Hydrogeology

Seismology und Seismics

Jakob Wilk

Institut für Geo- und Umweltnaturwissenschaften

Albert-Ludwigs-Universität Freiburg

Seismology

- Comprises all about earthquakes and the propagation of seismic waves in the Earth.
- One of the main fields of solid-earth geophysics.
- Has provided the majority of our knowledge on Earth's interior.

Seismics

- Exploration of the deep and shallow subsurface with the help of artificial seismic waves.
- The perhaps most important field of applied geophysics.

Seismic Waves

Waves in Elastic Media

- Propagating elastic deformation
- The same as sound waves in solids
- More complicated than sound waves in liquids and gases
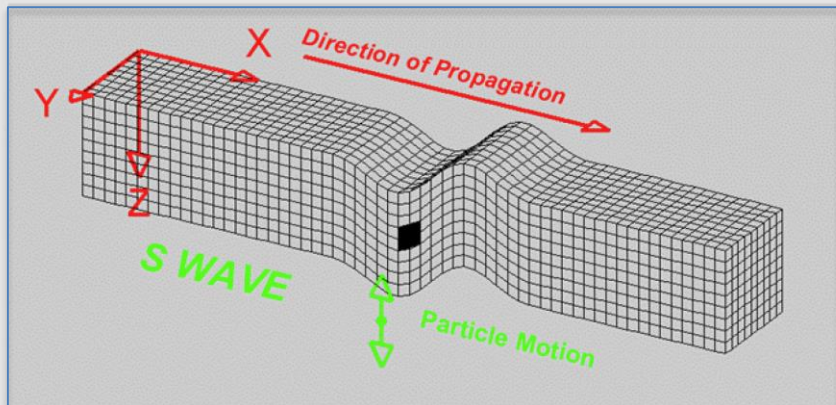
Seismic Waves

Basic Types of Body Waves

Two types of independent plane waves in an infinite, homogeneous elastic medium:

- Compressional wave (longitudinal wave, primary wave)

Source: L. Braile, Purdue University

Seismic Waves
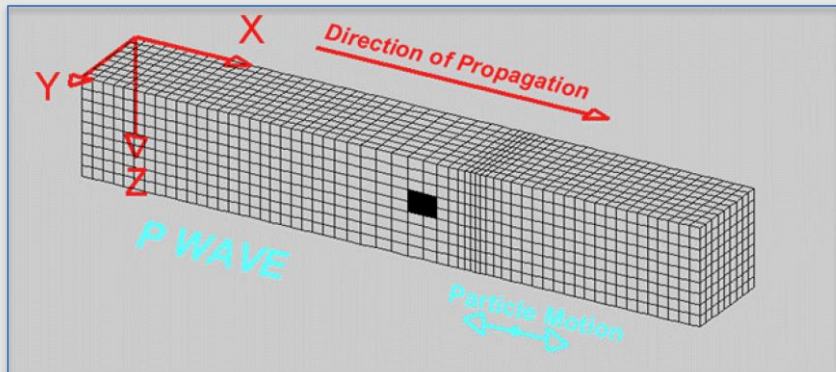
Basic Types of Body Waves

- Shear wave (transverse wave, secondary wave)

Source: L. Braile, Purdue University

Seismic Waves

Comparison with Sound Waves in Liquids and Gases

The compressional wave is similar to sound waves in liquids and gases, while the shear wave has no counterpart in liquids and gases.

Seismic Velocities

Medium	Compressional wave $[\frac{km}{s}]$	Shear wave $[\frac{km}{s}]$
air	0.34	–
water	1.45	–
wood	about 3	about 1.8
Earth	5.8–13.7	3.2–7.3

Seismic Waves

Seismic Velocities

Velocity v_p of the compressional wave is always higher than the velocity v_s of the shear wave.

⬇

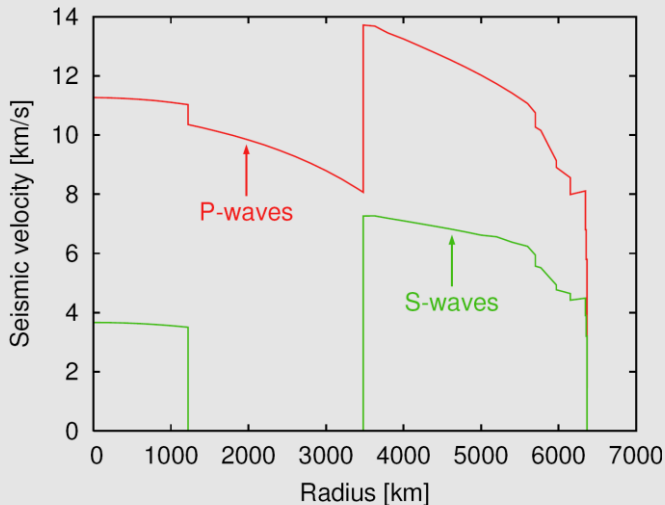Compressional wave always arrives prior to the shear wave.

⬇

compressional wave = primary wave (P-wave)
shear wave = secondary wave (S-wave)

Rules of Thumb

Solid rock: $v_s \approx 0.5\, v_p - 0.6\, v_p$

Soil / unconsolidated rock: $v_s \approx 0.4\, v_p$

Seismic Velocities according to the Preliminary Reference Earth Model

Seismic Waves

Seismic Velocities and Elastic Properties

$\vec{s}$ is the "slowness vector" of the wave ($|\vec{s}| = \frac{1}{v}$), so that

$$v_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{M}{\rho}} = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}}$$

$$v_s = \sqrt{\frac{\mu}{\rho}}$$

with

M = linear elastic modulus without lateral contraction

K = bulk modulus

μ = shear modulus

v_p und v_s are determined by the density and on the elastic modulus of the respective type of deformation.

Seismic Waves

Typical P-wave Velocities in the Shallow Subsurface

Medium	$v_p \left[\frac{km}{s}\right]$	Medium	$v_p \left[\frac{km}{s}\right]$
weathering zone	0.1–0.5	clay	1.2–2.8
dry sand	0.3–0.6	claystone	2.2–4.2
water-saturated sand	1.3–1.8	limestone	3–6
sandstone	1.8–4	halite	4.5–6.5
pit coal	1.6–1.9	granite	5–6.5

Seismic Waves

Propagation in Inhomogeneous Media

Two different approximations in analogy to optics:

- Computation of wavefronts (Huygens' principle, eikonal equation)
- Computation of ray paths normal to the wavefronts (ray optics)

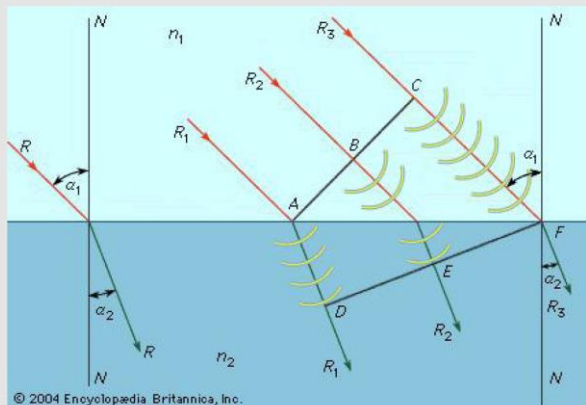## Ray Optics

Ray optics is rather simple in two limiting cases:

- Almost planar interface between two homogeneous media.
- Medium is almost homogeneous on the scale of the wavelength.

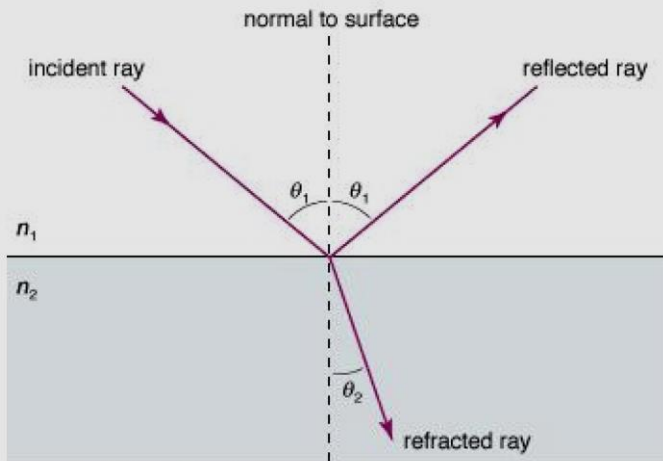# Seismic Waves

Shearer 2010, Introduction to Seismology

anisotropic layer

incident S pulse

Slow

Fast

transmitted S pulses

Seismic Waves
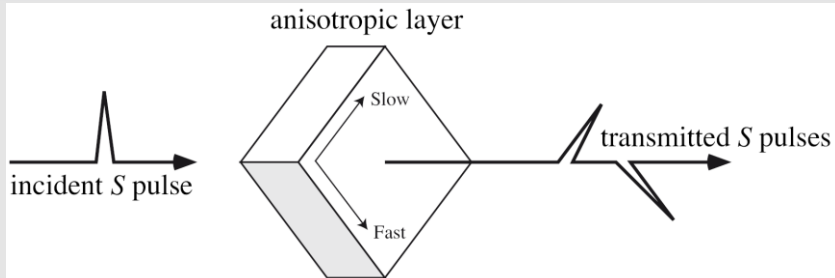
Reflection and Refraction

Simplest case: two homogeneous, isotropic halfspaces with different properties (λ, μ, ρ) and plane waves in each of them.

Snell's law:

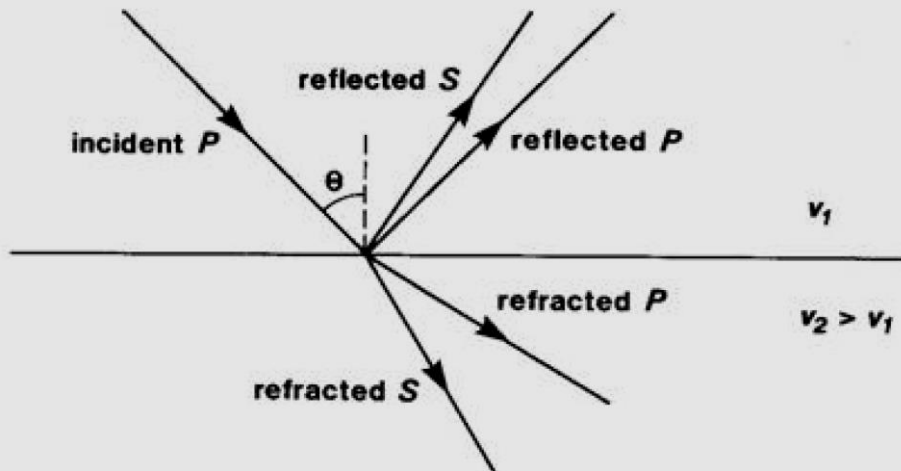$$\frac{\sin \alpha_1}{\sin \alpha_2} = \frac{v_1}{v_2}$$

© 2004 Encyclopædia Britannica, Inc.

Source: Encyclopaedia Britannica

Reflection and Refraction

normal to surface

incident ray

reflected ray

θ_1 θ_1

n_1

n_2

θ_2

refracted ray

© 2006 Encyclopædia Britannica, Inc.

Source: Encyclopaedia Britannica

Reflection and Refraction

Source: University College London

Reflected and Refracted Waves

P- and S-waves are merged when reflected or refracted.

⬇

Incoming P- or S-wave induces up to 4 reflected and refracted waves.

Snell's law holds for all involved pairs of waves

⬇

Ray parameter

$$p = \frac{\sin \alpha}{v}$$

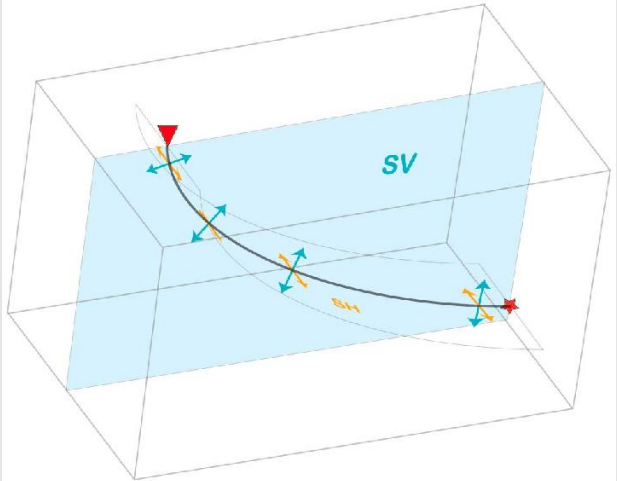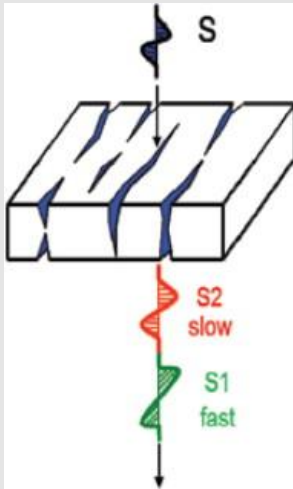(also called horizontal slowness) is the same for all involved waves.

Reflected and Refracted Waves

General form of Snell's law:

> Horizontal slowness remains constant in reflection and refraction.

- Horizontal velocity is not constant!
- Conservation of horizontal slowness is the main reason why slowness is preferred to veloctiy in seismology.

Seismic Waves

Shear-wave splitting after Long and Becker 2010,
Mantle dynamics and seismic anisotropy
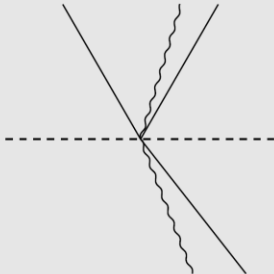
Seismic Waves

Conversion of Waves in Reflection and Refraction

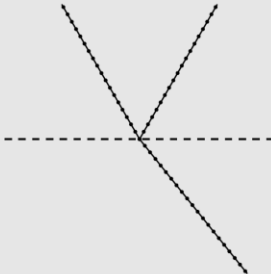Conversion of waves depends on the polarization of the S-waves.

Vertically polarized S-wave (SV) merges with P-waves.

Horizontally polarized S-wave (SH) is independent of P-waves and
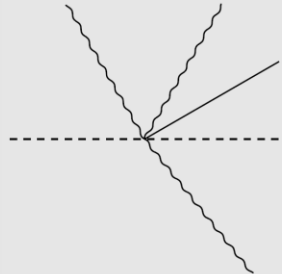SV-waves.

Incident P-wave   Incident SH-wave   Incident SV-wave

Seismics

Main Fields of Application

- Prospection and exploration of natural resources (oil, gas, ores, . . .).
- Exploration of the geological structure, e. g., for tunnel construction.
- Mapping of the interface between soil or unconsolidated rock and bedrock (ground investigation, slope instability, . . .).
- Mapping of aquifers.
- Mapping of residual waste sites.

Sources of Seismic Waves

While seismology uses the waves radiated by natural earthquakes, seismic uses artificial sources, e. g.,

- Hammer stroke
- Weight dropping
- Explosives
- Seismic vibrators ("Vibroseis")
- Airguns (in marine seismics)
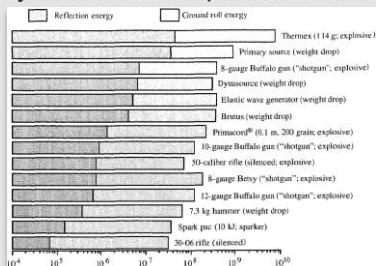
The different seismic sources differ concerning their energy and thus concerning their penetration depth.
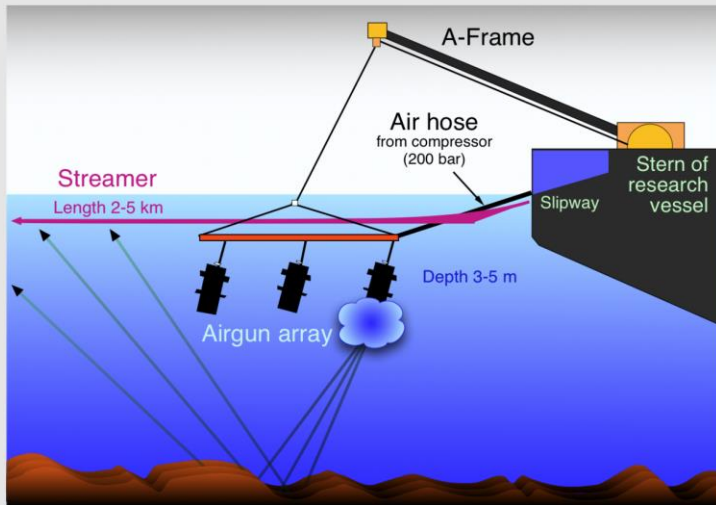
Hammer Stroke

Explosives

Source: Teaching material R. Scholger

Seismic Vibrators ("Vibroseis")

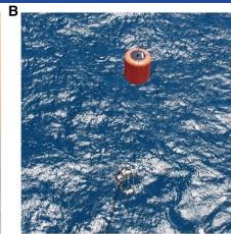Source: Wikipedia

Airguns (in Marine Seismics)

Source: Wikipedia

Airguns (in Marine Seismics)

A - VSP air gun

B - Air gun array

(source: IODP 2018)

Seismic Recording

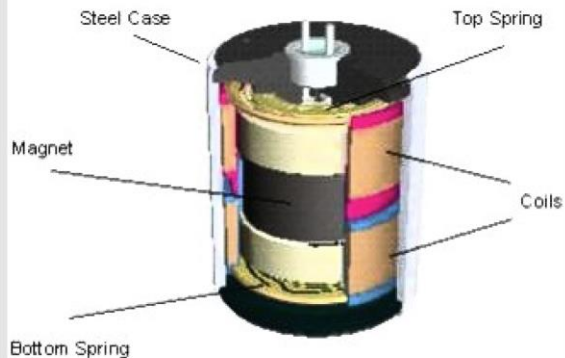Waves are recorded by either seismometers of geophones. Standard equipment:

Seismic source, e. g., hammer and plate.

Geophones
- In principle a simplified seismometer for small amplitudes.
- Similar to a microphone.
- Consists of a permanent magnet and a coil; one directly connected to the casing, while the other can move. the induced voltage is proportional to the relative velocity.

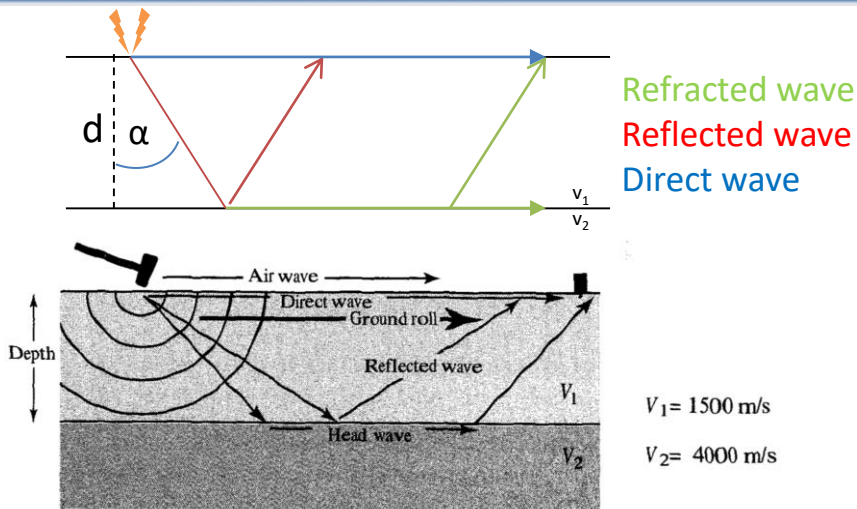**Seismograph unit** records the signals from 6–48 geophones (channels) and the seismic source.

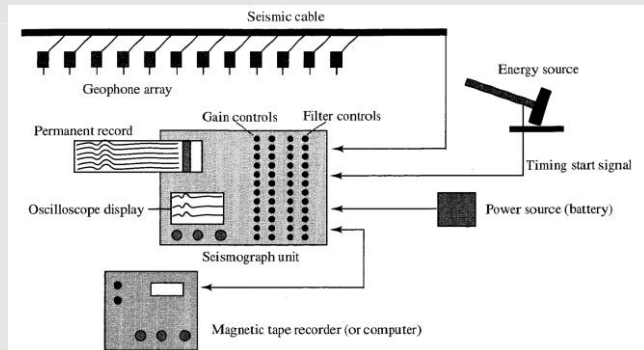## A Simple Geophone

Cut-a-way animation of SENSOR SM-24 Geophone

Typical Setup of Hammer Stroke Seismics

Refracted wave
Reflected wave
Direct wave

$V_1 = 1500$ m/s

$V_2 = 4000$ m/s

Source: Burger et al., Introduction to Applied Geophysics

Typical Setup of Hammer Stroke Seismics

De-Multiplexed Order	Channel		
Time	1	2	3
0.001	D001	E001	F001
0.002	D002	E002	F002
0.004	D003	E003	F003

Source: Burger et al., Introduction to Applied Geophysics

Signal form

Travel-time:
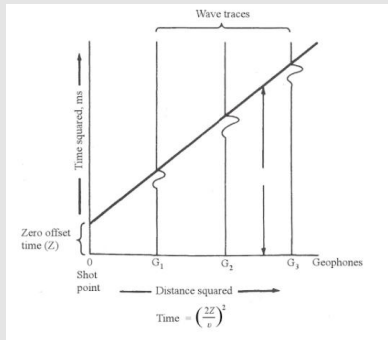
Impedanz contrast:
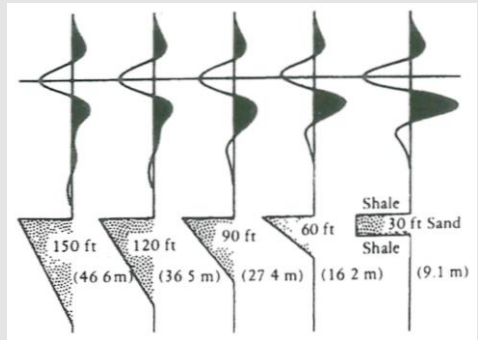
(source: Selley and Sonnenberg,1985)

Signal form

Reflectivitiy:

Center of trace

a

Wiggle only — Variable area — Variable density — Variable area/wiggle — Variable density/wiggle
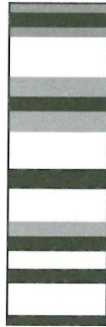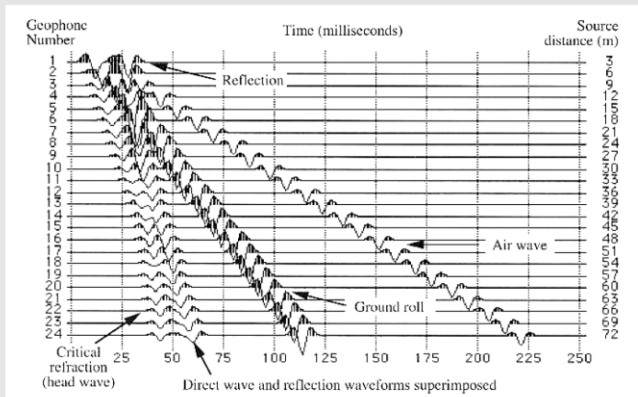
(source: Gluyas and Swarbrick, 2004)

Seismic Picking

In most cases, only the first arrival times of the different waves at the geophones are analyzed.

Source: Burger et al., Introduction to Applied Geophysics

Seismic Picking

The first arrival times are mostly considered to be

- the time when the signal significantly rises above the noise,
- the time of the first maximum minus a quarter of the period, or
- the time of the zero after the first maximum minus a half of the period.

Picking is often supported by software.

Seismics

Seismic Picking
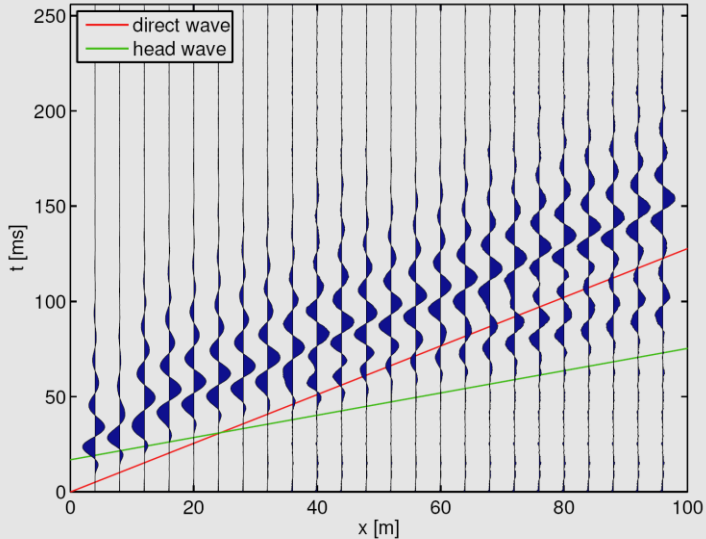
Signal attenuation / absorption

Distance from shot (m)	3	6	9	12	12	24	36	48	48	96	144	192	240
Spherical spreading los (db)	9,5	15,6	19,1	21,6	21,6	27,6	31,1	33,6	33,6	39,6	43,2	45,7	47,6
Loss from previous point (db)		6,02	3,52	2,50		6,02	3,52	2,50		6,02	3,52	2,50	1,94
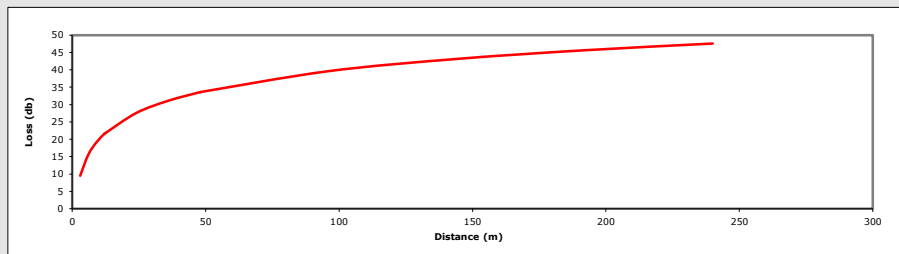
Seismic Methods

Reflection seismics: Evaluation of the waves reflected at discontinuities.

Refraction seismics: Evaluation of the head wave, i. e., the refracted wave at the limit to total internal refraction.

Seismic tomography: Inverting the signals of many sources and receivers. Mainly applied in large-scale seismology.

- Both reflection and refraction seismics address discontinuities where the seismic velocities change.
- Im most cases, only the first arrival times of the P-waves are evaluated.

Reflection Seismics

Basic Setup

Reflection Seismics
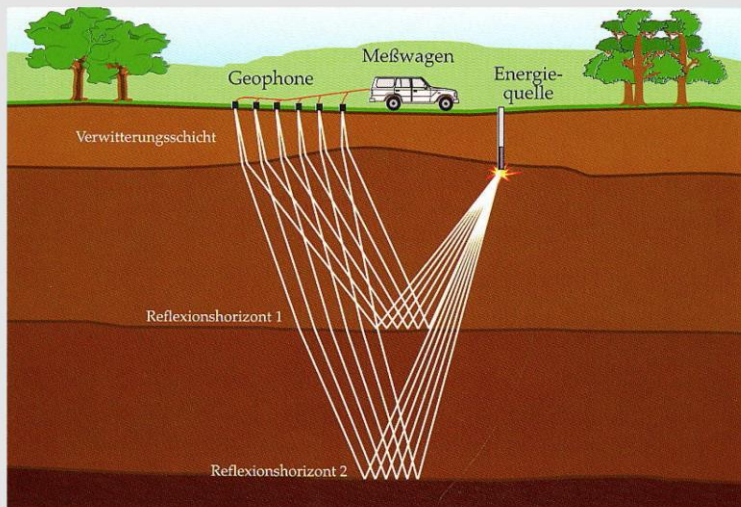
Reflection at the First, Horizontal Discontinuity

Travel time of the reflected wave to a geophone at distance x from the source (geophone offset):

$$t = \frac{\sqrt{4d_1^2 + x^2}}{v_1}$$
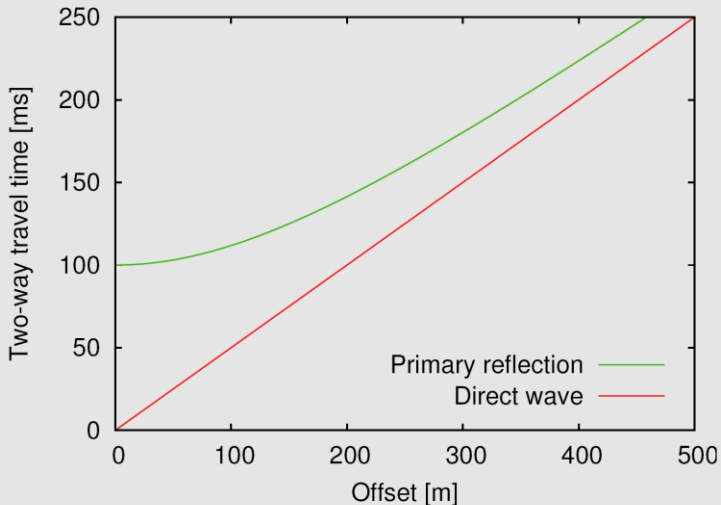
with

$$d_1 = \text{thickness of the upper layer}$$
$$v_1 = \text{P-wave velocity of the upper layer}$$

This equation describes a hyperbola:

$$\frac{t^2}{(2d_1/v_1)^2} - \frac{x^2}{(2d_1)^2} = 1$$

The Reflection Hyperbola

Evaluating the Reflection Hyperbola

- v_1 can be determined from the travel time curve of the direct wave.
- d_1 can be determined from v_1 and the hyperbola.

Question

What are v_1 and d_1 in the example on the previous page?

Evaluating the Reflection Hyperbola

v_1 can also be determined without the direct wave, either

- from the asymptotic slope of the hyperbola for $x \to \infty$ (requires large offsets), or
- from the curvature of the hyperbola.

Reflection Seismics

Multiple Reflections

Reflection Seismics
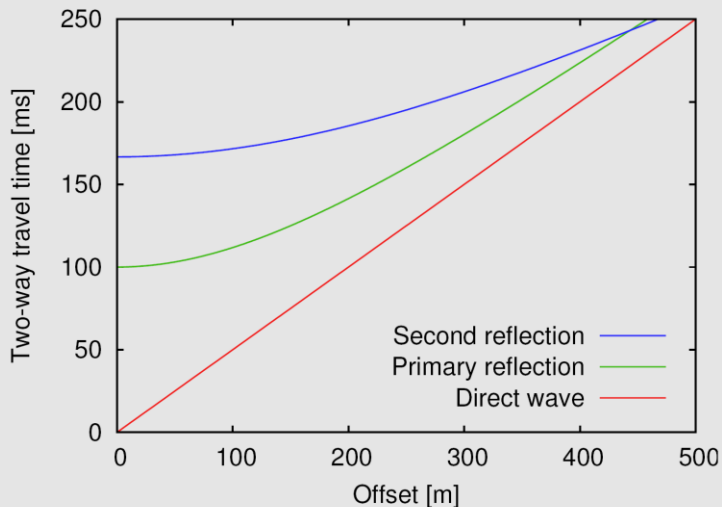
Multiple Reflections

- Even in horizontally layered media, secondary and later reflections are only approximately hyperbola.
- v_2, v_3, … can in principle be derived from the hyperbolas, but this is more complicated than for the primary reflection.
- The uncertainty increases from layer to layer.

Question

What is the depth of the second discontinuity in the example on the previous page?

Reflection at Tilted or Curved Interfaces

Transfer of travel times to depths (depth migration) can be done, e. g., by constructing spherical wave fronts.

Reflection Seismics

Intensity of the Reflected Waves

The intensity of the reflected waves depends on the contrast in the acoustic impedance

$$Z = \rho v.$$

Reflection coefficient R = amplitude ratio of reflected and incident wave

- In general, R increases with the contrast in Z.
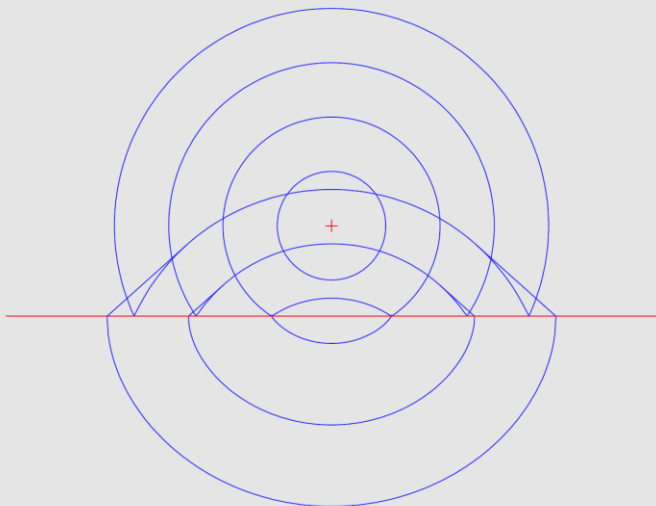- For normal incidence, P- and S-waves do not merge, and

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{\rho_2 v_2 - \rho_1 v_1}{\rho_2 v_2 + \rho_1 v_1}.$$

- Multiple reflexion costs much energy.

$$\Downarrow$$

Multiple-layer reflection seismics requires strong seismic sources.

Reflection and Refraction of a Spherical Wave at a Planar Interface
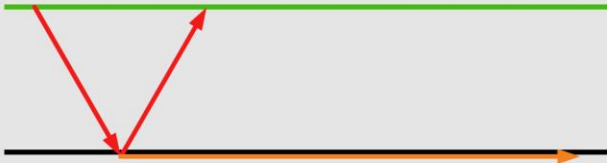
Refraction Seismics

Refraction at the Critical Angle

If the angle of incidence on a planar interface from a medium with v_1 to a medium with $v_2 > v_1$ achieves the critical value α_c with

$$\sin \alpha_c = \frac{v_1}{v_2}$$

the refracted waves propagates along the interface with the velocity v_2.

The Seismic Head Wave

Wave refracted at the critical angle = head wave = Mintrop wave (according to L. Mintrop, 1880–1956).

- Continuously radiates back into the first medium at the angle α_c.
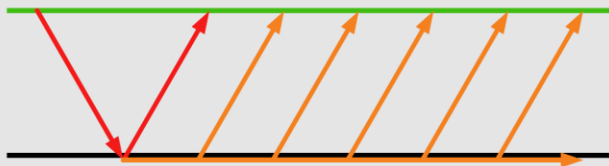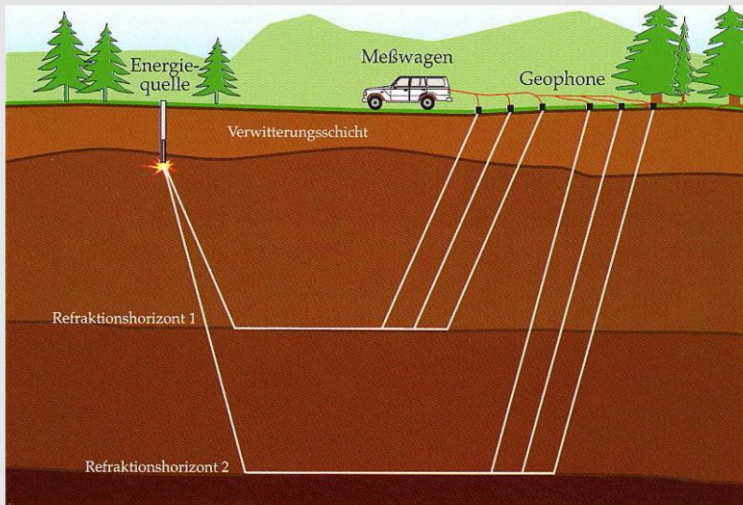

- Propagates along the surface with $v_2 > v_1$.

$\Downarrow$

Head wave finally outruns the direct wave (v_1)

Refraction Seismics

Basic Setup

Types of Waves

Source: Burger et al., Introduction to Applied Geophysics
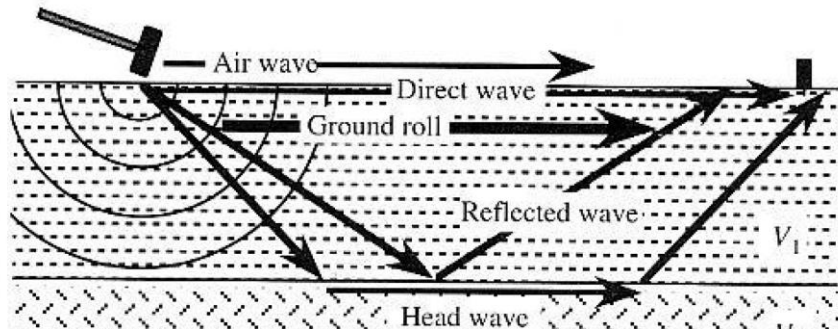
Travel Time of the Head Wave at a Horizontal Interface

The wave reflected at the critical angle α_c needs the time

$$t = \frac{2d}{v_1 \cos \alpha_c}$$

(d = layer's thickness) and arrives at the surface again at

$$x = 2d \tan \alpha_c.$$

⬇

The head wave reaches a receiver with offset x after

$$t = \frac{2d}{v_1 \cos \alpha_c} + \frac{x - 2d \tan \alpha_c}{v_2} = \frac{2d \cos \alpha_c}{v_1} + \frac{x}{v_2}$$

Refraction Seismics

Travel-Time Diagram for a Horizontal Interface
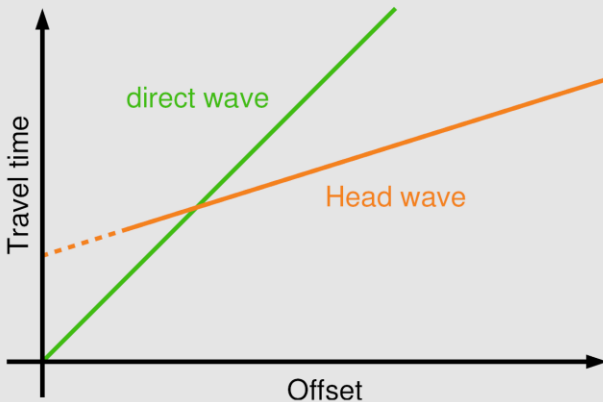

direct wave

Head wave

Travel time

Offset

Question

What would the travel time curve for the reflected wave look like?

Travel Time Diagram

- Mostly, the waves radiated at one source (shot point) are recorded with several geophones located on a straight line..
- Direct and refracted waves are straight lines in the diagram, while reflected waves are (approximately) hyperbola.
- Often only the earliest arriving wave is picked.

Quantitative Analysis in Case of a Horizontal Interface

1. Slope of the travel time curve of the direct wave $S_d = \frac{1}{v_1}$
2. Slope of the travel time curve of the refracted wave $S_r = \frac{1}{v_2}$
3. Intercept time = extrapolated time where the travel time curve of the refracted wave crosses the time axis

$$t_0 = \frac{2\,d\,\cos\alpha_c}{v_1} = 2\,d\,\sqrt{\frac{1}{v_1^2} - \frac{1}{v_2^2}}$$

so that

$$d = \frac{t_0\,v_1}{2\cos\alpha_c} = \frac{t_0}{2\sqrt{\frac{1}{v_1^2} - \frac{1}{v_2^2}}}$$

Example
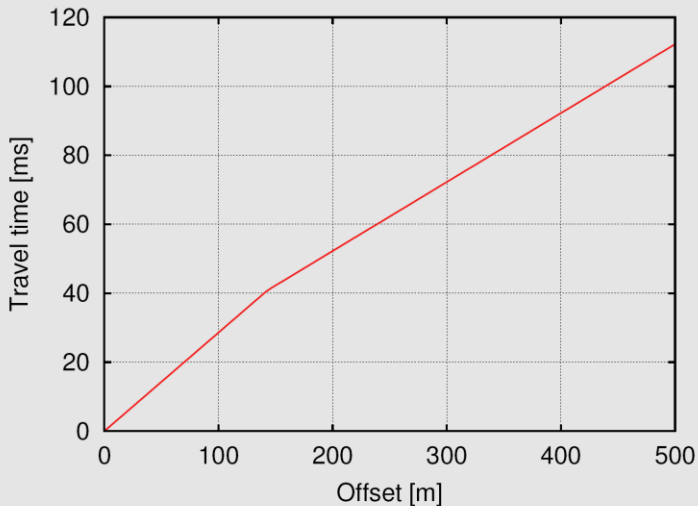
What is a Useful Offset?

The travel time curves of the direct wave and the head wave intersect at

$$\frac{x}{v_1} = \frac{2\,d\,\cos\alpha_c}{v_1} + \frac{x}{v_2}$$
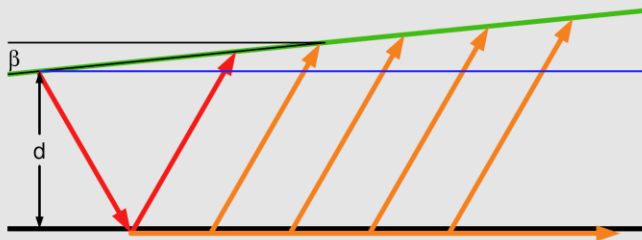
$$\Downarrow$$

$$x = \frac{2\,d\,\cos\alpha_c\,v_2}{v_2 - v_1} = 2\,d\,\sqrt{\frac{v_2 + v_1}{v_2 - v_1}}$$

The total offset (shot point to last geophone) should be a least two times x to recognize the travel time curves of both waves.

Question

What would be a reasonable offset for locating the ground water level in an aquifer of sand an gravel in about $12\,\mathrm{m}$ depth?

Travel Time Curve for a Sloping Interface

Intersection of the arriving rays with the blue line:

$$x_b = x \cos \beta - x \sin \beta \tan \alpha_c$$

Additional travel time from the blue line to the surface:

$$t - t_b = \frac{x \sin \beta}{v_1 \cos \alpha_c}$$

Travel Time Curve for a Sloping Interface

Head wave arrives at x_b on the blue line at

$$t_b = \frac{2\, d\, \cos \alpha_c}{v_1} + \frac{x_b}{v_2}$$

so that the total travel time is

$$t = \frac{2\, d\, \cos \alpha_c}{v_1} + \frac{x \cos \beta - x \sin \beta \tan \alpha_c}{v_2} + \frac{x \sin \beta}{v_1 \cos \alpha_c}$$

$$= \frac{2\, d\, \cos \alpha_c}{v_1} + \left(\frac{\cos \beta}{v_2} + \frac{\sin \beta \cos \alpha_c}{v_1} \right) x$$

Travel Time Curve for a Sloping Interface

Slope of the travel time curve:

$$S_r = \frac{\cos \beta}{v_2} + \frac{\sin \beta \cos \alpha_c}{v_1} = \frac{\sin (\alpha_c + \beta)}{v_1}$$

⬇

If the interface dips in shot direction (downdip), the head wave is apparently slower than for a horizontal interface and vice versa.

Intercept time

$$t_0 = \frac{2 \, d \cos \alpha_c}{v_1}$$

is the same as for a horizontal interface if the depth is taken perpendicularly to the interface (not to the surface).

Shot and Reverse Shot

For a sloping interface the travel time curve is not sufficient to determine v_2, d, and β.

Measure a second travel time curve opposite to the original one (reverse shot).

Shot and Reverse Shot

Steps of analysis:

1. Slope of the travel time curve of the direct wave $S_d = \frac{1}{v_1}$ (same as for a horizontal interface).

2. Use either

   (a) mean value of the absolute slopes of the travel time curves of both head waves:

   $$\overline{S}_r = \frac{\cos \beta}{v_2} \approx \frac{1}{v_2}$$

   if β is not too large or

   (b) absolute slopes of the travel time curves of both head waves:

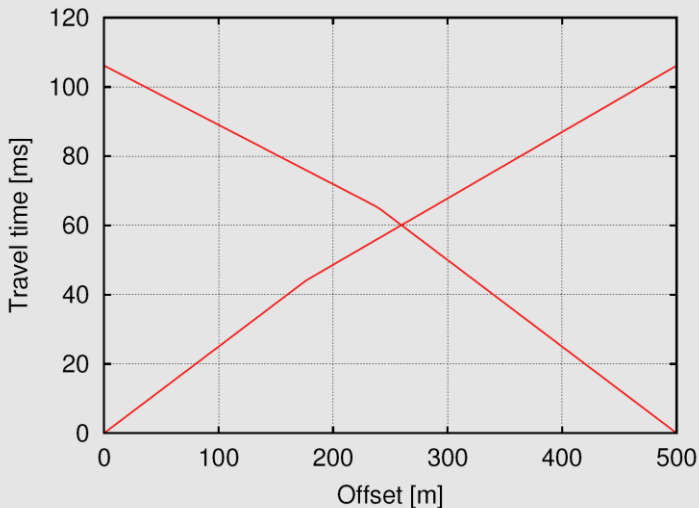 $$S_r = \frac{\sin (\alpha_c \pm \beta)}{v_1}.$$

Shot and Reverse Shot

3. Depth of the interface is similar to the horizontal case:

$$d = \frac{t_0\, v_1}{2 \cos \alpha_c} = \frac{t_0}{2 \sqrt{\frac{1}{v_1^2} - \frac{1}{v_2^2}}}$$

where the intercept times of shot and reverse shot yield the depth below the respective shot point (normal to the interface).

If β is not small, the (vertical) depths can be obtained by $\frac{d}{\cos \beta}$.

Example

Multiple Refraction

- Several head waves.
- Evaluation becomes more complicated, but without principal problems.
- Important limitation: Only interfaces where the velocity increases towards to lower layer can be detected (also applies to the case of two layers).

Refraction Seismics vs. Reflection Seismics

Advantages of Reflection Seismics

- High spatial resolution
- Complex geological structures (non-planar interfaces) can in principle be resolved.
- Layers with lower velocities can also be detected.

Advantages of Refraction Seismics

- Relatively simple evaluation.
- Moderate requirements on seismic energy.