Near-surface Geophysics

Resistivity Methods

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Introduction

Basic Idea

Measure electrical conductivities or resistivities using artificial fields.

Main Fields of Application

- Delimiting lithologic units and fault zones
- Determining depth and properties of aquifers
- Monitoring the impermeability of dams
- Exploration and monitoring of residual waste sites
- Monitoring the spread of pollutants
- Detecting potential slip surfaces (e. g., clay layers) in landslide-prone slopes

Electric Field and Potential

- An electric field $\vec{E}$ exerts a force

$$\vec{F} = q\,\vec{E}$$

  on a charge q.

- In absence of time-dependent magnetic fields, the electric field can be represented by the gradient of the electric potential U:

$$\vec{E}(\vec{x}) = -\nabla U(\vec{x}) = -\begin{pmatrix} \frac{\partial}{\partial x_1} U(\vec{x}) \\ \frac{\partial}{\partial x_2} U(\vec{x}) \\ \frac{\partial}{\partial x_3} U(\vec{x}) \end{pmatrix}$$

$$\Downarrow$$

$$\vec{F}(\vec{x}) = -q\,\nabla U(\vec{x})$$

Ohm's Law

Force on free electrons in a conductor

↓

Drift of electrons in direction of the force, velocity proportional to the force

↓

Current density (charge density $\times$ drift velocity)

$$\vec{j}(\vec{x}) \;=\; -\,\sigma\,\nabla U(\vec{x})$$

- Named after Georg Simon Ohm, 1789–1854.
- The constant of proportionality σ is a property of the material and is denoted electrical conductivity.

Questions

- What are the units of $\vec{j}$ und σ?
- Why is Ohm's law not completely reasonable at first sight? What should happen to free electrons exerted to a force?
- Which relationships with the same structure as Ohm's law have you met in previous classes?

Basics

Conductivity and Resistivity

Conductivity σ

$$[\sigma] = \frac{1}{\Omega \mathrm{m}} = \frac{\mathrm{S}}{\mathrm{m}},\ \Omega = \mathrm{Ohm} = \frac{\mathrm{V}}{\mathrm{A}},\ \mathrm{S} = \mathrm{Siemens} = \frac{\mathrm{A}}{\mathrm{V}}$$

Resistivity $\rho = \frac{1}{\sigma}$

$$[\rho] = \Omega \mathrm{m}$$

Conductance and resistance refer to objects and not to materials and are measured in S and Ω, respectively.

(Semi)Conductors	ρ [Ωm]
copper	1.7×10^{-8}
iron	10^{-7}
silicium	2300

Nonconductors	ρ [Ωm]
porcelain	10^{12}
rubber	10^{13}
silica glass	7.5×10^{17}

Conductivity / Resistivity of Rocks and Soils

- Rock forming minerals have very low conductivities.
- Many ores have considerably higher conductivities.
- The conductivity of pure water is rather low, but strongly increases by solving salts.

Solution	ρ [Ωm]
distilled water	10000
ocean water	0.5
10 % copper sulfate	0.3
10 % sodium chlorite	0.08
10 % sulfuric acid	0.025
10 % hydrochloric acid	0.015

Conductivity / Resistivity of Rocks and Soils

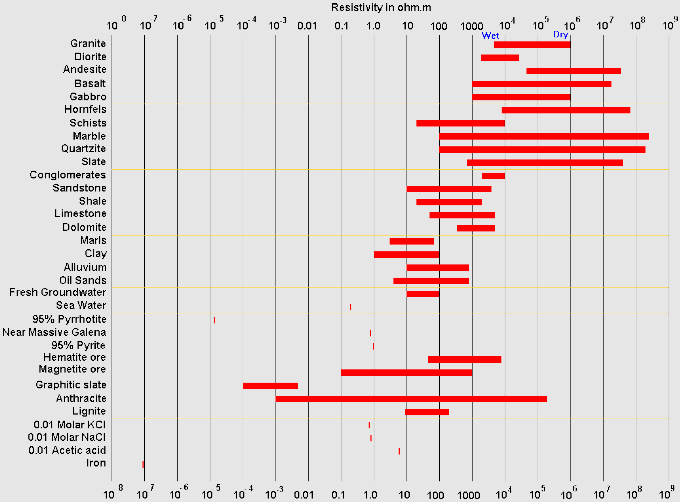
Thus, the total conductivity of a rock or a soil strongly depends on

- porosity
- water saturation
- connectivity of the pore space
- pureness of the contained water (in return depends on the properties of the rock/soil)

Question

Which are the main dependencies of the hydraulic conductivity of an aquifer?

Conductivity / Resistivity of Rocks and Soils



Source: Loke, Tutorial: 2-D and 3-D electrical imaging surveys

Conductivity / Resistivity of Rocks and Soils

Material	ρ [Ωm]
halite	$10^5 - 10^7$
dry sand	10^5
water satur. sand	$1000 - 10000$
quartzite	$3000 - 10^5$
ice	$1000 - 10^5$
granite	$300 - 30000$
sandy soils	$150 - 7000$
loamy soils	$50 - 9000$
clayey soils	$20 - 4000$

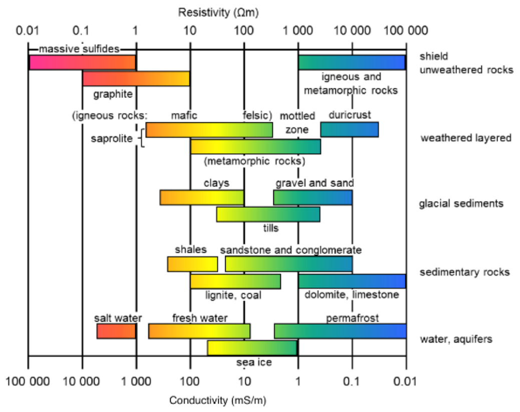
Material	ρ [Ωm]
limestone	$100 - 7000$
marsh	$30 - 700$
glacial moraine	$10 - 300$
clay shale	$10 - 1000$
marl	$5 - 200$
loam	$3 - 300$
dry clay	$30 - 1000$
wet clay	$1 - 30$
silt	$10 - 1000$

Source: Beblo (Ed.), Umweltgeophysik

DC Conductivity/Resistivity Values

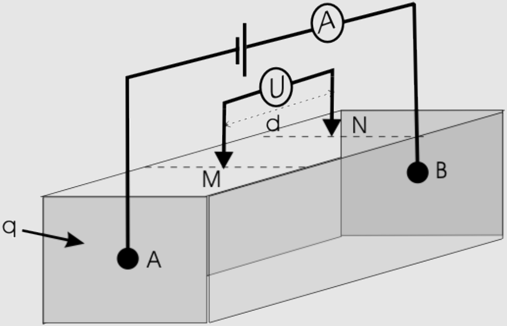
Telford 1990 Applied Geophysics

Rock	% H_20	P [Ohm m]
Siltstone	0.54	$1.5*10^4$
	0.38	$5.6*10^8$
Coarse grain SS	0.39	$9.6*10^5$
	0.18	10^8
Medium grain SS	1.0	$4.3*10^3$
	0.1	$1.4*10^3$
Dolomite	1.3	$6*10^3$
	0.96	$8*10^3$
Granite	0.31	$4.4*10^3$
	0.19	$1.8*10^6$
Basalt	0.95	$4*10^4$
	0	$1.3*10^8$
Graywacke SS	1.16	$4.7*10^3$
	0.45	$5.8*10^4$
Peridotite	0.1	$3*10^3$
	0	$1.8*10^7$



The Principle of Subsurface Resistivity Measurement

1. Two current electrodes A und B are plugged into the ground, and a voltage is applied, generating a current *I* from A to B.
2. Two potential electrodes M und N are plugged into the ground, and the voltage *U* between both is measured.



Source: Schmidt et al., Die Erde: Der dynamische Planet (CD-ROM)

Basics

Question

What are the analogies of these electrodes in subsurface hydrology?

The Potential Equation

The charge density in a conductor remains constant everywhere, so that

$$\operatorname{div} \vec{j}(\vec{x}) \;=\; \frac{\partial}{\partial x_1} j_1(\vec{x}) + \frac{\partial}{\partial x_2} j_2(\vec{x}) + \frac{\partial}{\partial x_3} j_3(\vec{x}) \;=\; 0.$$

$$\Downarrow$$

$$\operatorname{div}\left(\sigma \nabla U(\vec{x})\right) \;=\; 0.$$

For $\sigma = \text{const.}$ this reduces to

$$\operatorname{div}\left(\nabla U(\vec{x})\right) \;=\; \Delta U(\vec{x}) \;=\; \frac{\partial^2}{\partial x_1^2} U(\vec{x}) + \frac{\partial^2}{\partial x_2^2} U(\vec{x}) + \frac{\partial^2}{\partial x_3^2} U(\vec{x}) \;=\; 0.$$

Solutions of the Potential Equation in a Homogeneous Medium

Potential of a point source at the origin feeding a current I:

$$U(\vec{x}) = \frac{\rho I}{4\pi |\vec{x}|}$$

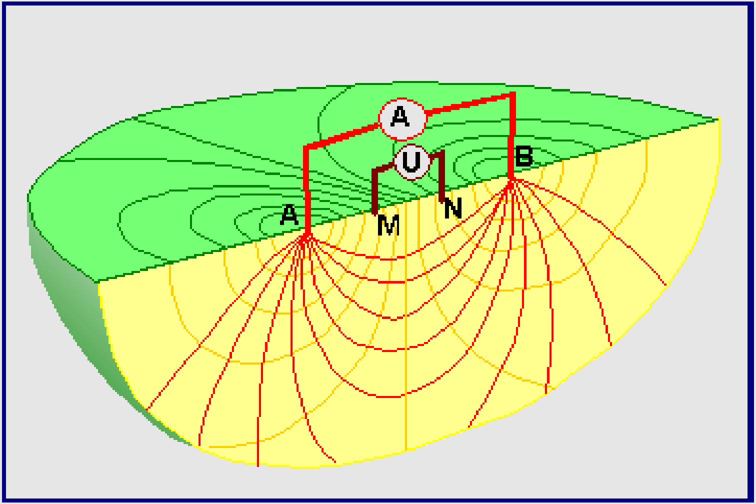
Potential of a point source at the point $\vec{x}_A$ if the current is distributed in a half space only:

$$U(\vec{x}) = \frac{\rho I}{2\pi |\vec{x} - \vec{x}_A|}$$

Feeding in a current I at $\vec{x}_A$ and extracting I at $\vec{x}_B$:

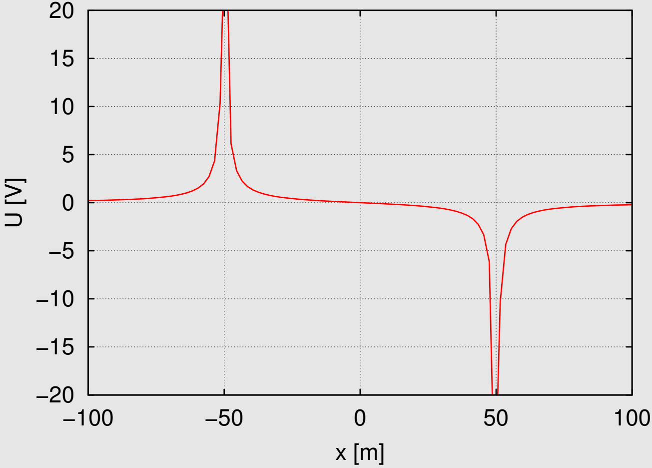
$$\begin{aligned} U(\vec{x}) &= \frac{\rho I}{2\pi |\vec{x} - \vec{x}_A|} - \frac{\rho I}{2\pi |\vec{x} - \vec{x}_B|} \\ &= \frac{\rho I}{2\pi} \left(\frac{1}{|\vec{x} - \vec{x}_A|} - \frac{1}{|\vec{x} - \vec{x}_B|} \right) \end{aligned}$$

Dipole Feed in a Homogeneous Half-Space



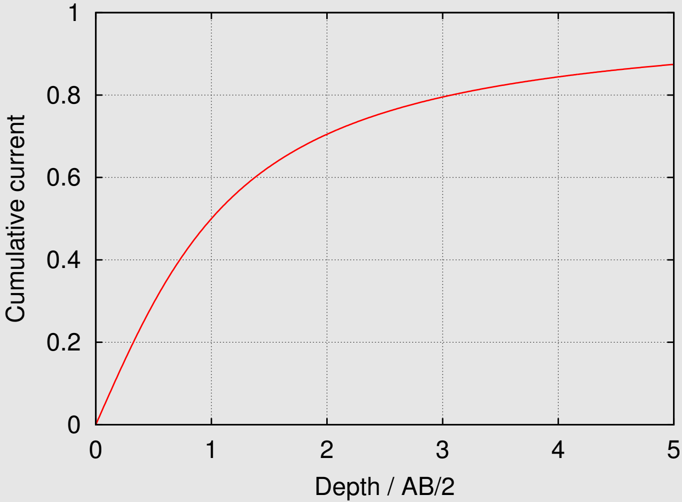
Source: Schmidt et al., Die Erde: Der dynamische Planet (CD-ROM)

The Potential between the Electrodes



$\rho = 1000\,\Omega\text{m},\ I = 100\,\text{mA},\ \text{offset} = 100\,\text{m}$

Penetration Depth of the Current



Four-Electrode Surveys

Inhomogeneous Media

Results obtained for large offsets AB are more sensitive to the resistivities at greater depth than results obtained for small offsets.

Types of Resistivity Measurements

Vertical sounding: same location, but different offsets

Horizontal profiling: constant electrode configuration used at different positions

Resistivity tomography: variable location and variable electrode spacing

⬇

Various types of electrode configurations more or less suitable for different purposes

Arbitrary Electrode Configuration in a Homogeneous Half-Space

- Voltage between M and N is the difference of the potentials at $\vec{x}_M$ and $\vec{x}_N$:

$$
\begin{aligned}
U &= U(\vec{x}_M) - U(\vec{x}_N) \\
&= \frac{\rho I}{2\pi}\left(\frac{1}{|\vec{x}_M - \vec{x}_A|} - \frac{1}{|\vec{x}_M - \vec{x}_B|} - \frac{1}{|\vec{x}_N - \vec{x}_A|} + \frac{1}{|\vec{x}_N - \vec{x}_B|}\right) \\
&= \frac{\rho I}{2\pi}\left(\frac{1}{r_{MA}} - \frac{1}{r_{MB}} - \frac{1}{r_{NA}} + \frac{1}{r_{NB}}\right)
\end{aligned}
$$

  where $r_{\dots}$ are the distances between the respective electrodes.
- Mostly, all electrodes are placed on a straight line.

The Geometric Factor

The resistivity of a homogeneous half-space can be determined according to

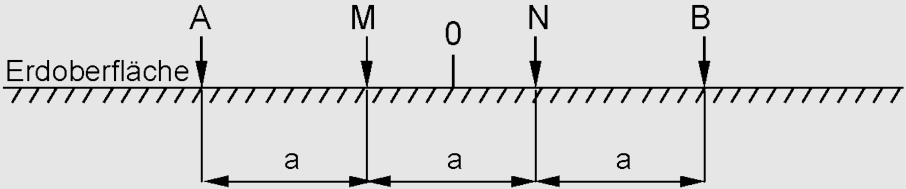
$$\rho = K \frac{U}{I}$$

with the geometric factor

$$K = \frac{2\pi}{\frac{1}{r_{MA}} - \frac{1}{r_{MB}} - \frac{1}{r_{NA}} + \frac{1}{r_{NB}}}$$

of the selected electrode configuration.

The Wenner (α) Configuration



Source: Wikipedia

$$K = 2\pi a$$

Widely used for horizontal profiling (a fixed)

Four-Electrode Surveys

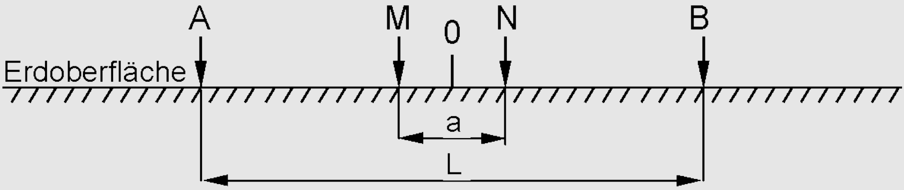
Variants of the Wenner Configuration

Configuration	Electrode sequence	Geometric factor
Wenner α	A-M-N-B	$K = 2\pi a$
Wenner β	A-B-M-N	$K = 6\pi a$
Wenner γ	A-M-B-N	$K = 3\pi a$

Wenner α is the standard configuration (Wenner without further specification).

Four-Electrode Surveys

The Schlumberger Configuration



Source: Wikipedia

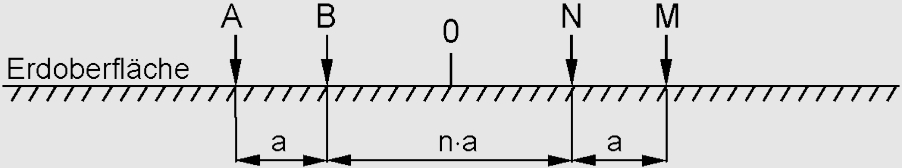
$$K = \frac{\pi\left(L^2 - a^2\right)}{4a} \approx \frac{\pi L^2}{4a} \quad \text{für } L \gg a$$

Widely used for vertical sounding (a fixed, L variable)

Caution: Sometimes L is used for AB/2 instead of the total offset AB.

Four-Electrode Surveys

The Dipole-Dipole Configuration



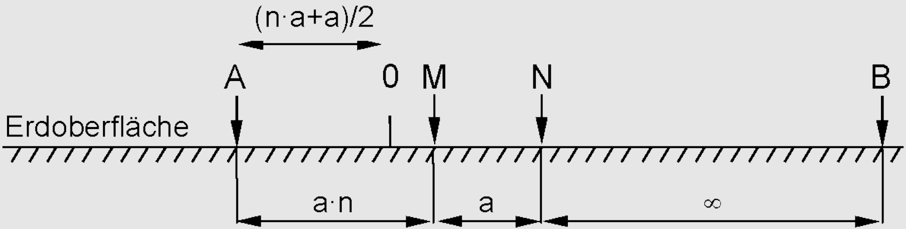
Source: Wikipedia

$$K = \pi\, n(n+1)(n+2)\, a$$

Particularly suitable for profiling of small-scale structures, but a requires high power input.

Four-Electrode Surveys

The Pole-Dipole Configuration



Source: Wikipedia

$$K = 2\pi \, n(n+1) \, a$$

Particularly suitable for investigating horizontal contrasts.

Typical field installation

Source: http://www.lgm.de/en/en_resistivity.html#electrodes

Source: http://www.gfinstruments.cz

Field Work Example

Four-Electrode Surveys

Apparent Resistivity

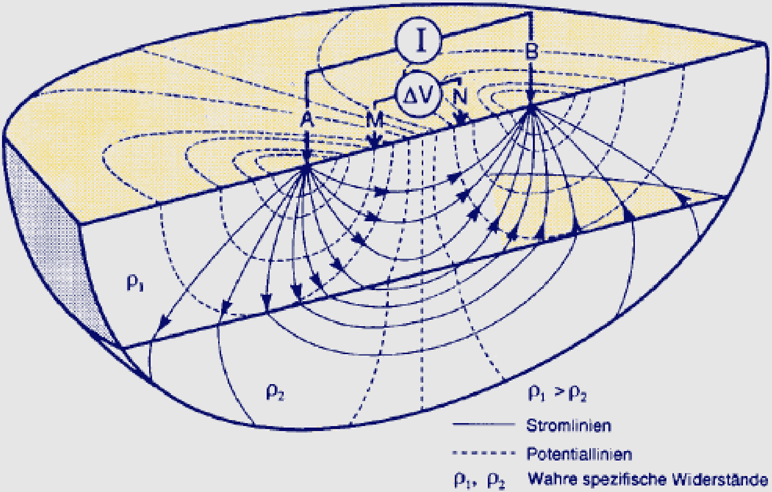
In a inhomogeneous medium,

$$\rho_a = K \frac{U}{I}$$

is called the apparent resistivity obtained from one measurement.

- ρ_a is the resistivity of a homogeneous medium that would yield the same result for the considered electrode configuration.
- ρ_a is not the real resistivity at any depth.
- The larger the offset is, the bigger is the contribution of deep regions to ρ_a.

Vertical Sounding in the Two-Layer Case



Source: Knödel et al., Handbuch zur Erkundung des Untergrundes von Deponien und Altlasten, Vol. 3

Layered Media

Vertical Sounding in the Two-Layer Case

Situation: Two homogeneous regions separated by a horizontal interface.

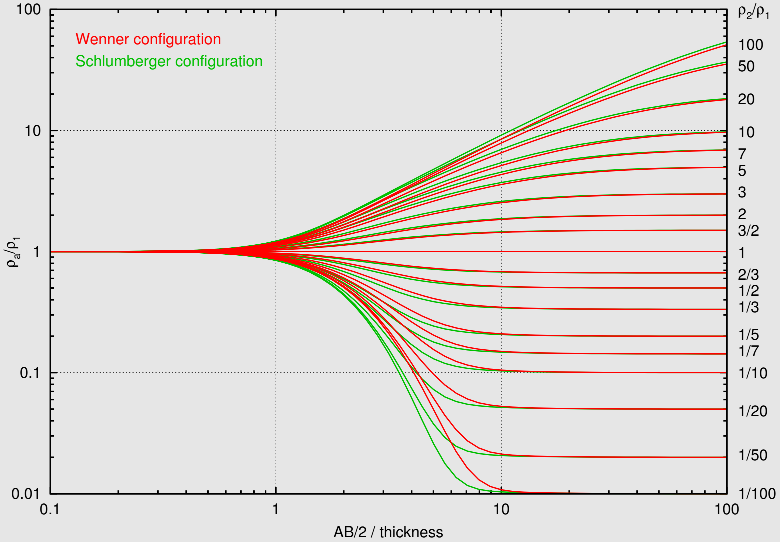
Target properties:

$$\begin{aligned} \rho_1 &= \text{resistivity of the upper layer} \\ \rho_2 &= \text{resistivity of the lower region} \\ d &= \text{thickness of the upper layer} \end{aligned}$$

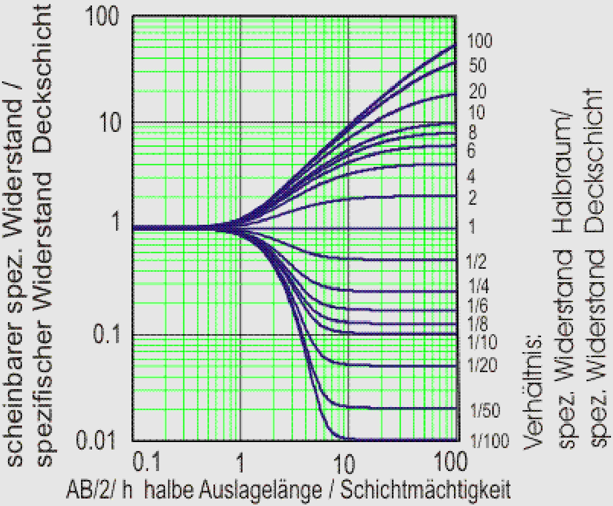
Procedure: ρ_a is measured for several offsets AB (Wenner or Schlumberger configuration).

Data analysis can be performed graphically because $\frac{\rho_a}{\rho_1}$ only depends on $\frac{\rho_2}{\rho_1}$ and $\frac{AB/2}{d}$.

Wenner and Schlumberger Configurations in the Two-Layer Case



Graphical Data Analysis in the Two-Layer Case



Source: Schmidt et al., Die Erde: Der dynamische Planet (CD-ROM)

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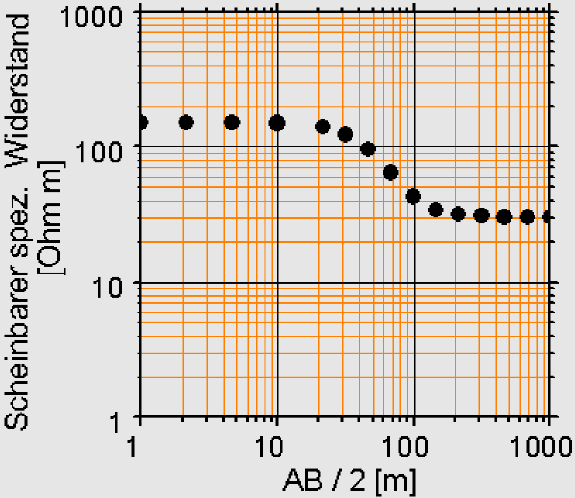

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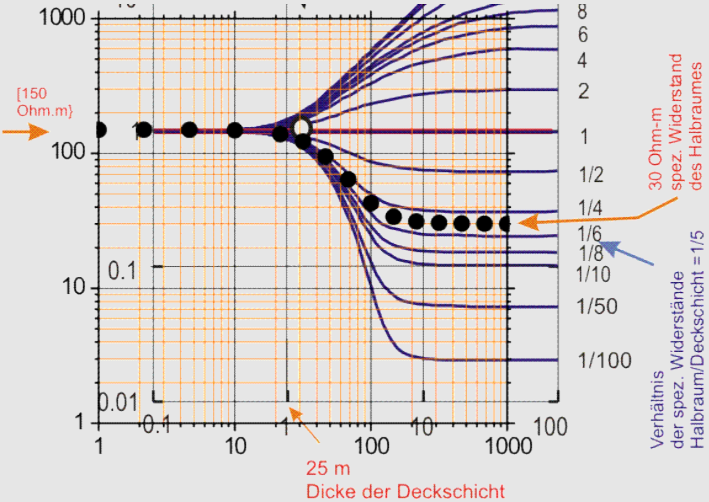
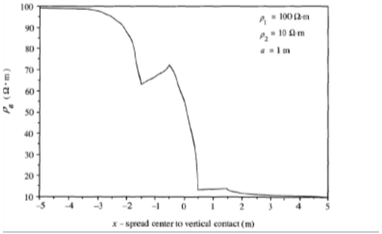
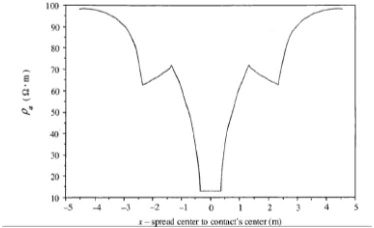


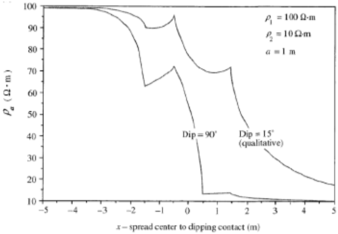
Source: Schmidt et al., Die Erde: Der dynamische Planet (CD-ROM)

Constant separation traversing (CST)

horizontal anomalies

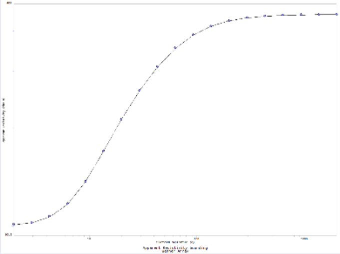




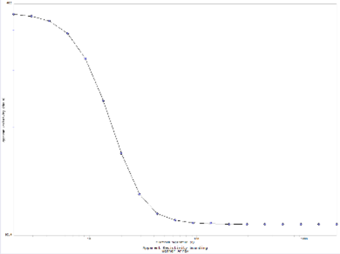


Master curves

2-layer case



- material with greater resistivity lies below the interface



- material with greater resistivity lies above the interface

Master curves

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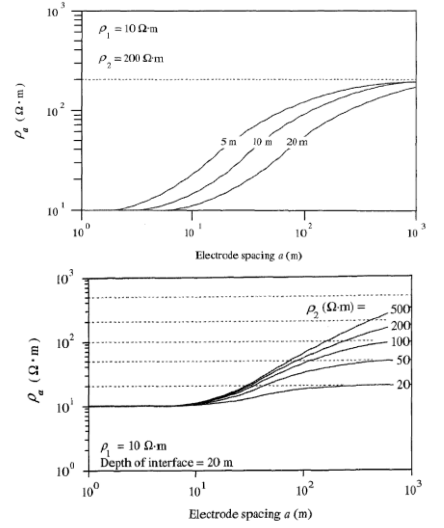
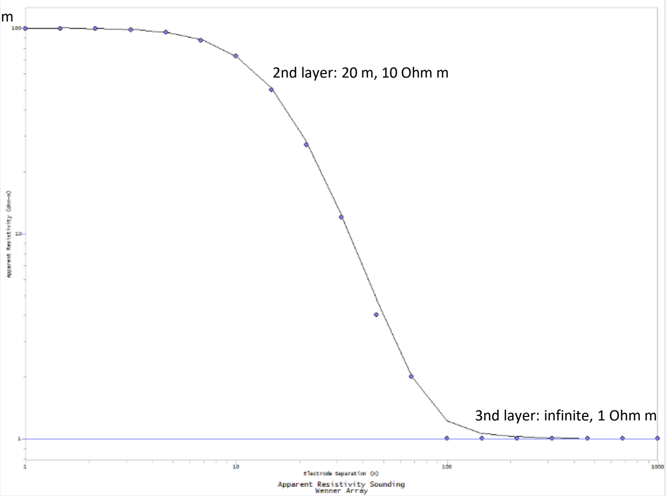


- constant resistivity
- constant depth

Master curves

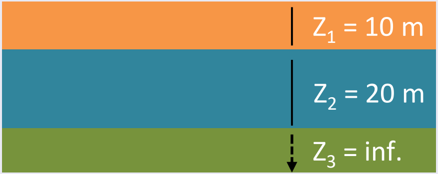
3-layer case

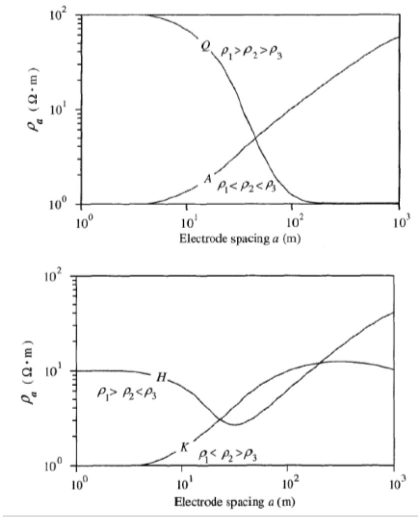
1st layer: 10 m, 100 Ohm m



Master curves

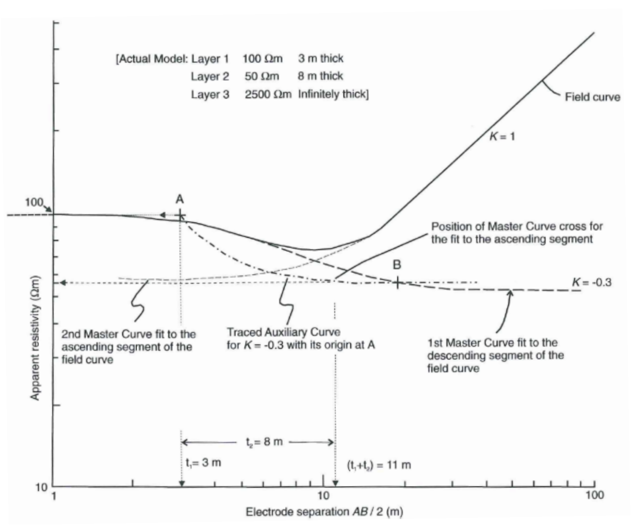
3-layer case





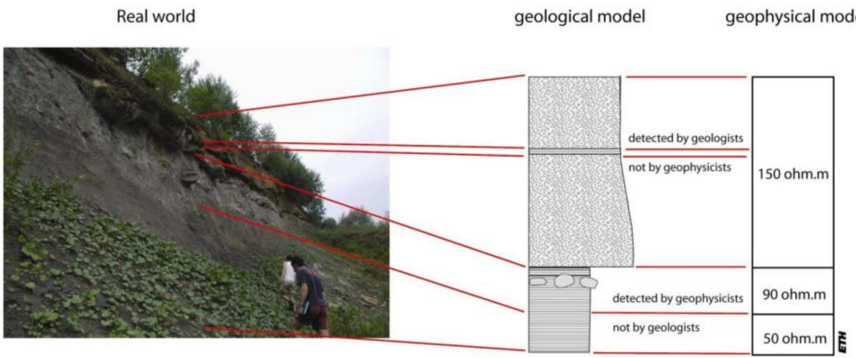
Master curves

multi-layer case



Master curves

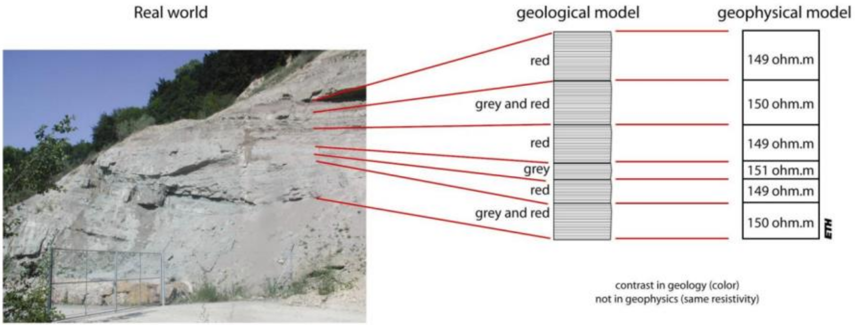
Source: Laurent Marescot, 2010



Master curves

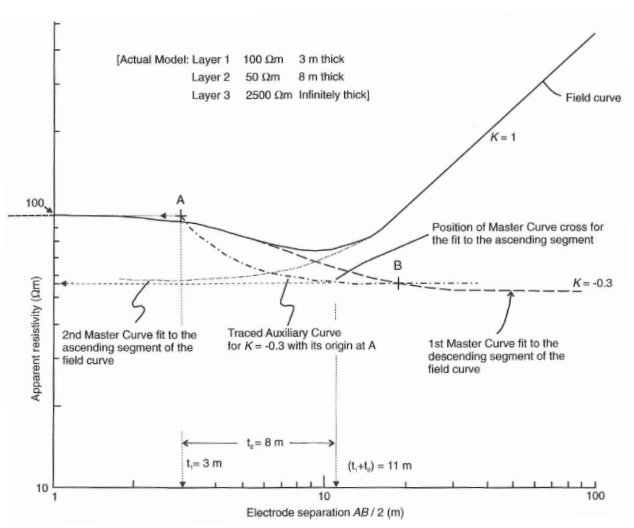
Source: Laurent Marescot, 2010

To characterize different material using geophysics, a contrast must exist (i.e. a difference in the physical properties)



Master curves

multi-layer case



The Two-Layer Case

- The result is more or less unique if a sufficient range of offsets is covered.
- The procedure can also be applied to gently dipping interfaces.
- This method has only historical and educational meaning. Practically, numerical inversion is preferred.

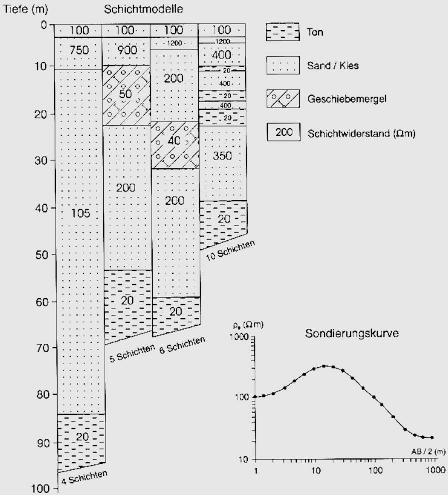
Multiple Layers

- Must be inverted numerically. Resistivities and thicknesses of the layers are adjusted to obtain the best fit to the measured apparent resistivities.
- The uppermost layer has a strong influence on the result.
- A deep, thin layer with a high contrast in resistivity may have a similar effect as a thicker layer with a lower contrast in resistivity.
- In the standard inversion procedure of vertical sounding, the number of layers is given, and thicknesses and resistivities are adjusted. Different numbers of layers may lead to strongly different results.

$$\Downarrow$$

Quantitative analysis often hinges on independent information, e. g., from seismics or boreholes.

Multiple Layers



Source: Knödel et al., Handbuch zur Erkundung des Untergrundes von Deponien und Altlasten, Vol. 3

Penetration Depth of the Current

Half of the current penetrates deeper than half of the total offset (AB/2), but

- the entire current must also pass shallow regions, and
- the potential electrodes are at the surface.

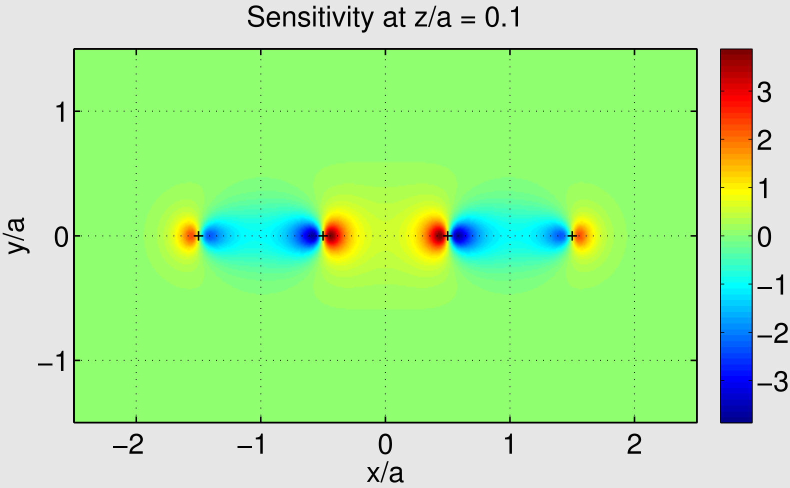
⇓

Typical depth of investigation is lower than AB/2.

Principle of the Sensitivity Analysis

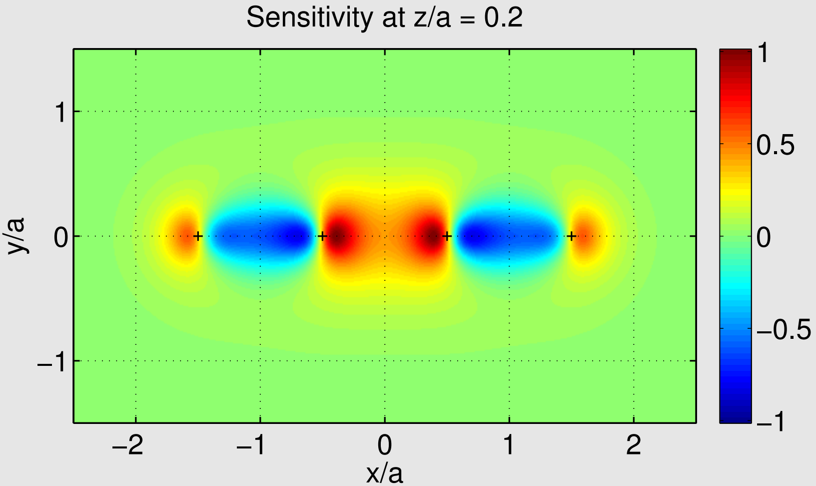
- Assume a given configuration of electrodes in a homogeneous medium with a resistivity ρ.
- Assume that ρ is increased (decreased) by a small amount $\delta\rho$ in a small region around a given point $\vec{x}$ in the subsurface.
- Determine how this small change affects the voltage between M and N if the current between A and B is given.

Sensitivity of the Wenner Configuration



Sensitivity Analysis

Sensitivity of the Wenner Configuration



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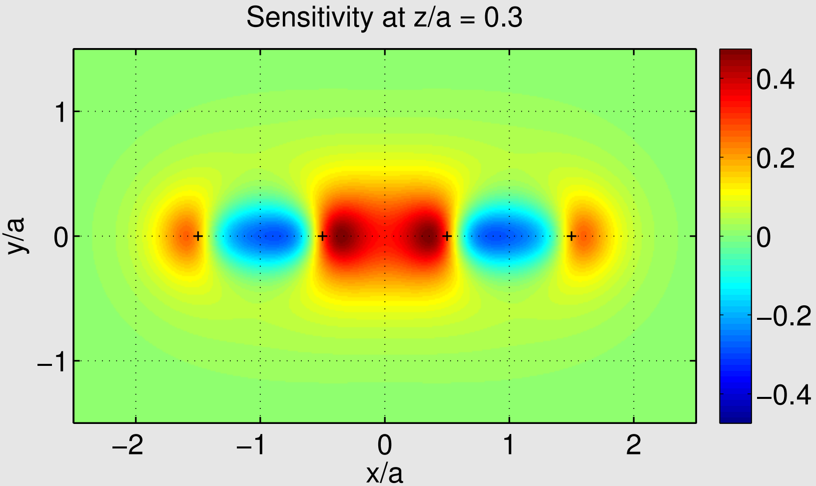

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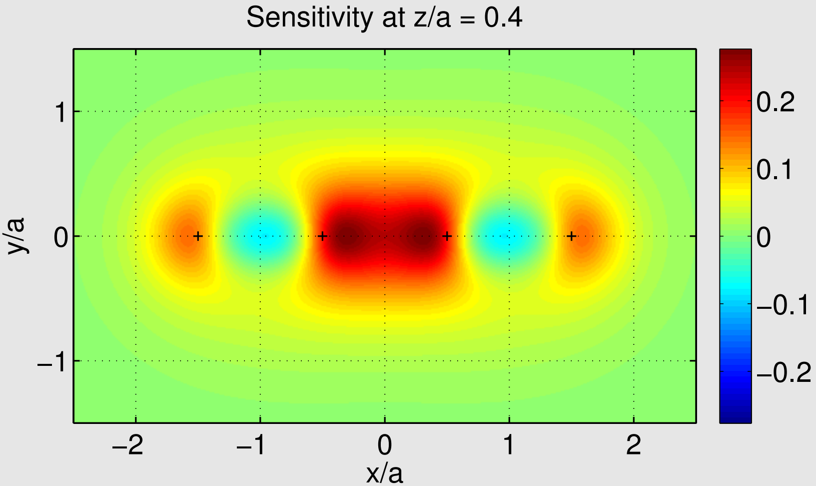

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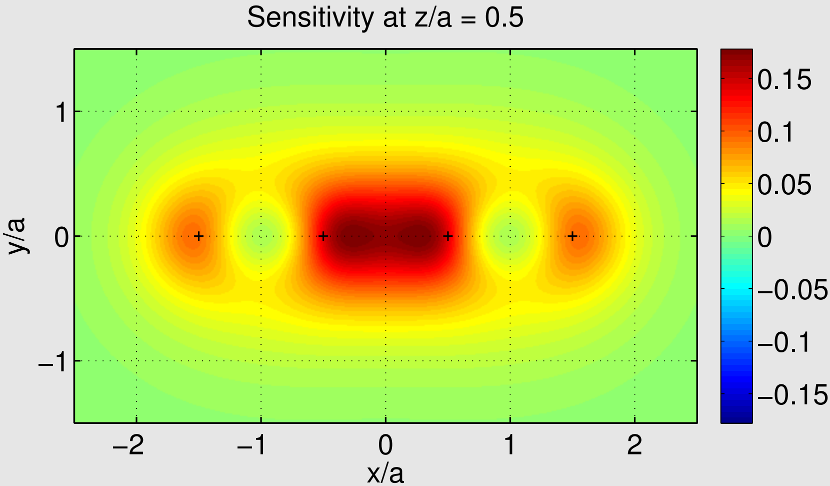

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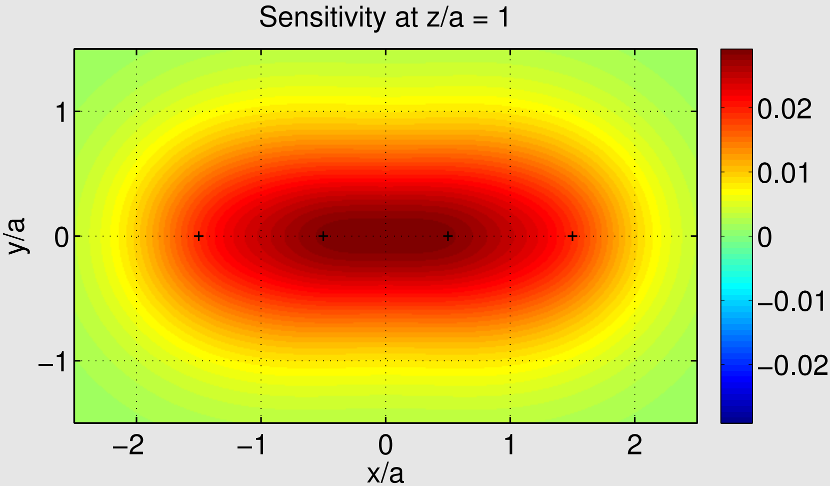

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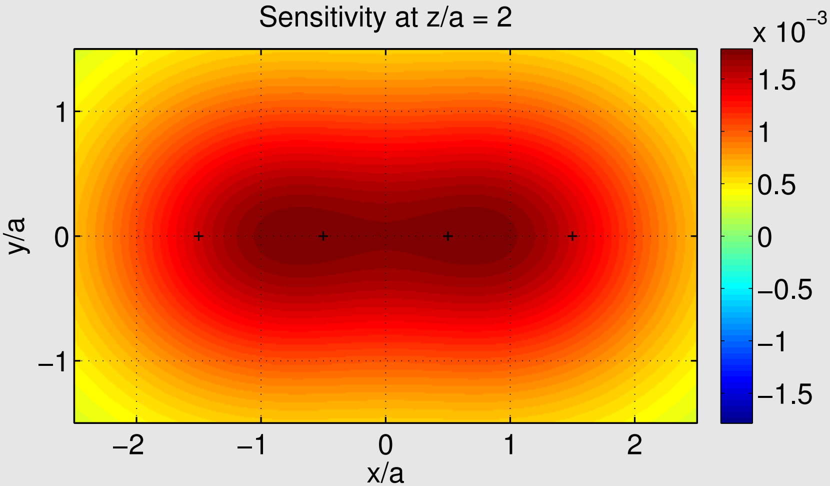

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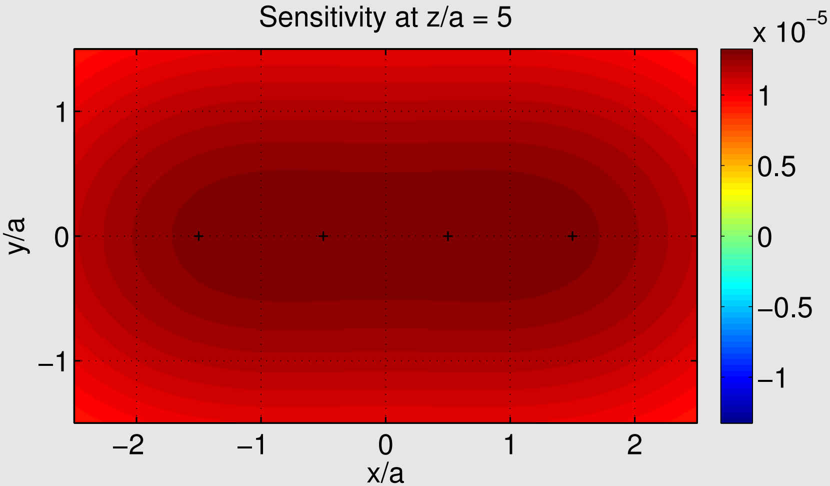
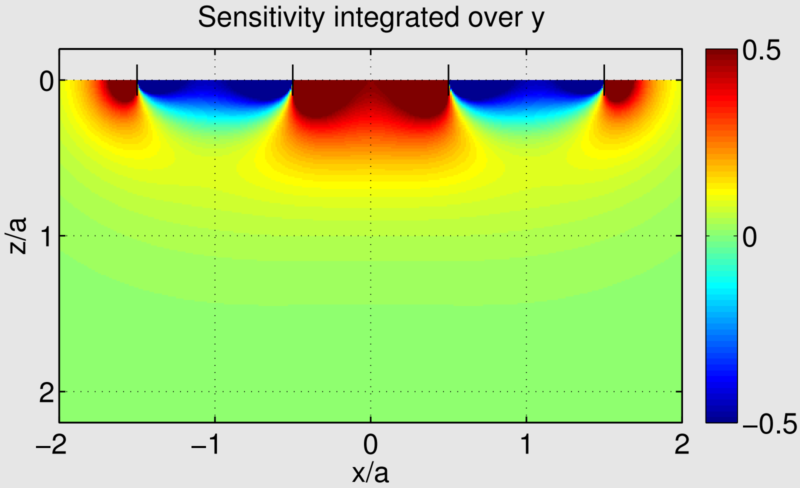
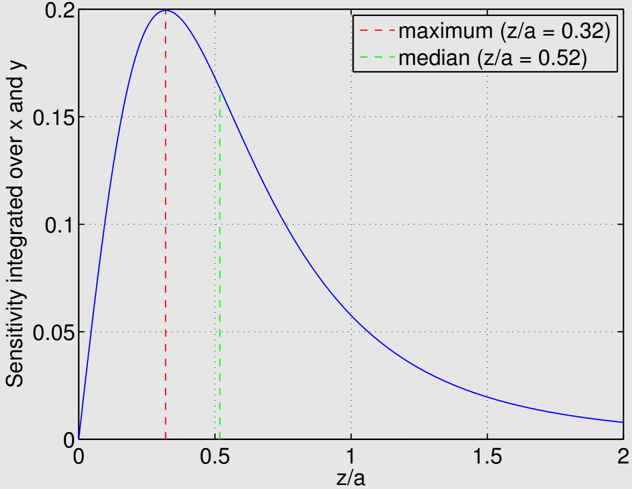


Sensitivity Analysis

Sensitivity of the Wenner Configuration



Sensitivity of the Wenner Configuration



Sensitivity of the Wenner Configuration

- Sensitivity is always highest at low depth, in particular close to the electrodes M and N.
- Sensitivity changes its sign at low depths.
- Horizontally integrated sensitivity is highest at $z \approx 0.32\ a$.
- Median of the horizontally integrated sensitivity distribution is at $z \approx 0.52\ a$.

$\Downarrow$

Regions with $z < 0.52\ a$ and $z > 0.52\ a$ contribute equally to the sensitivity in total.

$\Downarrow$

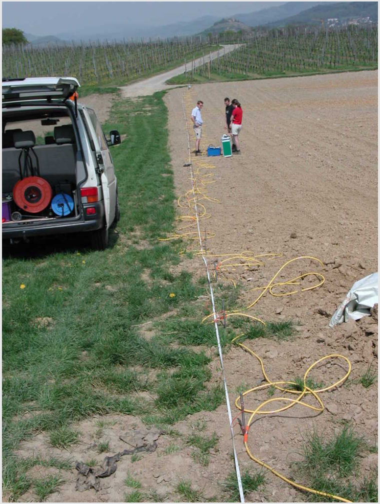
$0.52\ a$ is often assumed as the typical depth of investigation.

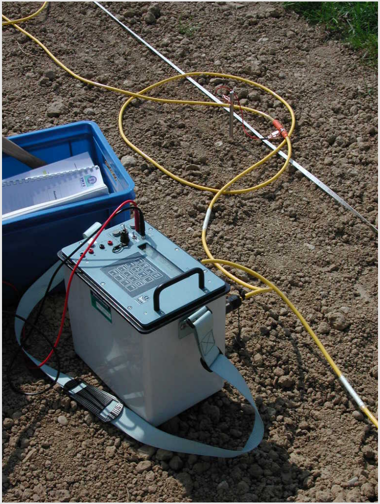
Multi-Electrode Arrays

Principle

- Several (up to some hundred) electrodes are plugged into the ground, either on a profile line or distributed in two dimensions.
- A programmable channel selector replays a defined sequence of usage of the electrodes as current or potential electrode pairs.
- The method is also called electric tomography, in particular if the electrodes are distributed in two dimensions.

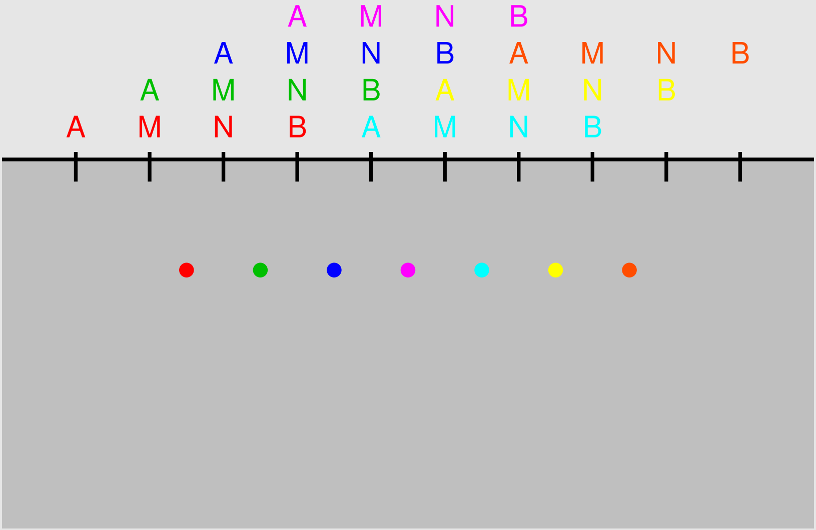
Example of Equipment and Configuration



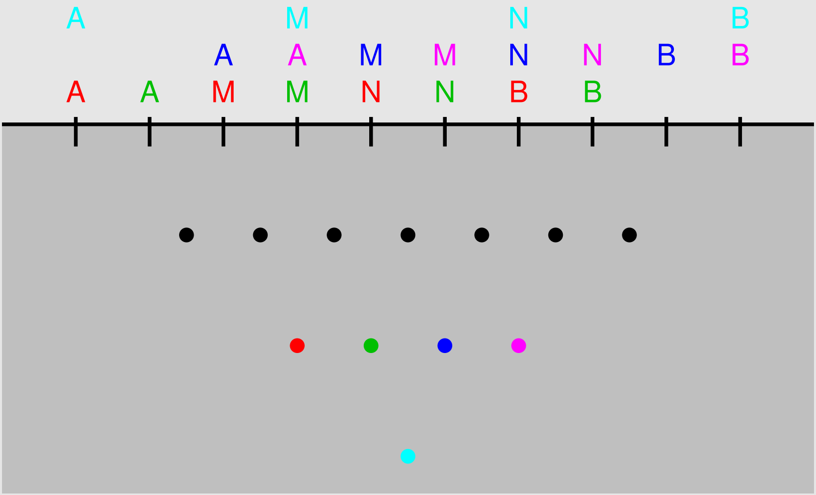


Source: Teaching material A. Henk

Pseudo-Depth Sections



Pseudo-Depth Sections



Pseudo-Depth Sections

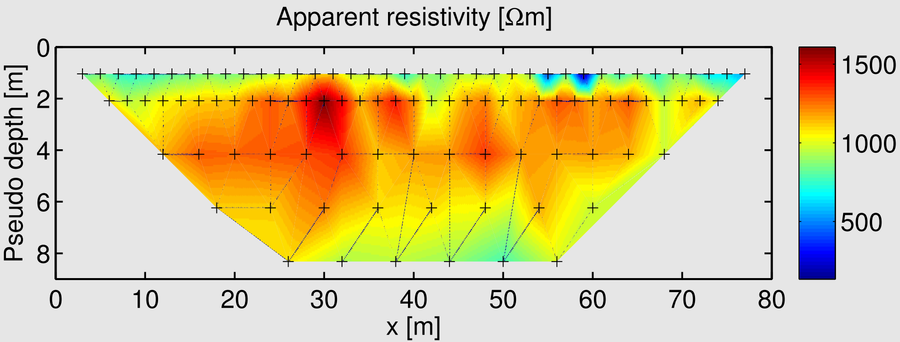
- ρ_a is registered in the middle between A and B and in a pseudo-depth corresponding to the typical depth of investigation, e. g., 0.52 a.
- The plot is often vertically exaggerated in such a way that the borders have 45° angles.
- A pseudo-depth section gives a first idea on the subsurface structure.
- ρ_a is not the resistivity at any point, but some kind of average over a larger region.
- ρ_a is strongly affected by near-surface heterogeneities.

$\Downarrow$

Deriving a realistic subsurface resistivity model requires a numerical inversion.

Multi-Electrode Arrays

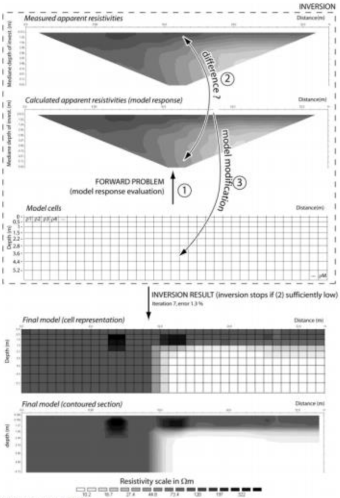
Example of a Pseudo-Depth Section



Other Configurations of Electrodes

The Wenner (α) configuration is most widely used, but all other configurations are also possible.

Electric resistivity tomography (ERT)



Laurent Marescot, 2010

Electric tomography inversion

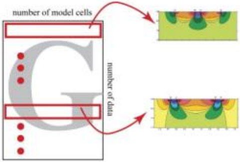
$$\Delta \mathbf{m} = \left[\mathbf{G}^{T}\mathbf{G} + \mathbf{W}\right]^{-1} \mathbf{G}^{T}\, \Delta \mathbf{d}$$

with:

$$\Delta \mathbf{m} = \log(\rho)$$

$$\Delta \mathbf{d} = \log(\rho_a^{meas}) - \log(\rho_a^{calc})$$

$$G_{ij} = \frac{\partial g(m)_i}{\partial m_j}$$ is the sensitivity matrix



Electrodes

Current and potential electrodes are technically identical. Criteria (in particular for the potential electrodes):

Contact resistance to the ground should be low.

Contact voltage should be small.

- Usage of nonpolarizable electrodes, e. g., copper core in $CuSO_4$ solution in a porous clay cylinder.
- Simple steel electrodes can be used with modern central units that are able to compensate contact voltages automatically.

The Central Unit

- Power source (constant current),
- voltmeter, and
- channel selector (for multi-electrode equipment)

are mostly combined in one unit.

Power up to about 1000 W

Currents mostly between 10 mA and 1 A

Voltages (between the current electrodes) up to some 1000 V

Types of current: DC, low-frequency AC or switched DC with changing polarity