

Geothermics and Geothermal Energy Geothermal Heating Systems

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Dependencies

The actual heating demand of a building depends on

- the thermal efficiency of the building
- the climatic conditions
- the number and the behavior of the residents.

Data Requirements

Fired heating systems (oil, gas, biofuels, ...): long-term mean power (yearly) and peak power

Geothermal and solar heating systems: time-resolved data (e. g., on a monthly scale)

Thermal Efficiency

The **U value** of an element of the building's surface (wall, window, ...) quantifies the heat flux density per temperature difference:

$$U = \frac{q}{T_i - T_o} \quad (1)$$

where

q = heat flux density = power per area [$\frac{W}{m^2}$]

T_i = inside temperature [K]

T_o = outside temperature [K]

Unit: $\frac{W}{m^2K}$

Thermal Efficiency

The R value of an element of the building's surface is

$$R = \frac{1}{U} = \frac{T_i - T_o}{q} = \frac{d}{\lambda} \quad (2)$$

for a homogeneous material of thickness d and thermal conductivity λ .

Actual required heating power for the entire building:

$$P = \sum_j q_j A_j = \left(\begin{array}{c} T_i \\ \uparrow \\ \text{residents} \end{array} - \begin{array}{c} T_o \\ \uparrow \\ \text{climate} \end{array} \right) \sum_j \begin{array}{c} U_j A_j \\ \uparrow \\ \text{building} \end{array} \quad (3)$$

for $T_i > T_o$ where the A_j are the surface areas of the elements.

The Influence of the Climate

Total energy required for heating during a given time span:

$$E = \int_{T_i > T_o} (T_i - T_o(t)) dt \sum_j U_j A_j \quad (4)$$

The integral

$$\text{HDD} = \int_{T_i > T_o} (T_i - T_o(t)) dt \quad (5)$$

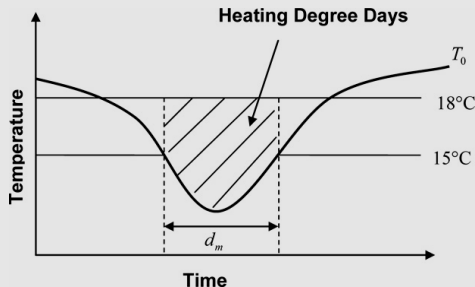
for a fixed value of T_i (independent of the specific residents' behavior) measured in units of (K days) is called **number of heating degree days** within the given time span.

Several slightly different ways of calculating the HDD.

The Influence of the Climate

Definition established in the EU:

- $T_i = 18^\circ\text{C}$
- Use mean temperatures over one day periods for T_o instead of continuous time.
- Take into account only days below a heating threshold of 15°C .



Source: Global CCS Institute

The Influence of the Climate

Total energy required for heating during a given time span:

$$E = \text{HDD} \sum_j U_j A_j \quad (6)$$

- Obtained unit is W days.
- If the total energy demand, $\sum E$, and total HDD, $\sum \text{HDD}$, for one year are given instead of $\sum_j U_j A_j$:

$$E = \sum E \frac{\text{HDD}}{\sum \text{HDD}} \quad (7)$$

Why?

Domestic heating systems require $T \geq 35^\circ\text{C}$ (old systems even much more)



Cannot be achieved by shallow geothermal systems.



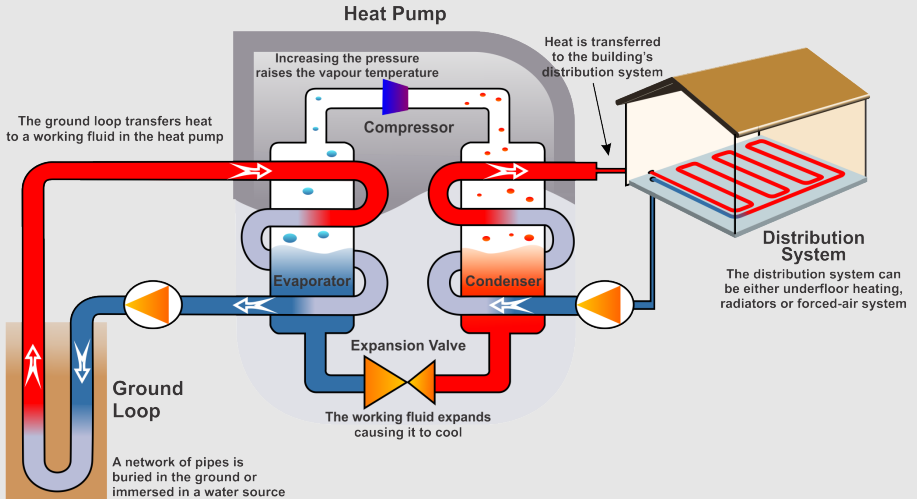
Temperature of the fluid in the heat exchanger must be increased using mechanical work.

Principle

Heat engine: heat (hot reservoir) \rightarrow mechanical work + heat (cold res.)

Heat pump: heat (cold reservoir) + mechanical work \rightarrow heat (hot res.)

Principle

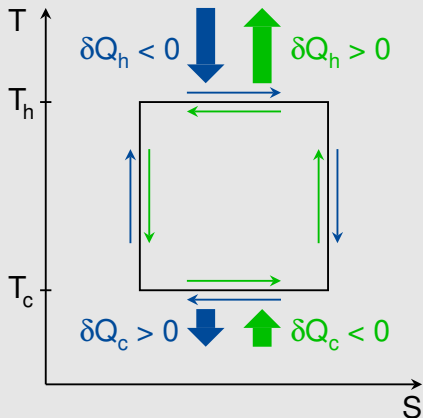


Source: Geothermal heat-pump association of New Zealand

Carnot Cycle and Inverse Carnot Cycle

Carnot cycle

Inverse Carnot cycle



Directions:

- isothermal expansion
(coupled to large reservoir)
- ← isothermal compression
(coupled to large reservoir)
- ↓ isentropic cooling
(by rapid expansion)
- ↑ isentropic heating
(by rapid compression)

The Thermodynamic Limit of the Carnot Cycles

$$\delta S = \frac{\delta Q_h}{T_h} + \frac{\delta Q_c}{T_c} \geq 0 \quad (8)$$

where

δQ_h = thermal energy supplied to to the hot system

T_h = temperature of the hot system

δQ_c = thermal energy supplied to the cold system

T_c = temperature of the cold system

$\delta Q < 0$ describes extraction of energy from the system.

The Thermodynamic Limit of a Geothermal Heating System

Hot system: heating system, $\delta Q_h > 0$

Cold system: geothermal reservoir, $\delta Q_c < 0$

Thermodynamic limit of the heat pump (Eq. 8) written in terms of total power P_{tot} (to the heating system) and thermal power P_{th} (from the geothermal reservoir):

$$\frac{P_{\text{tot}}}{T_h} - \frac{P_{\text{th}}}{T_c} \geq 0 \quad (9)$$

The Thermodynamic Limit of a Geothermal Heating System

The difference between P and P_{th} must be supplied as mechanical (electrical) power by the compressor, $P_{el} = P_{tot} - P_{th}$.

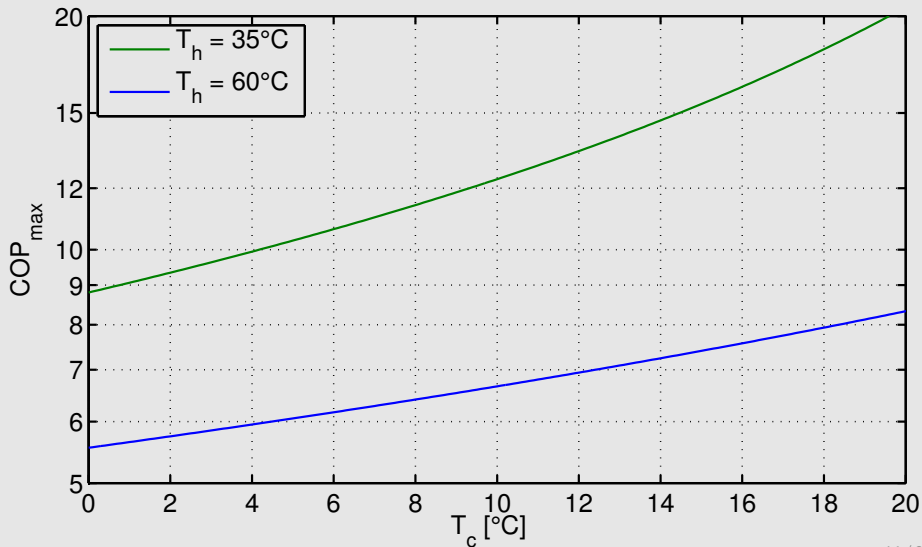
$$\frac{P_{tot}}{T_h} - \frac{P_{tot} - P_{el}}{T_c} \geq 0 \quad (10)$$

$$P_{tot} \leq \frac{T_h}{T_h - T_c} P_{el} \quad (11)$$

The Coefficient of Performance

$$\text{COP} = \frac{P_{tot}}{P_{el}} \leq \frac{T_h}{T_h - T_c} \quad (12)$$

The Upper Thermodynamic Limit of the COP



The COP of Real Heat Pumps

- Real heat pumps achieve a significantly lower performance than the thermodynamic limit, e. g., $\text{COP} = 5$ is very good for $T_h = 35^\circ\text{C}$ and $T_c = 0^\circ\text{C}$ (instead of $\text{COP}_{\text{max}} = 8.8$).
- Data sheets with the COP for different temperatures are provided by some suppliers.
- If not, use the concept of relative efficiency.

The Relative Efficiency

General concept:

$$\eta = \eta_{\max} \eta_{rel} \quad (13)$$

where

η = total efficiency (output/input)

η_{\max} = theoretically possible maximum efficiency

η_{rel} = relative efficiency of the specific device

For a heat pump: η_{\max} defined by the thermodynamic limit (inverse Carnot Cycle)

The Relative Efficiency

Two ways to apply the concept of relative efficiency to a heat pump:

Electrical power \rightarrow total power:

$$\text{COP} = \frac{P_{\text{tot}}}{P_{\text{el}}} = \eta_{\text{rel}} \left(\frac{P_{\text{tot}}}{P_{\text{el}}} \right)_{\text{max}} = \eta_{\text{rel}} \frac{T_h}{T_h - T_c} \quad (14)$$

Electrical power \rightarrow thermal power:

$$\frac{P_{\text{th}}}{P_{\text{el}}} = \eta_{\text{rel}} \left(\frac{P_{\text{th}}}{P_{\text{el}}} \right)_{\text{max}} = \eta_{\text{rel}} \left(\frac{P_{\text{tot}} - P_{\text{el}}}{P_{\text{el}}} \right)_{\text{max}} \quad (15)$$

$$= \eta_{\text{rel}} \left(\frac{T_h}{T_h - T_c} - 1 \right) = \eta_{\text{rel}} \frac{T_c}{T_h - T_c} \quad (16)$$

The Relative Efficiency



$$\text{COP} = \frac{P_{\text{tot}}}{P_{\text{el}}} = 1 + \frac{P_{\text{th}}}{P_{\text{el}}} = 1 + \eta_{\text{rel}} \frac{T_c}{T_h - T_c} \quad (17)$$

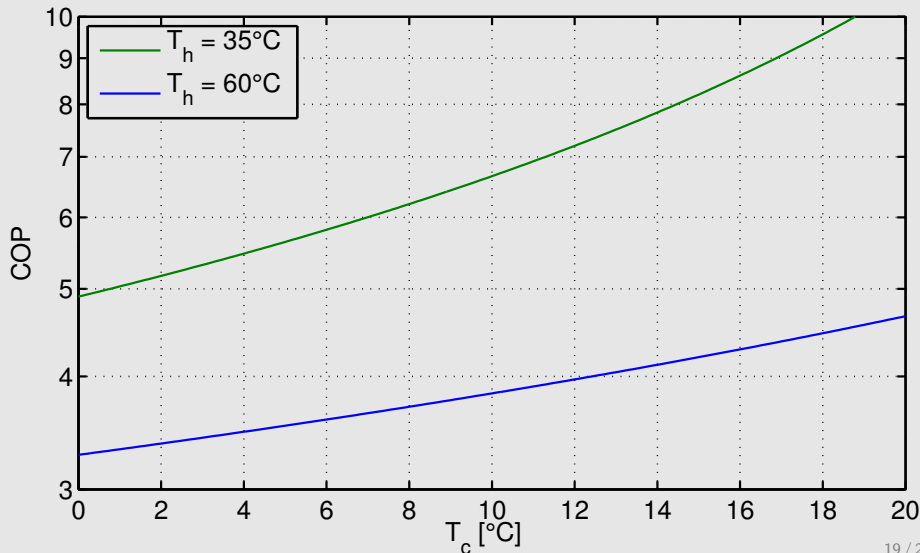
- The second concept is physically better.



Use it in the following.

- Typical value: $\eta_{\text{rel}} \approx 0.5$ for good heat pumps

Typical COP of a Good Heat Pump ($\eta_{rel} = 0.5$)



Actual Energy Prices in Germany

Energy source	Price [$\frac{\text{ct}}{\text{kWh}}$]
wood	6
gas	6
oil	7
electricity	28



Heat pump driven by electricity makes sense only if COP \gtrsim 4.5 in the mean.

Alternative: gas heat pump

The Heat Pump in Geothermal Calculations

- P_{tot} given by the heating demand
- P_{th} required for the calculation of the geothermal system
- P_{el} required for calculating the costs of heating

$$P_{\text{el}} = \frac{P_{\text{tot}}}{1 + \eta_{\text{rel}} \frac{T_c}{T_h - T_c}} \quad (18)$$

$$\begin{aligned} P_{\text{th}} &= P_{\text{tot}} - P_{\text{el}} = P_{\text{tot}} - \frac{P_{\text{tot}}}{1 + \eta_{\text{rel}} \frac{T_c}{T_h - T_c}} \\ &= \frac{P_{\text{tot}}}{1 + \frac{1}{\eta_{\text{rel}}} \frac{T_h - T_c}{T_c}} \end{aligned} \quad (19)$$