Geothermics and Geothermal Energy
Closed Geothermal Systems

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Closed Geothermal Systems

Principle

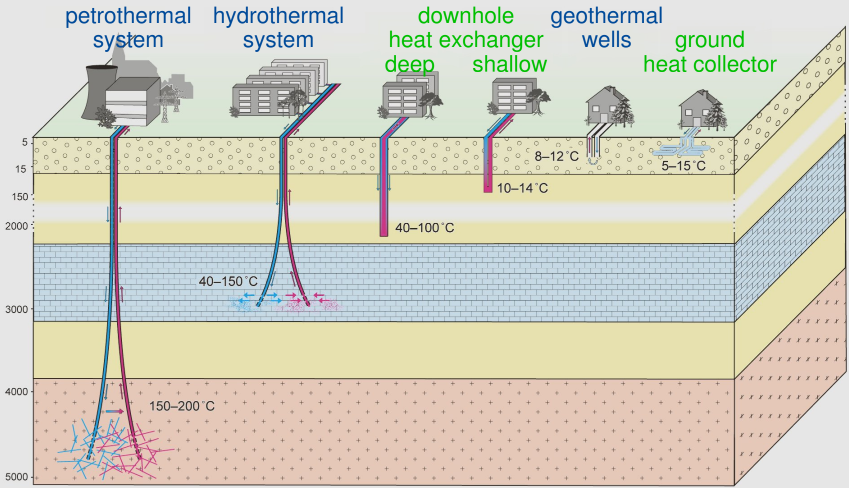
- Fluid circulates in a closed heat exchanger.
- Heat is transported to the fluid by heat conduction.
- Heat transport in the surrounding rock or soil by heat conduction; in some cases also by advection (groundwater).

Fluids

- Water or alcohol-water mixtures.
- Water has the best properties (heat capacity, thermal conductivity, viscosity) as long as $T > 0^\circ\mathrm{C}$.

Closed Geothermal Systems

Types of Geothermal Systems



Source: Ingolstädter Kommunalbetriebe (modified)

Closed Geothermal Systems

Limitation

Heat is transported to the heat exchanger by conduction.

↓

Requires a temperature gradient towards the exchanger.

↓

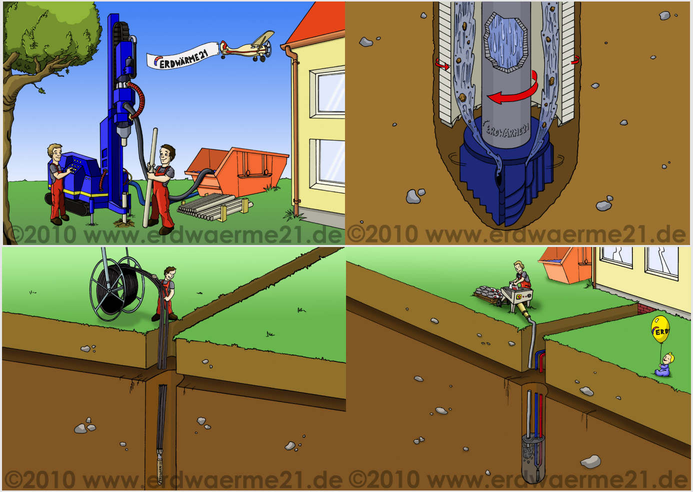
Temperature in the exchanger is lower than the undisturbed subsurface temperature; temperature drop depends on

- extracted power
- thermal properties of the subsurface (mainly the thermal conductivity)
- properties of the exchanger (size, shape, material)

↓

Production of electricity is economically not reasonable (so far?).

Downhole Heat Exchangers (Borehole Heat Exchangers)

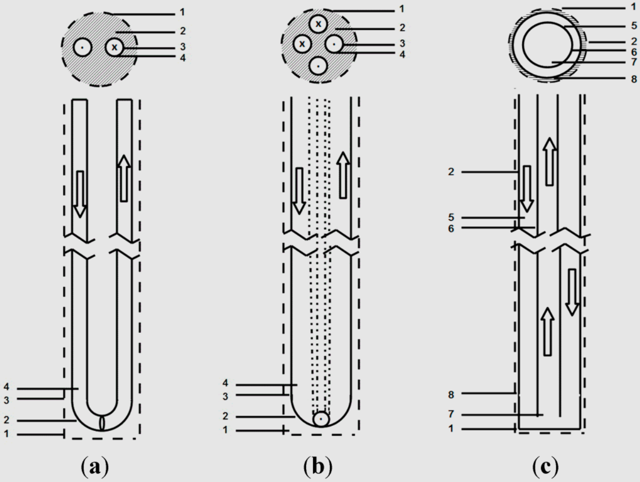


Source: Erdwärme21

Downhole Heat Exchangers (Borehole Heat Exchangers)

Source: Baublog: Villa Lugana in Teltow

Downhole Heat Exchangers (Borehole Heat Exchangers)



(a) single U-tube
(b) double U-tube
(c) coaxial
1 borehole wall
2 grout
3 interior of U-tube
4 U-tube wall
5 annulus space
6 inner column (pipe)
7 interior of inner column (pipe)
8 external column

Source: Sliwa & Rosen (2015), *Sustainability*, 7(10), 13104, doi:10.3390/su71013104

Application of Downhole Heat Exchangers

Shallow heat exchangers ($d \lessapprox 400\,\mathrm{m}$, $T \lessapprox 25^\circ\mathrm{C}$): heating of buildings with the help of heat pumps

Deep heat exchangers ($d \gtrapprox 1000\,\mathrm{m}$, $T \gtrapprox 40^\circ\mathrm{C}$): direct heating

- down to depths of about 3000 m so far (coaxial type only)
- mostly reuse or deepen abandoned hydrocarbon boreholes
- economically still questionable

Geothermal Heat Exchangers

Ground Heat Collectors

Source: www.bauweise.net

Ground Heat Collectors

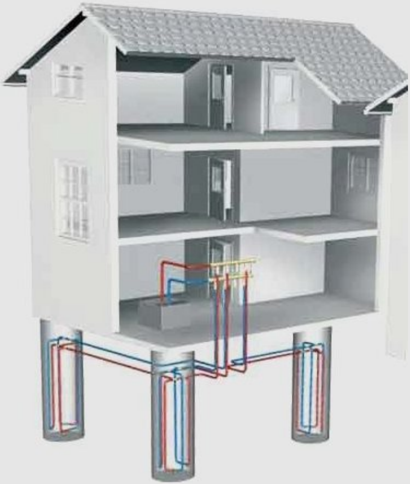
Source: Rehau AG & Co

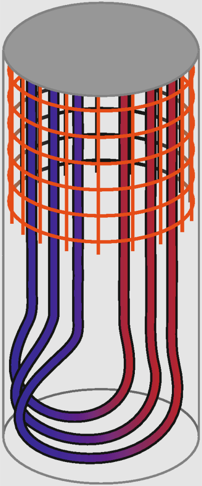
Geothermal Baskets

Source: Heizungsjournal

Geothermal Heat Exchangers

Geothermal Energy Piles





Sources: Energy Systems Research Unit, University of Strathclyde; Stober & Bucher, Geothermie

Superposition of Solutions

The heat conduction equation is linear.

$$\Downarrow$$

Can be solved by superposing individual components:

$$T(\vec{x}, t) \;=\; T_m(\vec{x}) + T_y(\vec{x}, t) + T_1(\vec{x}, t) + T_2(\vec{x}, t) + \ldots \tag{1}$$

with

$$\begin{aligned} T_m(\vec{x}) &= \text{steady-state geotherm} \\ T_y(\vec{x}, t) &= \text{natural seasonal variation} \\ T_i(\vec{x}, t) &= \text{temperature change caused by the } i^{\text{th}} \text{ heat exchanger,} \\ & \text{usually} < 0 \end{aligned}$$

We use T instead of T_i for the rest of the chapter.

Analytical Approximations

All three components can be approximated by analytical solutions of the heat conduction equation reasonably well in most cases.

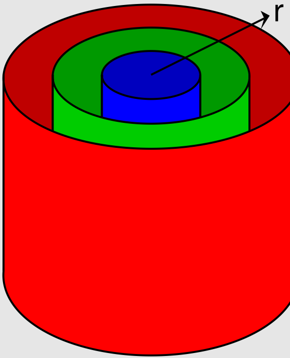
↓

No need for numerical simulations and / or specific software

Analytical solutions use

- symmetries for reducing the spatial dimension (mainly from 3 to 1) and
- scaling properties (length vs. time).

Cylindrical Symmetry



$T(x, y, z, t)$ only depends on $r = \sqrt{x^2 + y^2}$ and t.

Limitations:

- Fluid temperature in the heat exchanger must increase with the geothermal gradient.

  ↓

  Only applicable to shallow boreholes and to deep coaxial heat exchangers under specific conditions.
- Seasonal temperature variation in the upper region cannot be taken into account.
- Borehole length l must be much larger than $L(t) = \sqrt{\kappa t}$.

Analogy to Darcy Flow

Pressure around an injection or extraction well:

$$p(r,t) = \frac{\eta}{4\pi kl}\, Q\, E_1\left(\frac{r^2}{4L(t)^2}\right) = \frac{\eta}{4\pi kl}\, Q\, E_1\left(\frac{r^2}{4\tilde{\kappa} t}\right) \tag{2}$$

with

$$\begin{aligned}
\eta &= \text{dynamic viscosity of the fluid } [\mathrm{Pa\,s}] \\
k &= \text{hydraulic permeability } [\mathrm{m}^2] \\
l &= \text{length of the well } [\mathrm{m}] \\
Q &= \text{rate of injection } [\tfrac{\mathrm{m}^3}{\mathrm{s}}] \\
\tilde{\kappa} &= \text{diffusivity } [\tfrac{\mathrm{m}^2}{\mathrm{s}}]
\end{aligned}$$

and the exponential integral $E_1(v) = \int\limits_v^\infty \frac{e^{-x}}{x} dx$

Analogy to Darcy Flow

Equivalence of properties:

	Darcy flow	Heat conduction
variable	p	T
parameters	$\rho_f \frac{k}{\eta}$	λ
	S	ρc
	$\tilde{\kappa} = \frac{\rho_f k}{\eta S}$	$\kappa = \frac{\lambda}{\rho c}$
balanced property	mass	energy
total input / output	$\rho_f Q$	P (power)

↓

$$T(r,t) = \frac{P}{4\pi\lambda l} E_1\left(\frac{r^2}{4L(t)^2}\right) = \frac{P}{4\pi\lambda l} E_1\left(\frac{r^2}{4\kappa t}\right) \tag{3}$$

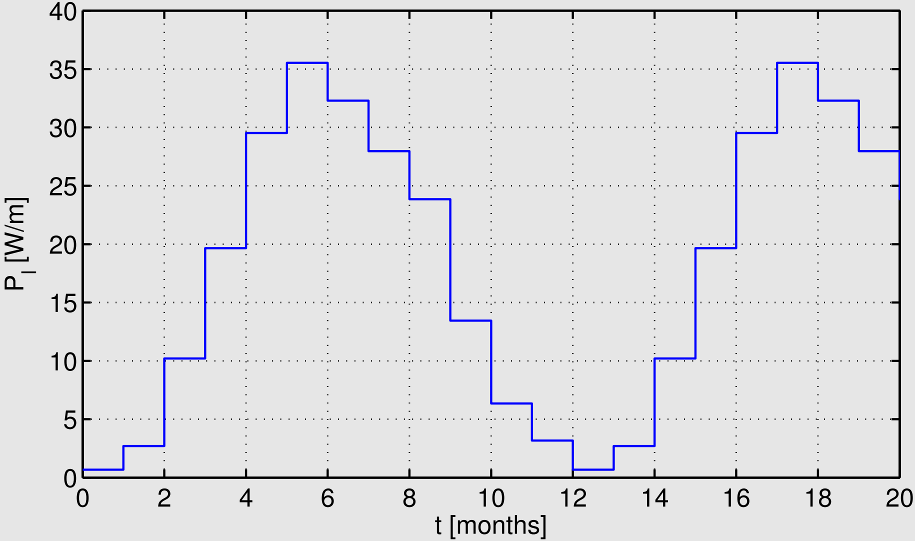
Solution for Constant Heat Extraction

Define $P_l = -\frac{P}{l}$ as the extracted power per borehole length.

$$\Downarrow$$

$$T(r,t) = -\frac{P_l}{4\pi\lambda} E_1\left(\frac{r^2}{4L(t)^2}\right) = -\frac{P_l}{4\pi\lambda} E_1\left(\frac{r^2}{4\kappa t}\right) \tag{4}$$

Solution for Step-Like Heat Extraction



Solution for Step-Like Heat Extraction

Superposition of several heat exchangers switched on at different times:

Month #	P_l [$\frac{W}{m}$]
1	0.7
2	2.7
3	10.2
4	19.7
5	29.5
6	35.5
7	32.3
8	28.0
9	23.8
10	13.4
11	6.3
12	3.2
...	...

Exchanger #	Starting time [mon]	P_l [$\frac{W}{m}$]
1	0	0.7
2	1	2.0
3	2	7.5
4	3	9.5
5	4	9.8
6	5	6.0
7	6	−3.2
8	7	−4.3
9	8	−4.2
10	9	−10.4
11	10	−7.1
12	11	−3.1
...	...	...

Solution for Step-Like Heat Extraction

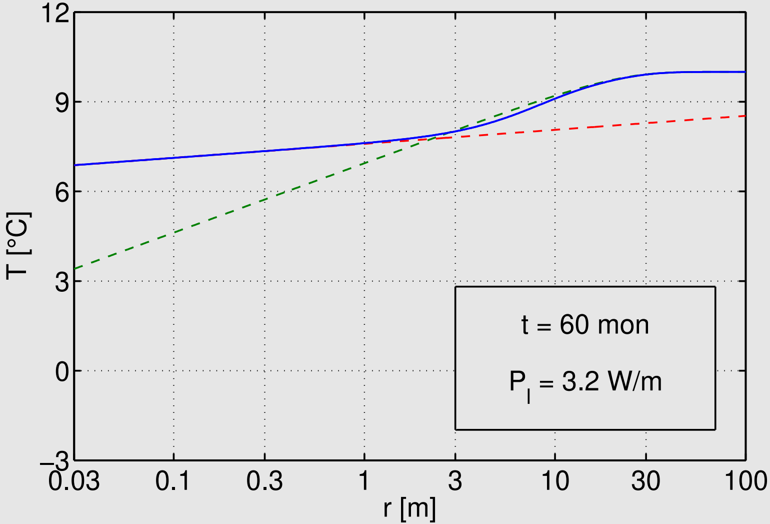
Formally:

$$P_l(t) = \begin{cases} 0 & \text{for} \quad t < t_0 \\ P_{l,i} & \quad\quad\; t_{i-1} \leq t < t_i \end{cases} \tag{5}$$

$$\Downarrow$$

$$\begin{aligned} T(r,t) &= -\frac{P_{l,1}}{4\pi\lambda} E_1\left(\frac{r^2}{4\kappa(t-t_0)}\right) \\ &- \frac{(P_{l,2}-P_{l,1})}{4\pi\lambda} E_1\left(\frac{r^2}{4\kappa(t-t_1)}\right) \\ &- \ldots \\ &- \frac{(P_{l,n+1}-P_{l,n})}{4\pi\lambda} E_1\left(\frac{r^2}{4\kappa(t-t_n)}\right) \end{aligned} \tag{6}$$

Solution for Step-Like Heat Extraction



Solution for Step-Like Heat Extraction

- Behavior for $r \gg L(t)$ is determined by the long-term mean heat extraction $\overline{P}_l$:

$$T(r,t) \approx -\frac{\overline{P}_l}{4\pi\lambda} E_1\left(\frac{r^2}{4L(t)^2}\right) \qquad (7)$$

- Behavior for $r < L(\delta t)$ is determined by the actual heat extraction P_l if P_l has been constant for a time interval δt:

$$T(r,t) \approx -\frac{P_l}{4\pi\lambda} E_1\left(\frac{r^2}{4L(\delta t)^2}\right) + f(t) \qquad (8)$$

$$\approx \frac{P_l}{4\pi\lambda}\left(\ln\left(\frac{r^2}{4L(\delta t)^2}\right) + 0.5772\right) + f(t) \qquad (9)$$

where the function $f(t)$ depends on the history of P_l.

The Thermal Resistance

Consider two boreholes of different radii r_1 and r_2.

↓

$$T(r_1, t) - T(r_2, t) = \frac{P_l}{4\pi\lambda}\left(\ln\left(\frac{r_1^2}{4L(\delta t)^2}\right) - \ln\left(\frac{r_2^2}{4L(\delta t)^2}\right)\right) \quad (10)$$

$$= \frac{P_l}{2\pi\lambda}\ln\left(\frac{r_1}{r_2}\right) \quad (11)$$

for $r_1 \ll L(\delta t)$ and $r_2 \ll L(\delta t)$.

↓

Temperature difference is proportional to the actual P_l.

The Thermal Resistance

Basically the same result for the effect of the borehole's filling and the walls of the heat exchanger:

$$T_f(t) = T(r_b, t) - R\,P_l \tag{12}$$

where

$$\begin{aligned} T_f(t) &= \text{temperature of the fluid in the heat exchanger} \\ r_b &= \text{radius of the borehole} \\ R &= \text{resistance of the borehole / heat exchanger} \end{aligned}$$

- Borehole resistance depends on the geometry (single U-tube, double U-tube, coaxial) and on the material used for filling.
- Typical values for double U-tube heat exchangers:
  $R_b \approx 0.1\,\frac{\mathrm{m\,K}}{\mathrm{W}}$ (standard filling)
  $R_b \approx 0.08\,\frac{\mathrm{m\,K}}{\mathrm{W}}$ (thermally improved filling)

Including the Thermal Resistance in the Calculation

Define an apparent borehole radius r_a in such a way that

$$T_f(t) = T(r_a, t) \tag{13}$$

↓

$$T(r_b, t) - T_f(t) = T(r_b, t) - T(r_a, t) = \frac{P_l}{2\pi\lambda} \ln\left(\frac{r_b}{r_a}\right) = R\,P_l \tag{14}$$

with

$$R = \frac{1}{2\pi\lambda} \ln\left(\frac{r_b}{r_a}\right) \tag{15}$$

↓

$$r_a = r_b\, e^{-2\pi\lambda R} \tag{16}$$

The Thermal Resistance of a Single Tube

Assume a single tube of outer radius r_b, wall thickness d and thermal conductivity λ_t (in general smaller than λ of the surrounding rock or soil).

$$\Downarrow$$

$$R = \frac{1}{2\pi\lambda_t} \ln\left(\frac{r}{r-d}\right) \tag{17}$$

or in terms of an apparent radius r_a:

$$r_a = r_b\, e^{-2\pi\lambda R} = r_b\, e^{-\frac{\lambda}{\lambda_t}\ln\left(\frac{r_b}{r_b-d}\right)} = r_b \left(\frac{r_b-d}{r_b}\right)^{\frac{\lambda}{\lambda_t}} \tag{18}$$

Thermal Response Tests

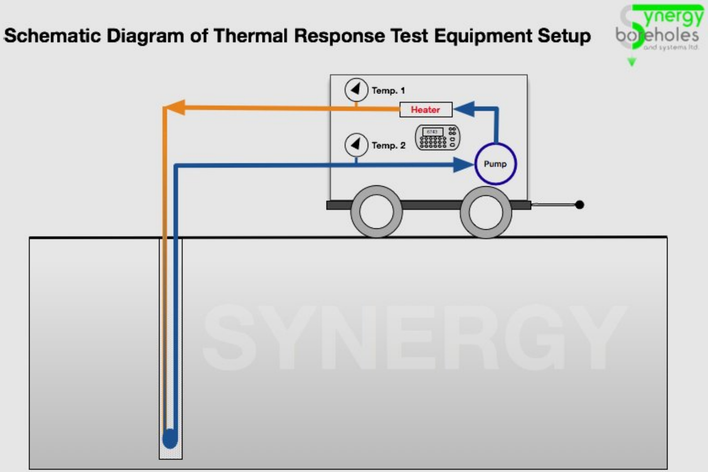
Principle

Measure how feeding thermal energy into the geothermal system changes the temperature.

Simplest Implementation

1. Let water circulate until its temperature is in equilibrium with the surrounding rock / soil.
2. Supply a well-defined thermal power to the water cycle and measure the temperature of the returning water (if possible, also in the borehole at different depths) through time.

Simplest Implementation



Source: Synergy boreholes and systems ltd.

Thermal Response Tests

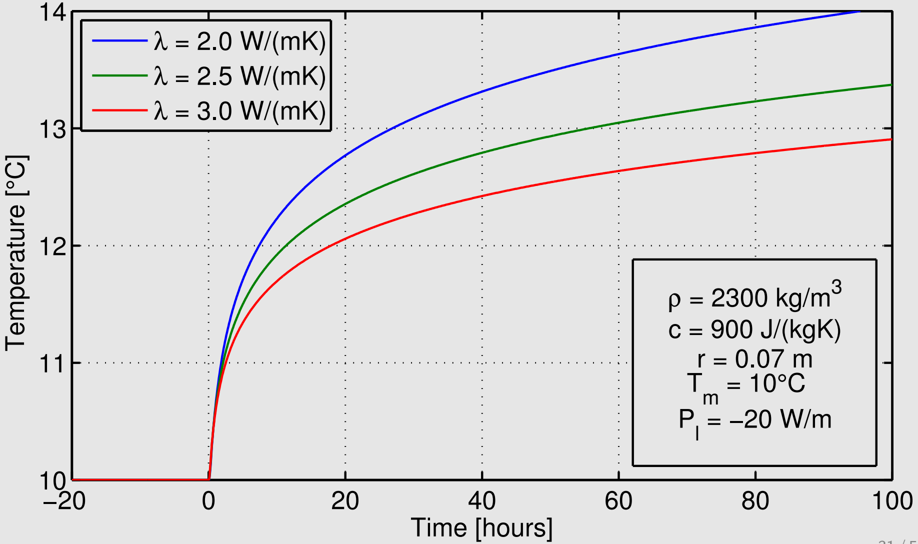
Result

- Thermal parameters of the surrounding rock / soil and of the borehole.
- Reveals whether the thermal properties are as assumed (e. g., whether filling has been done correctly).

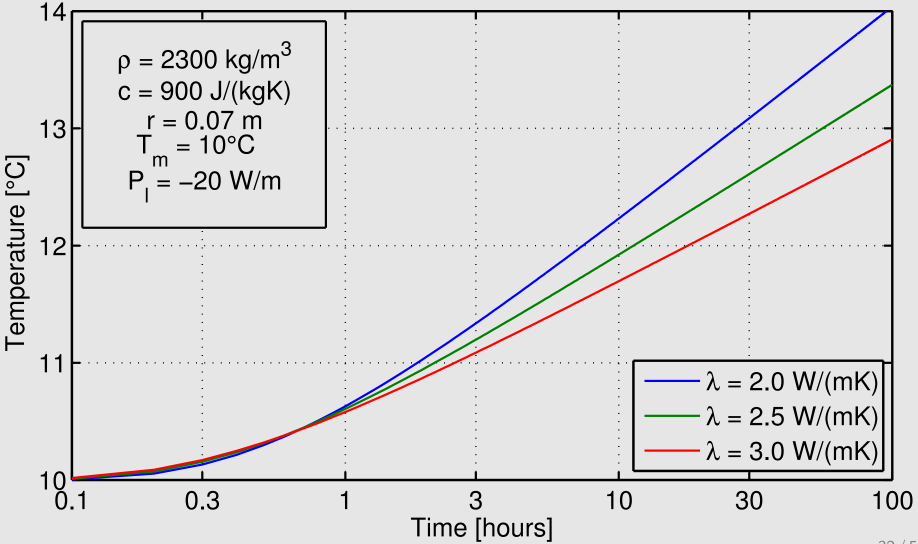
Limitation

Can only be applied after the heat exchanger has been installed.

Example of a Response Curve Without Thermal Resistance



Example of a Response Curve Without Thermal Resistance



Analysis of Thermal Response Tests

Fluid temperature relative to the undisturbed rock temperature for large times:

$$\begin{aligned}
T_f(t) &= T(r,t) - R\,P_l && (19)\\
&= -\frac{P_l}{4\pi\lambda}\,E_1\left(\frac{r^2}{4\kappa t}\right) - R\,P_l && (20)\\
&\approx \frac{P_l}{4\pi\lambda}\left(\ln\left(\frac{r^2}{4\kappa t}\right) + 0.5772\right) - R\,P_l && (21)\\
&= \frac{P_l}{4\pi\lambda}\left(\ln\left(\frac{r^2}{4\kappa}\right) - \ln t + 0.5772\right) - R\,P_l && (22)\\
&= \frac{-P_l}{4\pi\lambda}\ln t - P_l\left(R - \frac{\ln\left(\frac{r^2}{4\kappa}\right) + 0.5772}{4\pi\lambda}\right) && (23)
\end{aligned}$$

Thermal Response Tests

Analysis of Thermal Response Tests

1. Fit a straight line to the T_f vs. $\ln t$ data at large t. For a T_f vs. $\log t$ plot, use $\ln t = \ln 10 \log t$.
2. Compute λ from the slope.
3. Compute κ from λ (only possible if ρc is known).
4. Compute R from the offset.

Ground Heat Collectors

Modeling Approaches

- Infinite horizontal plane
- Set of parallel pipes

Too small

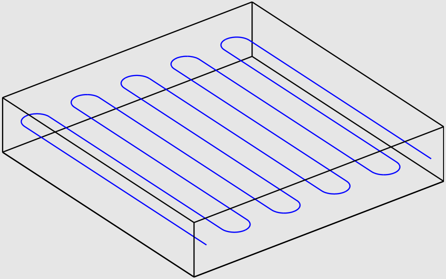
Source: www.bauweise.net

Large enough

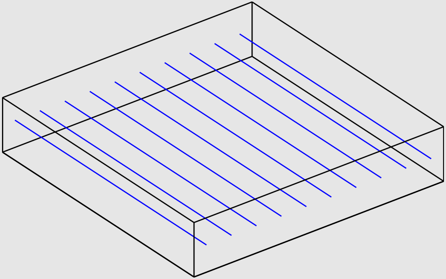
Source: Rehau AG & Co

Modeling as a Set of Parallel Pipes

Typical configuration



Simplified model

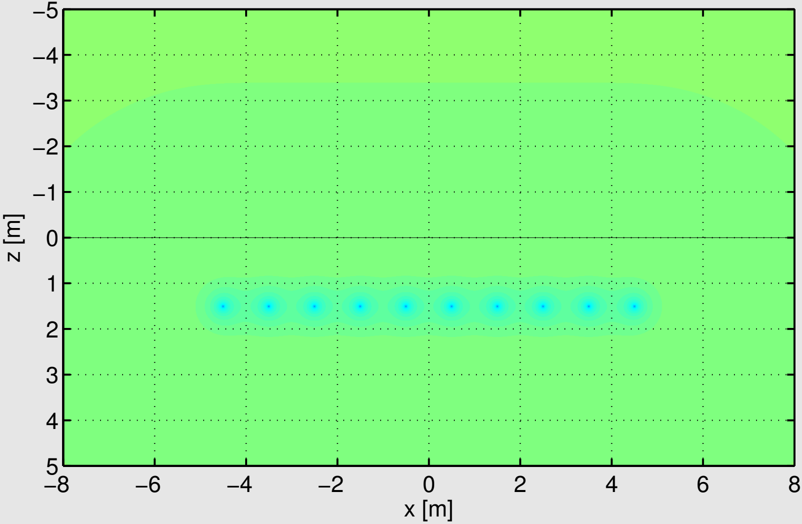


- Parallel horizontal pipes of infinite length

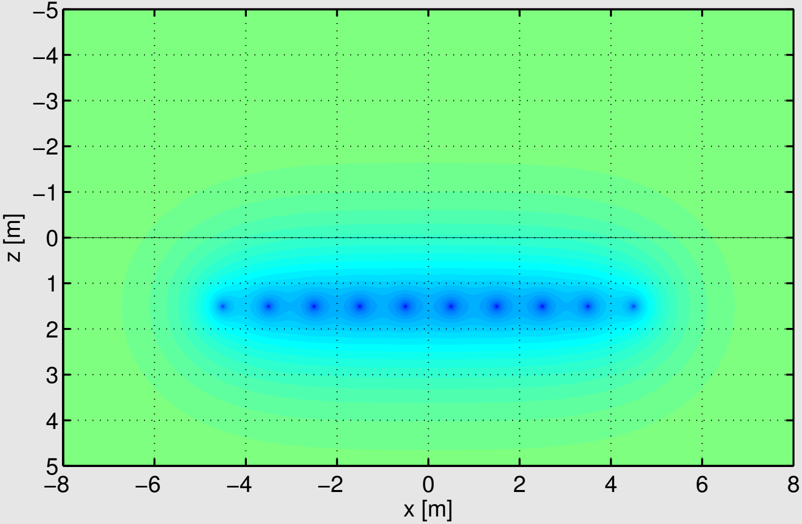
  ↓

  Theory of shallow downhole heat exchangers can be applied.
- Assume the same power per length for all pipes.

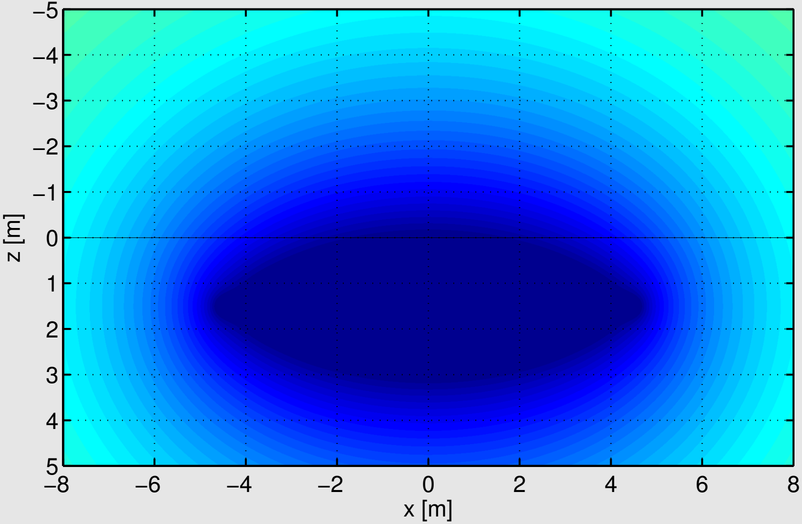
10 Parallel Pipes With 1 m Spacing in 1.5 m Depth After 3 Days



10 Parallel Pipes With 1 m Spacing in 1.5 m Depth After 1 Month



10 Parallel Pipes With 1 m Spacing in 1.5 m Depth After 1 Year



Ground Heat Collectors

Limitations

- Temperatures of the pipes are not the same.

↓

Same P_l for each pipe is not correct.

↓

Not a big problem, use mean temperature.

- Surface temperature is affected by the heat collector as if there was no surface, while solar radiation and rapid heat transport in the atmosphere keep the temperature more constant in reality.

↓

Simplest model: keep surface temperature constant by introducing virtual pipes with $-P_l$ (supplying energy) above the surface.

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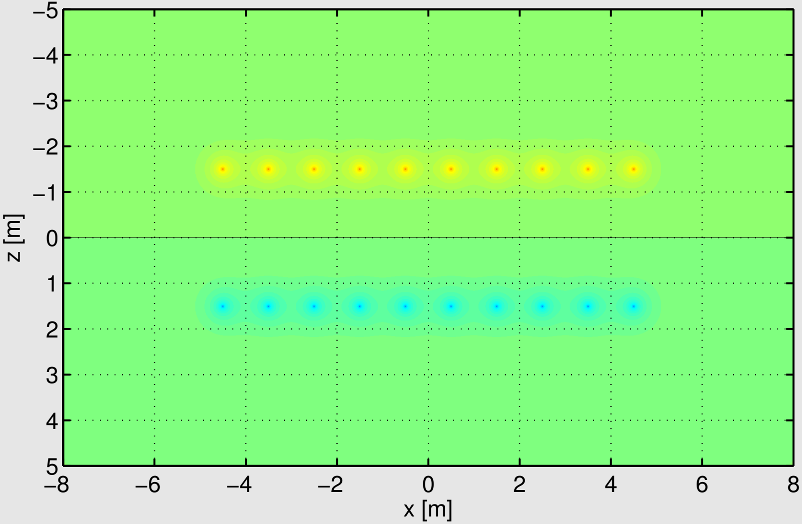

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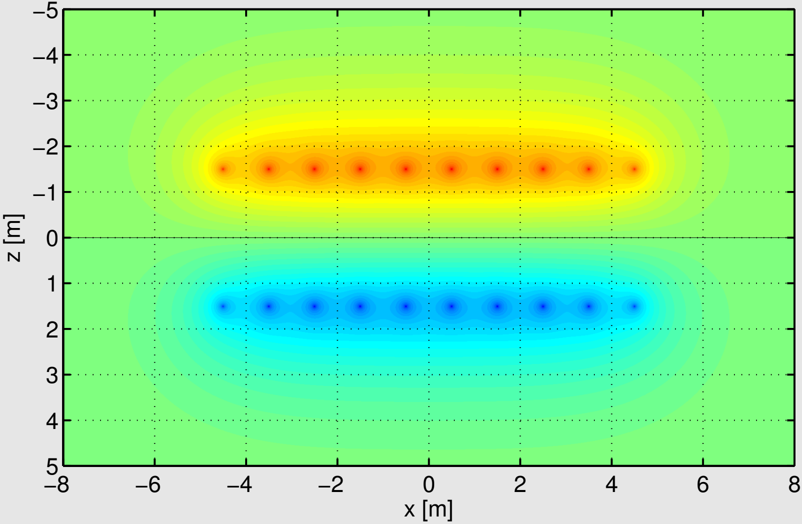

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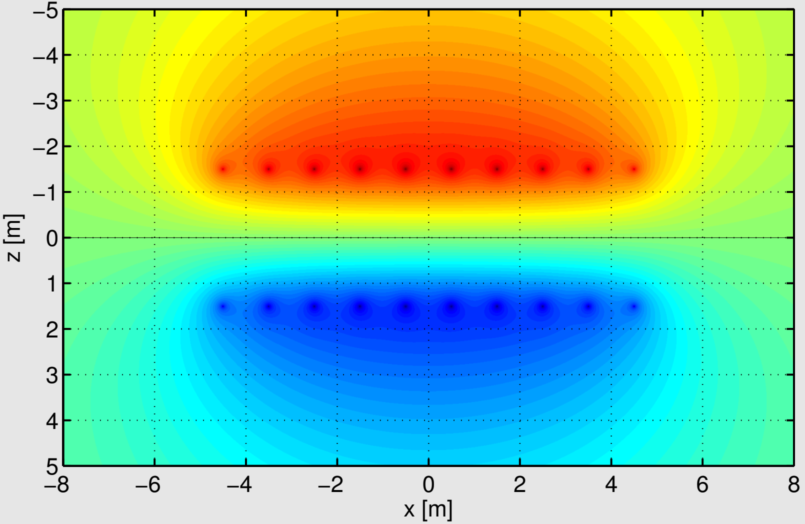


Computing the Average Temperature of Parallel Pipes

Assume n parallel pipes in a depth d below the surface and a horizontal spacing s.

Dist. r	Sign	Mean num.
r_a	$+$	1
s	$+$	$2\left(1-\frac{1}{n}\right)$
$2s$	$+$	$2\left(1-\frac{2}{n}\right)$
$3s$	$+$	$2\left(1-\frac{3}{n}\right)$
...	...	...

Distance r	Sign	Mean num.
$2d$	$-$	1
$\sqrt{s^2+(2d)^2}$	$-$	$2\left(1-\frac{1}{n}\right)$
$\sqrt{(2s)^2+(2d)^2}$	$-$	$2\left(1-\frac{2}{n}\right)$
$\sqrt{(3s)^2+(2d)^2}$	$-$	$2\left(1-\frac{3}{n}\right)$
...	...	...

Deep Downhole Heat Exchangers

Main Difference towards Shallow Heat Exchangers

Significant variation in temperature along the borehole

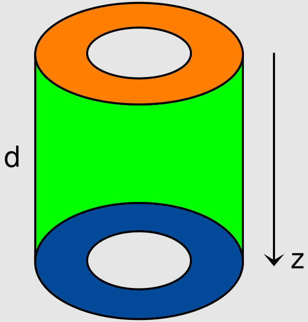
$\Downarrow$

Only coaxial heat exchangers can be used.

Main Field of Application

Direct (district) heating without heat pumps.

Energy Balance for the Fluid in the Heat Exchanger



Steady-state energy balance for the fluid:

$$\rho_f c_f Q T_f - \rho_f c_f Q T_f + P_l d = 0 \quad (24)$$

with

$$\begin{aligned} \rho_f &= \text{density of the fluid} \\ c_f &= \text{specific heat capacity of the fluid} \\ Q &= \text{flow rate } [\tfrac{m^3}{s}] \\ T_f &= \text{fluid temperature} \end{aligned}$$

Energy Balance for the Fluid in the Heat Exchanger

Introduce $f = \rho_f c_f Q$.

↓

$$f \, \frac{T_f - T_f}{d} = P_l \quad (25)$$

↓ for $d \to 0$

$$f \, \frac{\partial T_f}{\partial z} = P_l \quad (26)$$

Simplest Estimate

P_l depends on the temperature difference between undisturbed rock and fluid, $T_m - T_f$.

$$\Downarrow$$

P_l can be constant for all z if the fluid temperature follows the undisturbed rock temperature:

$$\frac{\partial T_f}{\partial z} = \frac{\partial T_m}{\partial z} = \text{const} \tag{27}$$

$$\Downarrow$$

$$P_l = f \frac{\partial T_m}{\partial z} = \rho_f c_f Q \frac{\partial T_m}{\partial z} \tag{28}$$

Simplest Estimate

If P_l is also constant through time:

$$T_m - T_f \;=\; \frac{P_l}{4\pi\lambda} E_1\left(\frac{r_a^2}{4\kappa t}\right) \;=\; \frac{f}{4\pi\lambda}\frac{\partial T_m}{\partial z} E_1\left(\frac{r_a^2}{4\kappa t}\right) \qquad (29)$$

Total power extracted between z_1 and z_2:

$$P \;=\; \rho_f c_f Q\left(T_f(z_2) - T_f(z_1)\right) \;=\; f\left(T_f(z_2) - T_f(z_1)\right) \qquad (30)$$

Practically, $T_f(z_2)$ follows Eq. 29, but $T_f(z_1)$ is given by the return temperature of the heating system (higher than predicted by Eq. 29).

Numerical Implementation of the General Model

Switch the power per length at time t to a value $P_l(z,t)$ in such a way that the temperature at time $t+\delta t$ is consistent with Eqs. 6 and 26.

Use a difference quotient for Eq. 26:

$$f\,\frac{T_f(z,t+\delta t)-T_f(z-\delta z,t+\delta t)}{\delta z} = \frac{P_l(z,t)+P_l(z-\delta z,t)}{2} \quad (31)$$

Fluid temperature according to Eq. 6:

$$\begin{aligned} T_f(z,t+\delta t) &= T_m(z)+T(z,t+\delta t) \\ &= T_m(z)+T_0(z,t+\delta t)-\frac{P_l(z,t)}{4\pi\lambda}\,E_1\left(\frac{r_a^2}{4\kappa\delta t}\right) \quad (32) \end{aligned}$$

where $T_0(z,t+\delta t)$ is the temperature that would occur if we switched $P_l(z,t)$ to 0.