \;=\; F(u(r,t)) \tag{23}$$

with the nondimensional variable

$$u(r,t) \;=\; \frac{r}{2L(t)} \;=\; \frac{r}{2\sqrt{\tilde{\kappa}t}} \tag{24}$$

Same procedure as in 1D, except that the outer derivative (div) is

$$\frac{1}{r}\frac{\partial}{\partial r}r \;=\; \frac{1}{u}\frac{\partial}{\partial r}u \tag{25}$$

instead of $\frac{\partial}{\partial z}$.

Solution of the Darcy Equation for Cylindrical Symmetry

⬇

$$\frac{1}{u}\,\frac{\partial}{\partial u}\left(u\frac{\partial}{\partial u}F(u)\right) \;=\; -\,2\,u\,\frac{\partial}{\partial u}F(u) \tag{26}$$

Define $G(u) = u\frac{\partial}{\partial u}F(u)$:

$$\frac{\partial}{\partial u}G(u) \;=\; -\,2\,u\,G(u) \tag{27}$$

(even the same as in 1D)

Solution:

$$G(u) \;=\; a\,e^{-u^2} \tag{28}$$

with an arbitrary constant a.

Solution of the Darcy Equation for Cylindrical Symmetry

Solution for $F(u)$ with the condition $F(u) \to 0$ for $u \to \infty$ ($p(\infty, t) = 0$):

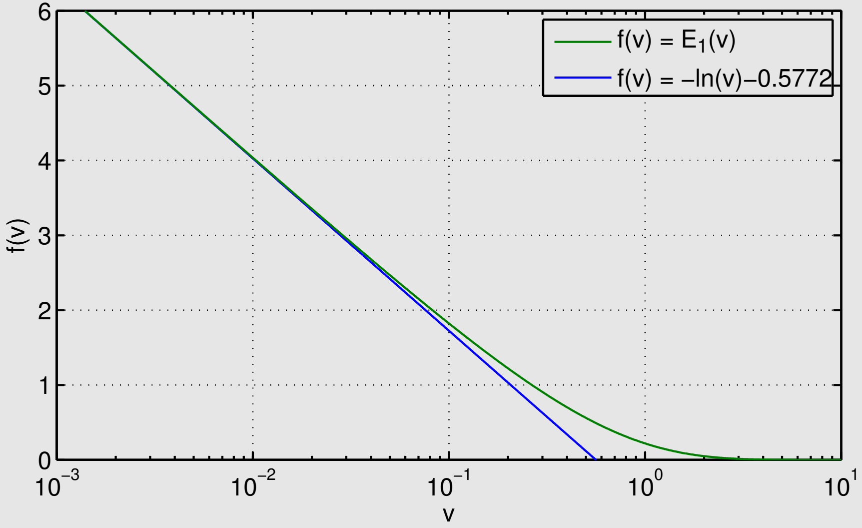
$$
\begin{aligned}
F(u) &= -\int_u^\infty \frac{G(\xi)}{\xi}\, d\xi = -\int_u^\infty \frac{a\, e^{-\xi^2}}{\xi} d\xi \\
&= -\frac{a}{2}\int_{u^2}^\infty \frac{e^{-x}}{x} dx = -\frac{a}{2} E_1(u^2) \qquad (29)
\end{aligned}
$$

with the function

$$
E_1(v) = \int_v^\infty \frac{e^{-x}}{x} dx \qquad (30)
$$

called exponential integral.

Solution of the Darcy Equation for Cylindrical Symmetry



Solution of the Darcy Equation for Cylindrical Symmetry

- The exponential integral can be approximated by

$$E_1(v) \approx -\ln(v) - 0.5772 \qquad (31)$$

for $v \ll 1$.

- $E_1(v) \to 0$ for $u \to \infty$

↓

$$E_1(v) \approx \max\{-\ln(v) - 0.5772, 0\} \qquad (32)$$

is a reasonable approximation if E_1 is not available.

Solution of the Darcy Equation for Cylindrical Symmetry

Meaning of $G(u)$:

$$\begin{aligned} G(u) &= u\frac{\partial}{\partial u}F(u) = r\frac{\partial}{\partial r}p(r,t) \\ &= -r\,\frac{\eta}{k}\,v(r,t) = -\frac{\eta}{2\pi kl}\,(2\pi r\,l\,v(r,t)) \end{aligned} \tag{33}$$

In the limit $r \to 0$ (at the walls of a thin well):

$$G(0) = -\frac{\eta}{2\pi kl}Q = a \tag{34}$$

with the total rate of injection

$$Q = 2\pi r\,l\,v(r,t) \tag{35}$$

(volume per time, $Q < 0$ for extraction)

Solution of the Darcy Equation for Cylindrical Symmetry

$$\Downarrow$$

$$F(u) = -\frac{a}{2} E_1(u^2) = \frac{\eta}{4\pi k l} Q E_1(u^2) \tag{36}$$

$$\Downarrow$$

$$p(r,t) = \frac{\eta}{4\pi k l} Q E_1\left(\frac{r^2}{4L(t)^2}\right) = \frac{\eta}{4\pi k l} Q E_1\left(\frac{r^2}{4\tilde{\kappa} t}\right) \tag{37}$$

Well Doublets

$\tilde{\kappa} \gtrapprox 1\,\frac{\mathrm{m}^2}{\mathrm{s}}$ for highly permeable rocks ($k \gtrapprox 0.01\,\mathrm{D}$) required for hydrothermal systems if the rock is fully saturated with water.

↓

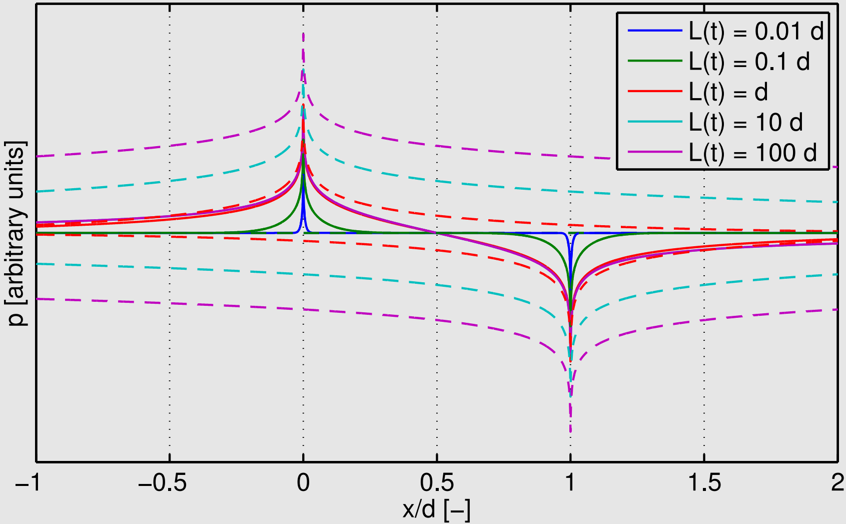
Pressure rapidly decreases around an extraction well.

Solution: Use a well doublet consisting of an injection well and an extraction well working at the same flow rate.

$$p(x,y,t) \;=\; \frac{\eta Q}{4\pi k l}\left(E_1\left(\frac{r_i^2}{4\tilde{\kappa}t}\right) - E_1\left(\frac{r_e^2}{4\tilde{\kappa}t}\right)\right) \tag{38}$$

where $r_{i/e}$ is the distance of the considered point from the injection / extraction well.

Pressure Distribution of a Well Doublet Compared to a Single Well



Well Doublets

Use the approximation

$$E_1(v) \approx -\ln(v) - 0.5772 \quad \text{for} \quad v \ll 1 \tag{39}$$

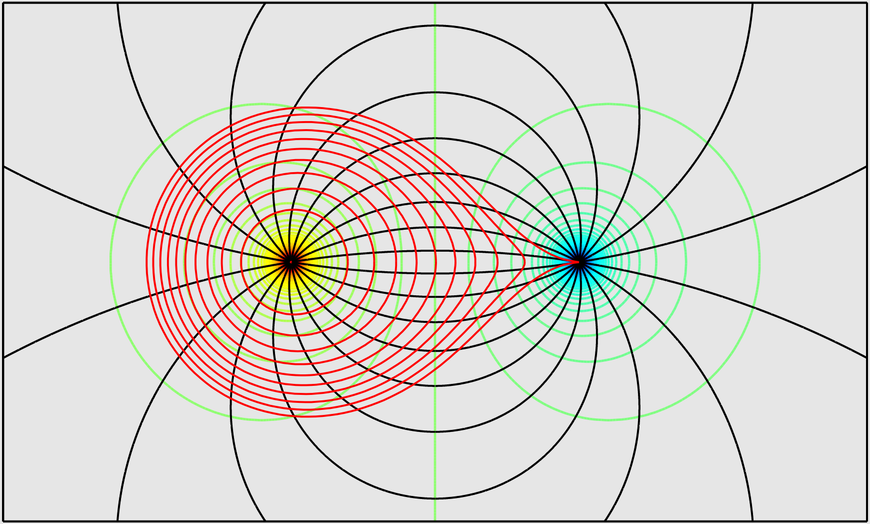
$$\Downarrow$$

$$p(x,y,t) \approx \frac{\eta Q}{4\pi k l}\left(-\ln\left(\frac{r_i^2}{4\tilde{\kappa}t}\right) + \ln\left(\frac{r_e^2}{4\tilde{\kappa}t}\right)\right) \tag{40}$$

$$= \frac{\eta Q}{2\pi k l}\ln\frac{r_e}{r_i} \tag{41}$$

is independent of t (steady-state flow conditions).

Pressure and Flow Lines of a Simple Well Doublet



The Simplest Model for a Well Doublet

Limitation: In principle only valid

- for confined aquifers or
- if the horizontal distance of the wells is much smaller than the open borehole length l

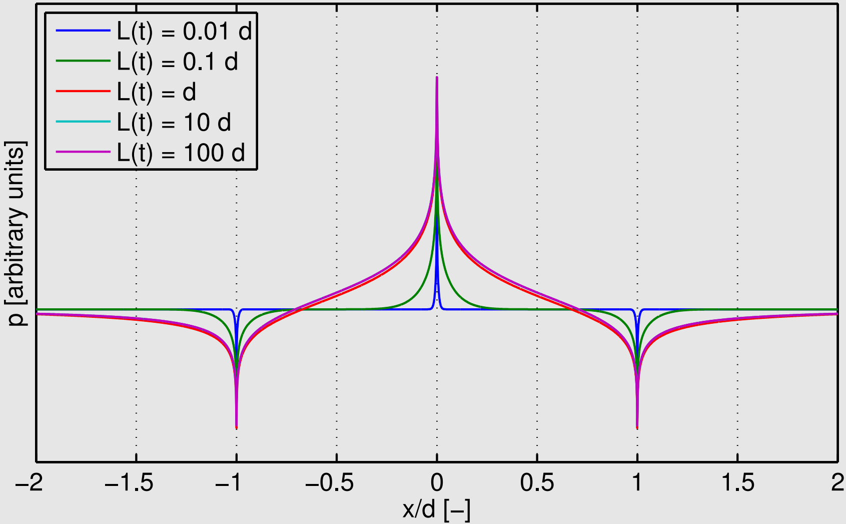
Mechanical power required for maintaining the flow:

$$P = (p_i - p_e)\, Q \tag{42}$$

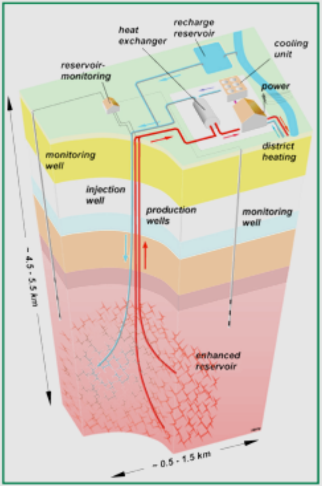
where

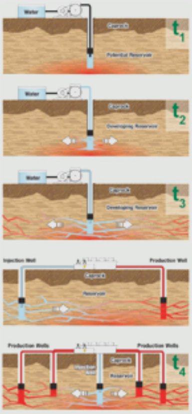
$$\begin{aligned} p_i &= \text{pressure at the walls of the injection well} \\ p_e &= \text{pressure at the walls of the extraction well} \end{aligned}$$

Well Triplet



Hydraulic Fracturing for Increasing the Permeability





Drill a well to explore

Inject water to cause slip on faults (high water pressure pushes fractures open)

Injection extends a network of connected fractures

Inject water to sweep heat to a production well

Maximize production rate and lifetime

Source: NewEnergyNews

Enhanced Geothermal Systems

Environmental Issues related to Hydraulic Fracturing

- Large amounts of contaminated water if fracturing is supported by additional chemicals
- Fluid-induced seismicity

Mechanisms of Heat Transport in Porous Media

Solid matrix: conduction

Fluid: conduction and advection

Heat Exchange Between Fluid and Matrix

Length scale of heat conduction:

$$L(t) = \sqrt{\kappa t} \tag{43}$$

Water: $\kappa = 1.4 \times 10^{-7} \, \frac{\mathrm{m}^2}{\mathrm{s}}$

Rocks: $\kappa \approx 10^{-6} \, \frac{\mathrm{m}^2}{\mathrm{s}}$

$\Downarrow$

Fluid and matrix rapidly adjust to the same temperature locally.

The Heat Equation for a Fluid

Heat flux density for a fluid moving at a velocity $\vec{v}$:

$$\vec{q} = \underset{\text{conduction}}{\underset{\uparrow}{-\lambda \nabla T}} + \underset{\text{advection}}{\underset{\uparrow}{\rho c T \vec{v}}} \tag{44}$$

$$\Downarrow$$

$$\rho c \frac{\partial T}{\partial t} = -\mathrm{div}(\vec{q}) \tag{45}$$

$$= \mathrm{div}\left(\lambda \nabla T - \rho c T \vec{v}\right) \tag{46}$$

The Heat Equation for a Porous Medium

$$\left(\rho_m c_m + \phi \rho_f c_f\right) \frac{\partial T}{\partial t} \;=\; \operatorname{div}\left(\left(\lambda_m + \phi \lambda_f\right) \nabla T - \rho_f c_f T \vec{v}\right) \qquad (47)$$

where

$$\begin{aligned}
\rho_f, c_f, \lambda_f &= \text{parameters of the fluid} \\
\rho_m, c_m, \lambda_m &= \text{parameters of the dry matrix (not the solid!)} \\
\phi &= \text{porosity} \\
\vec{v} &= \text{Darcy velocity}
\end{aligned}$$

⬇

Effective velocity of heat advection:

$$\vec{v}_a \;=\; \frac{\rho_f c_f}{\rho_m c_m + \phi \rho_f c_f} \vec{v} \;\approx\; 1.7\, \vec{v} \qquad (48)$$

Velocities of Fluid Flow and Heat Transport

Mean interstitial velocity of the water particles

$$\vec{v}_p = \frac{\vec{v}}{\phi} \tag{49}$$

is significantly higher than the flow rate (Darcy velocity) $\vec{v}$.

Effective velocity of heat advection

$$\vec{v}_a = \frac{\rho_f c_f}{\rho_m c_m + \phi \rho_f c_f} \vec{v} \approx 1.7 \, \vec{v} \tag{50}$$

is also higher than the flow rate $\vec{v}$, but lower than the mean interstitial velocity $\vec{v}_p$.

Velocities of Fluid Flow and Heat Transport

$$\vec{v}_a = \frac{\phi \rho_f c_f}{\rho_m c_m + \phi \rho_f c_f} \vec{v}_p = \frac{\vec{v}_p}{R} \quad (51)$$

where

$$R = \frac{\rho_m c_m + \phi \rho_f c_f}{\phi \rho_f c_f} = 1 + \frac{\rho_m c_m}{\phi \rho_f c_f} \quad (52)$$

is the coefficient of retardation.

$$\Downarrow$$

Water circulates R times between the wells until the cold temperature front breaks through (if heat conduction is neglected).