

# Seismology and Seismic Hazard

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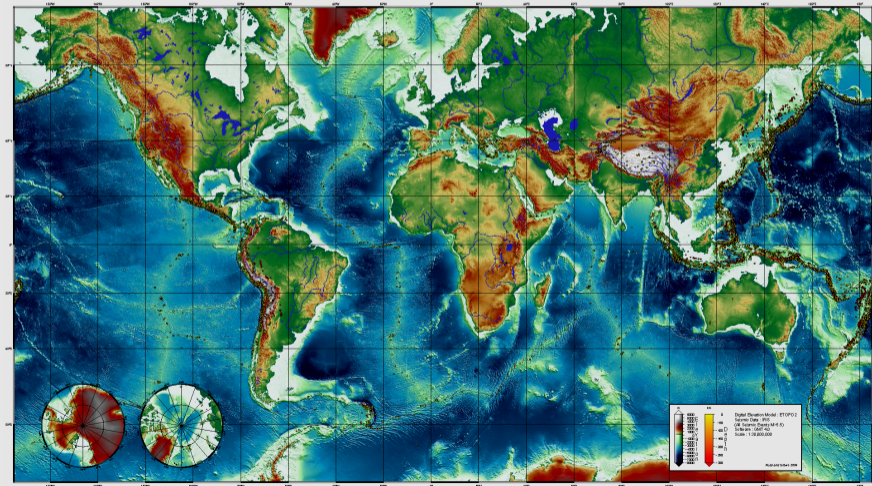
## Seismology

- Comprises all about earthquakes and the propagation of seismic waves in the Earth.
- One of the main fields of solid-earth geophysics.
- Has provided the majority of our knowledge on Earth's interior.

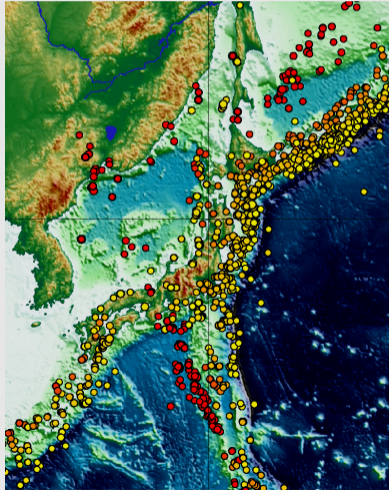
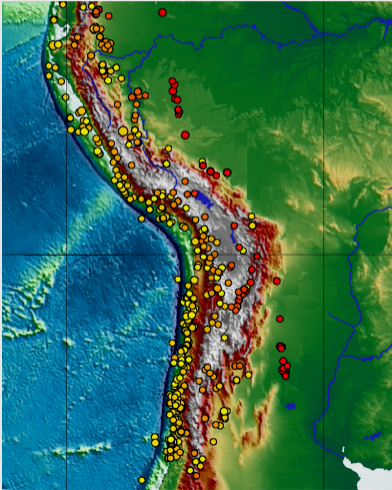
## Seismics

- Exploration of the deep and shallow subsurface with the help of artificial seismic waves.
- The perhaps most important field of applied geophysics.

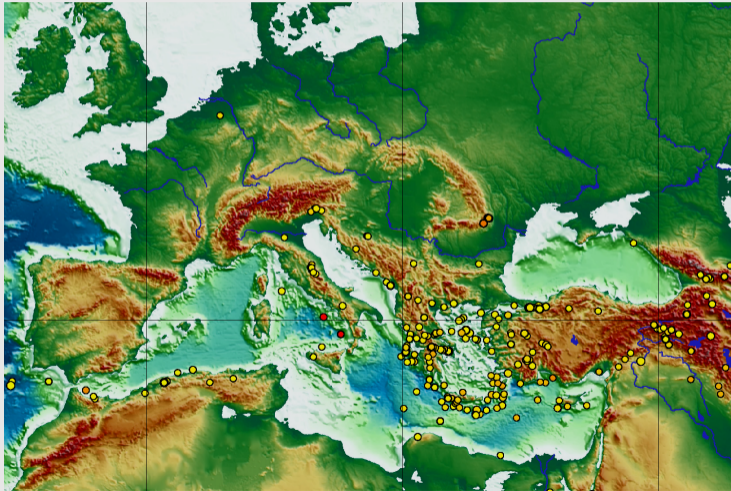
## Worldwide Distribution of Earthquakes



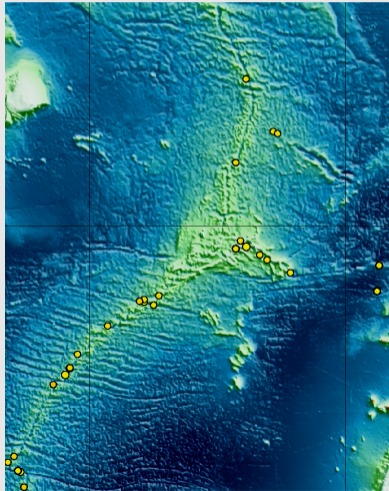
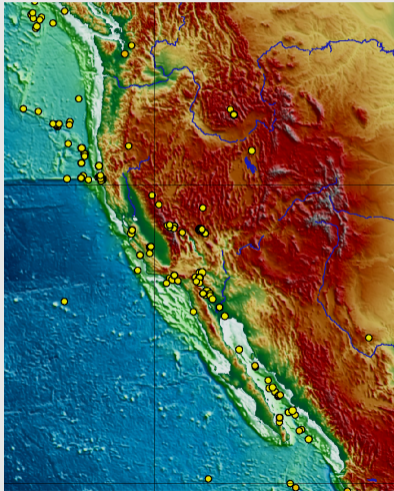
## Worldwide Distribution of Earthquakes



## Worldwide Distribution of Earthquakes



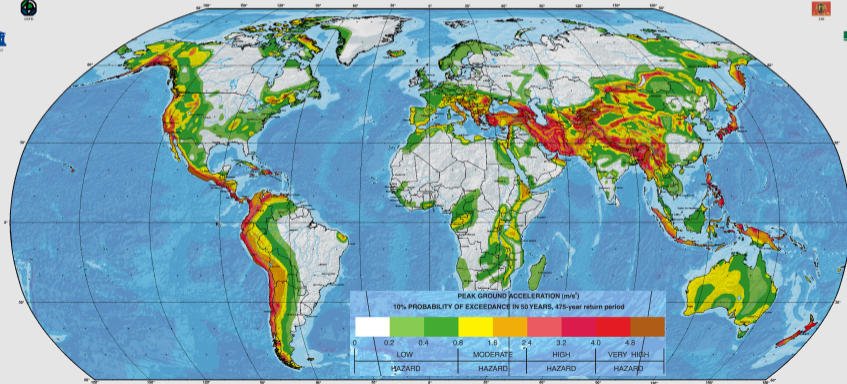
## Worldwide Distribution of Earthquakes



## Earthquake Hazard

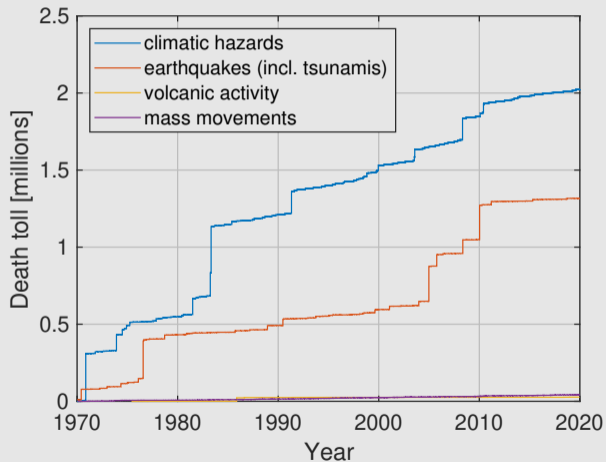
### GLOBAL SEISMIC HAZARD MAP

Produced by the Global Seismic Hazard Assessment Program (GSHAP),  
a demonstration project of the UN International Decade of Natural Disaster Reduction, conducted by the International Lithosphere Program.  
Global map assembled by D. Giardini, G. Grünthal, K. Shedlock, and P. Zhang  
1999



Source: Global Seismic Hazard Assessment Program

## Cumulative Death Toll Since 1970



Data: Centre for Research on the Epidemiology of Disasters



## The First "Seismometer" (132 a.D.)



## History of Seismology

1660	basic law of elasticity	R. Hooke
1821–22	differential equations of elasticity	C. Navier, A. L. Cauchy
1830	theory of two fundamental types of elastic waves (P- and S-wave)	S. D. Poisson
1875	First “serious” seismometer	F. Gecchi
1887	theory of the first type of surface waves	J. W. Strutt (3. Lord Rayleigh)
1889	first recording of a distant earthquake	
1892	first compact seismometer, used at about 40 stations	J. Milne
1894	statistics of aftershocks	F. Omori
1903	12 degree scale for the intensity of earthquakes based on the damage	G. Mercalli

## History of Seismology

1906–1913	detection of the liquid core of the earth and determination of its size	R. D. Oldham, B. Gutenberg
1909	detection of the crust-mantle discontinuity	A. Mohorovičić
1911	theory of a second type of surface waves	A. E. H. Love
1935	local magnitude as an “objective” measure of earthquake intensity	C. F. Richter
1936	detection of the inner, solid core	I. Lehmann
1954	frequency-magnitude relation of earthquakes	B. Gutenberg, C. F. Richter
1975	first successful short-term prediction of a strong earthquake	
1977	moment magnitude as a measure of earthquake source strength	H. Kanamori

## The Navier-Cauchy Equations in Seismology

- Small, but spatially and temporally variable displacement  $\vec{u}(\vec{x}, t)$
- Neglect gravity
- Sign convention as in mathematics, physics, and engineering
- Elastic deformation



$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} = \begin{pmatrix} \frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} + \frac{\partial \sigma_{13}}{\partial x_3} \\ \frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{23}}{\partial x_3} \\ \frac{\partial \sigma_{31}}{\partial x_1} + \frac{\partial \sigma_{32}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3} \end{pmatrix} = \text{div}(\boldsymbol{\sigma}) \quad (1)$$

## The Navier-Cauchy Equations in Seismology

with the stress tensor

$$\boldsymbol{\sigma} = \lambda \epsilon_v \mathbf{1} + 2\mu \boldsymbol{\epsilon}, \quad (2)$$

the strain tensor  $\boldsymbol{\epsilon}$  consisting of the components

$$\epsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad (3)$$

the volumetric strain

$$\epsilon_v = \epsilon_{11} + \epsilon_{22} + \epsilon_{33}, \quad (4)$$

the density  $\rho$ , and

the Lamé parameters of the medium  $\lambda$  and  $\mu$

## The Navier-Cauchy Equations in 1D

Displacement  $u(x, t)$  instead of  $\vec{u}(\vec{x}, t)$ .



$$\rho \frac{\partial^2}{\partial t^2} u = \frac{\partial}{\partial x} \sigma \quad (5)$$

with

$$\sigma = (\lambda + 2\mu) \epsilon = (\lambda + 2\mu) \frac{\partial}{\partial x} u \quad (6)$$



$$\rho \frac{\partial^2}{\partial t^2} u(x, t) = \frac{\partial}{\partial x} \left( (\lambda + 2\mu) \frac{\partial}{\partial x} u(x, t) \right) \quad (7)$$

## The Navier-Cauchy Equations in 1D

Solution if  $\lambda$  and  $\mu$  are constant:

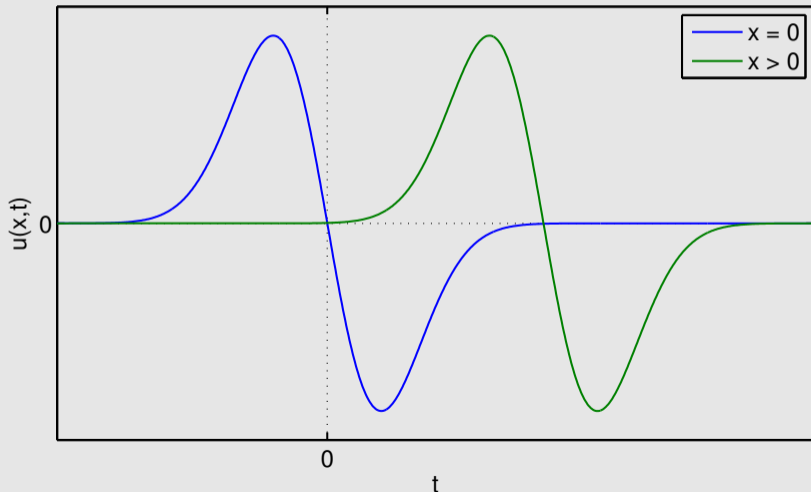
$$u(x, t) = f(t \pm sx) \quad (8)$$

where

$f$  = arbitrary function describing  
= the shape of the wave

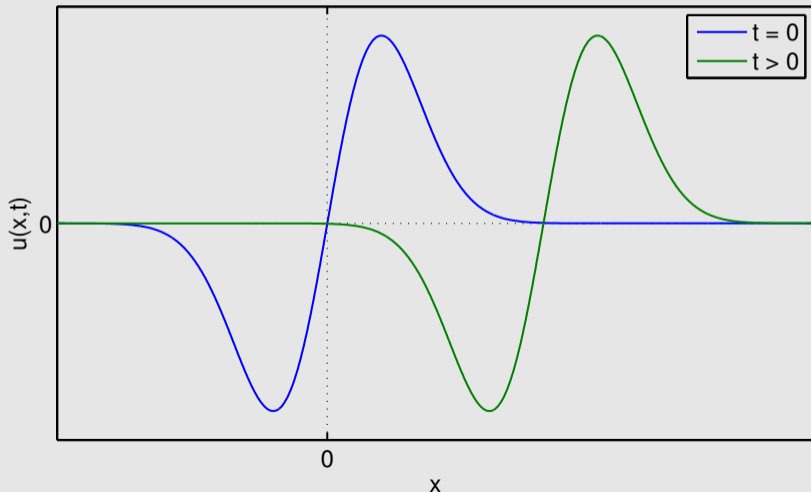
$$s = \sqrt{\frac{\mu}{\lambda + 2\mu}} = \text{slowness} \left[ \frac{\text{m}}{\text{s}} \right]$$

## Example





## Example



## The Retarded Time

$\tau = t \pm sx$  is called retarded time.

Meaning: At the position  $x$  and time  $t$ , we observe what happened at the origin ( $x = 0$ , e. g. earthquake focus) at the retarded time  $\tau = t \pm sx$ .

## The Shape of the Wave

Examples for the function  $f$ :

- $f(\tau) = \begin{cases} 1 & \text{for } \tau \geq T \\ 0 & \text{else} \end{cases}$

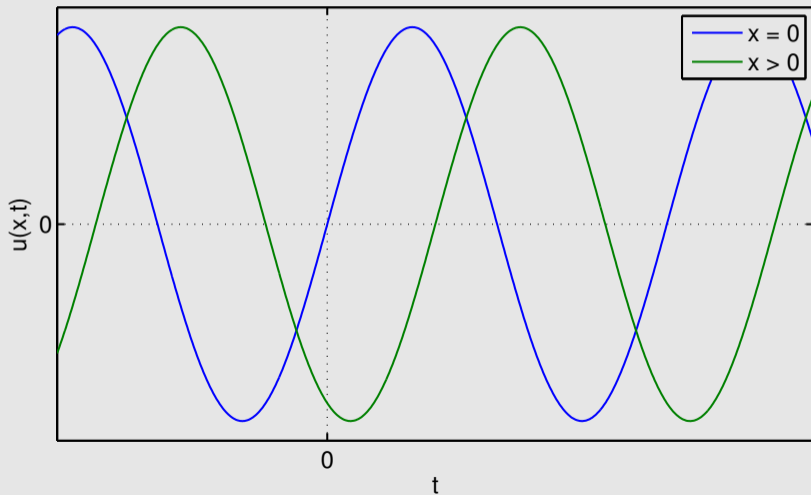
describes a step-like shape (shock wave).

- $f(\tau) = \begin{cases} 1 & \text{for } |\tau| \leq \frac{T}{2} \\ 0 & \text{else} \end{cases}$

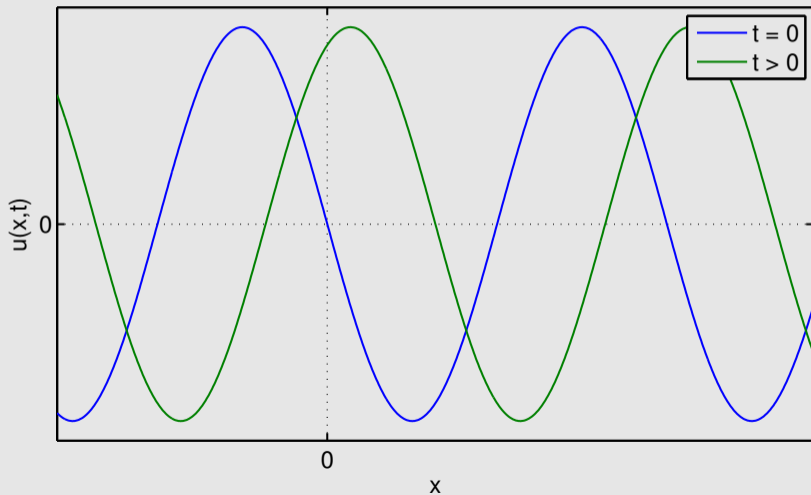
describes a boxcar-shaped wave.

- $f(\tau) = a \cos(\omega\tau)$ ,  $f(\tau) = a \sin(\omega\tau)$  or  $f(\tau) = a e^{i\omega\tau}$  describes a harmonic wave with angular frequency  $\omega$  and amplitude  $a$ .

## Basic Terms



## Basic Terms



## Basic Terms

- Time domain:

Angular frequency:  $\omega$  [ ]

Frequency:  $\nu = \frac{\omega}{2\pi}$  [ ]

Period:  $T = \frac{1}{\nu} = \frac{2\pi}{\omega}$  [ ]

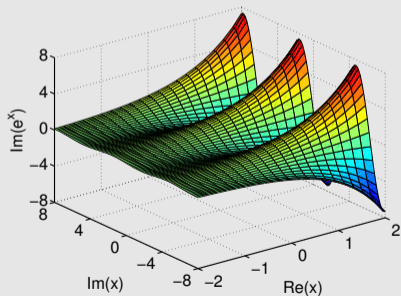
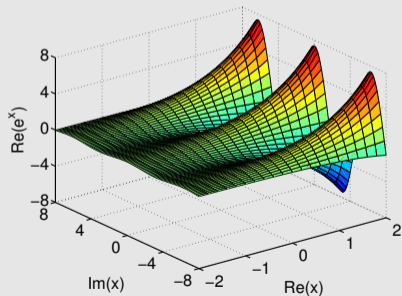
- Spatial wave pattern:

Wave number:  $k = \omega s$  [ ]

Wavelength:  $L = \frac{2\pi}{k} = \frac{1}{\nu s}$  [ ]

## The Complex Exponential Function vs. Sine and Cosine

With  $e^{i\phi} = \cos \phi + i \sin \phi$ , the complex exponential function combines the real exponential function with the sine and cosine functions.



## The Complex Exponential Function vs. Sine and Cosine

$$\operatorname{Re}(e^{i\omega\tau}) = \cos(\omega\tau) \quad (9)$$

$$\operatorname{Im}(e^{i\omega\tau}) = \sin(\omega\tau) \quad (10)$$



Real part and imaginary part of the complex solution can be considered as independent real solutions.



## The Complex Exponential Function vs. Sine and Cosine

Derivatives of the complex solutions are simpler than those of the real solutions:

$$\frac{\partial}{\partial \tau} e^{i\omega\tau} = \text{[redacted]} \quad (11)$$

while

$$\frac{\partial}{\partial \tau} \cos(\omega\tau) = \text{[redacted]} \quad (12)$$

$$\frac{\partial}{\partial \tau} \sin(\omega\tau) = \text{[redacted]} \quad (13)$$

## Seismic Waves in 3D

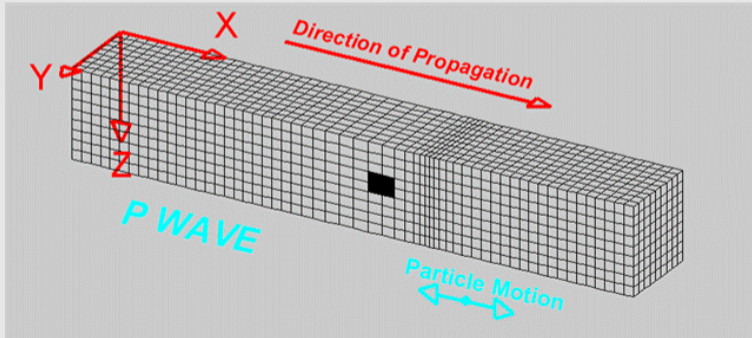
Complications towards the 1D case:

- Displacement  $\vec{u}(\vec{x}, t)$  is a vector.
- Propagation in arbitrary direction in space instead of the positive or negative  $x$  axis only.

## Fundamental Types of Body Waves

Two types of independent plane waves in an infinite, homogeneous elastic medium:

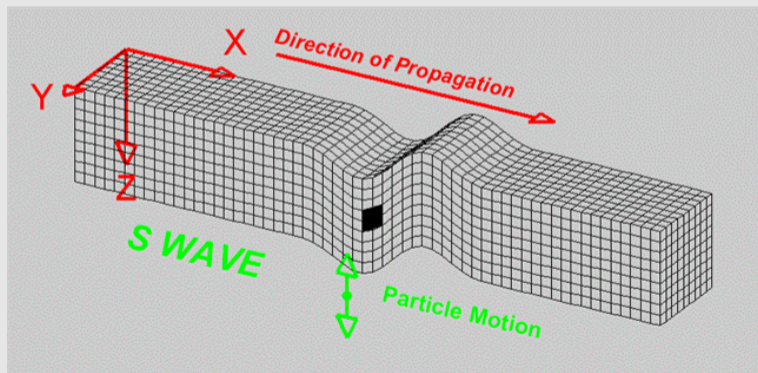
- Compressional wave (longitudinal wave, primary wave)



Source: L. Braille, Purdue University

## Fundamental Types of Body Waves

- Shear wave (transverse wave, secondary wave)



Source: L. Braille, Purdue University

## Plane Waves in Homogeneous, Isotropic Media

Plane wave:  $\vec{u}(\vec{x}, t)$  is constant on parallel planes.

Mathematical description:

$$\vec{u}(\vec{x}, t) = f(t - \vec{s} \cdot \vec{x}) \vec{a} \quad (14)$$

or for a harmonic wave:

$$\vec{u}(\vec{x}, t) = e^{i\omega(t - \vec{s} \cdot \vec{x})} \vec{a} \quad (15)$$

where

$\vec{s}$  = slowness vector

$\vec{a}$  = amplitude vector (constant)

## Plane Waves in Homogeneous, Isotropic Media

Simplest version: propagation in  $x_1$  direction,  $\vec{s} = \begin{pmatrix} s \\ 0 \\ 0 \end{pmatrix}$





$$\vec{u}(\vec{x}, t) = f(t - sx_1) \vec{a} \quad (16)$$



$$\epsilon = -s f'(t - sx_1) \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix} \quad (17)$$

## Plane Waves in Homogeneous, Isotropic Media


$$\sigma = -s f'(t - sx_1) \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix} \quad (18)$$


$$\operatorname{div}(\sigma) = s^2 f''(t - sx_1) \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix} \quad (19)$$

if  $\lambda$  and  $\mu$  are constant.

## Plane Waves in Homogeneous, Isotropic Media

Insert into the Navier-Cauchy equations:

$$\rho \frac{\partial^2}{\partial t^2} \vec{u} = \rho f''(t - sx_1) \vec{a} \quad (20)$$

$$= \operatorname{div}(\boldsymbol{\sigma}) = s^2 f''(t - sx_1) \begin{pmatrix} (\lambda + 2\mu) a_1 \\ \mu a_2 \\ \mu a_3 \end{pmatrix} \quad (21)$$

Can only be satisfied if

- = 0 and  $s = \sqrt{\text{input}}$  (longitudinal polarization)

or

- = 0 and  $s = \sqrt{\text{input}}$  (transverse polarization).



## Plane Waves in Homogeneous, Isotropic Media

General case: Navier-Cauchy equations can be satisfied only if either  $\vec{a}$  is parallel (or opposite) to  $\vec{s}$  or normal to  $\vec{s}$ .

**Transverse wave:**  $\vec{a}$  is normal to  $\vec{s}$  ( $\vec{a} \cdot \vec{s} = 0$ )

$$|\vec{s}|^2 = \frac{\rho}{\mu}, \quad v_s = \frac{1}{|\vec{s}|} = \sqrt{\frac{\mu}{\rho}} \quad (22)$$

**Longitudinal wave:**  $\vec{a}$  is parallel or opposite to  $\vec{s}$

$$|\vec{s}|^2 = \frac{\rho}{\lambda + 2\mu}, \quad v_p = \frac{1}{|\vec{s}|} = \sqrt{\frac{\lambda + 2\mu}{\rho}} \quad (23)$$

## Comparison with Sound Waves in Liquids and Gases

The longitudinal wave is similar to sound waves in liquids and gases, while the transverse wave has no counterpart in liquids and gases.

## Seismic Velocities

Medium	Longitudinal wave [ $\frac{\text{km}}{\text{s}}$ ]	Transverse wave [ $\frac{\text{km}}{\text{s}}$ ]
air	0.34	–
water	1.45	–
wood	about 3	about 1.8
Earth*	5.8–13.7	3.4–7.2

\*Parametric Earth Models (PEM), not valid for the shallow subsurface

## Seismic Velocities

$$\frac{v_p}{v_s} = \sqrt{\frac{\lambda + 2\mu}{\mu}} \geq \sqrt{\frac{4}{3}} \approx 1.15 \quad (24)$$



Longitudinal wave always arrives prior to the transverse wave.



longitudinal wave = primary wave (P-wave)  
transverse wave = secondary wave (S-wave)

## Typical $v_p$ - $v_s$ Ratios

- For solid rocks:

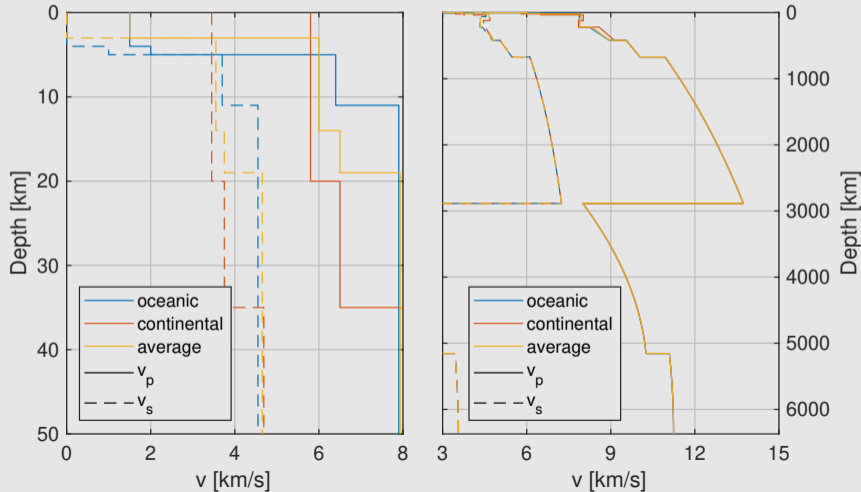
$$\frac{v_p}{v_s} = \sqrt{\frac{\lambda + 2\mu}{\mu}} \approx \sqrt{3} \approx 1.7 \quad (25)$$

for  $\lambda \approx \mu$ .

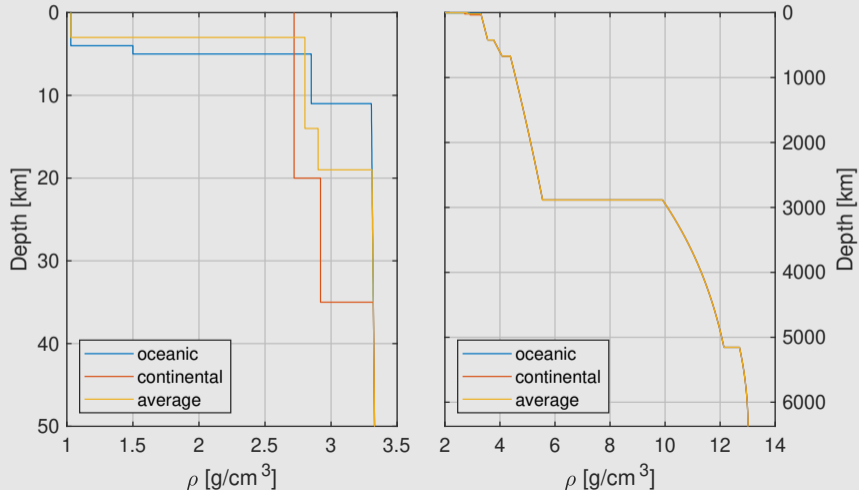
- For soil or unconsolidated rocks:

$$\frac{v_p}{v_s} \approx 2.5 \quad (26)$$

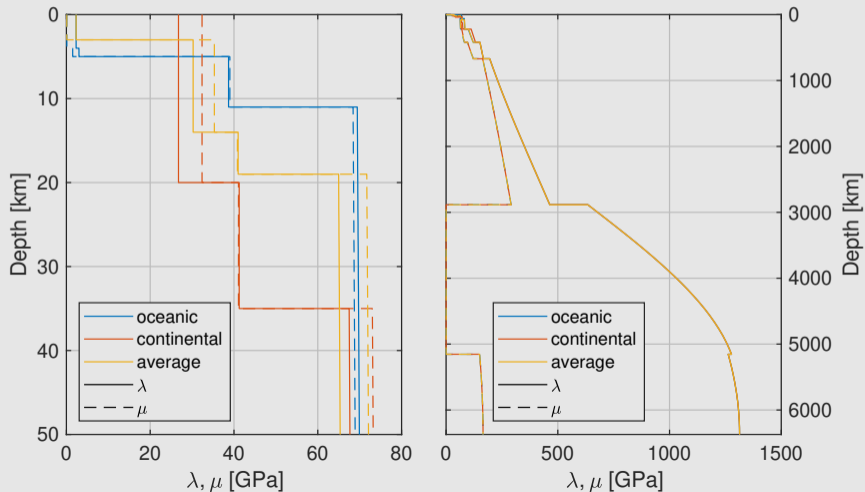
## Seismic Velocities According to the Parametric Earth Models (PEM)



## Density According to the Parametric Earth Models (PEM)



## Lamé Parameters According to the Parametric Earth Models (PEM)



## Typical P-wave Velocities in the Shallow Subsurface

Medium	$v_p$ [ $\frac{\text{km}}{\text{s}}$ ]	Medium	$v_p$ [ $\frac{\text{km}}{\text{s}}$ ]
weathering zone	0.1–0.5	clay	1.2–2.8
dry sand	0.3–0.6	claystone	2.2–4.2
water-saturated sand	1.3–1.8	limestone	3–6
sandstone	1.8–4	halite	4.5–6.5
pit coal	1.6–1.9	granite	5–6.5



## The Point-Force Solution

- Infinite, homogeneous medium with parameters  $\rho$ ,  $\lambda$ , and  $\mu$  (like plane wave consideration).
- Assume that a given force  $\vec{F}(t)$  acts at  $\vec{x} = \vec{0}$ .

Solution of the Navier-Cauchy equations:

$$\begin{aligned} \vec{u}_f(\vec{x}, t) = & \frac{s_p^2}{4\pi\rho r} \mathbf{P} \vec{F}(t - s_p r) + \frac{s_s^2}{4\pi\rho r} (\mathbf{1} - \mathbf{P}) \vec{F}(t - s_s r) \\ & + \frac{1}{4\pi\rho r^3} (3\mathbf{P} - \mathbf{1}) \int_{s_p r}^{s_s r} \tau \vec{F}(t - \tau) d\tau \quad (27) \end{aligned}$$

where

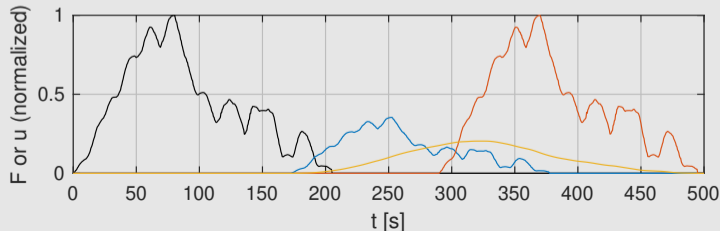
$$r = |\vec{x}|$$

$\mathbf{P}$  = projection on the radial direction

## The Point-Force Solution

$$\begin{aligned} \vec{u}_f(\vec{x}, t) = & \frac{s_p^2}{4\pi\rho r} \mathbf{P}\vec{F}(t - s_p r) + \frac{s_s^2}{4\pi\rho r} (\mathbf{1} - \mathbf{P})\vec{F}(t - s_s r) \\ & + \frac{1}{4\pi\rho r^3} (3\mathbf{P} - \mathbf{1}) \int_{s_p r}^{s_s r} \tau \vec{F}(t - \tau) d\tau \quad (28) \end{aligned}$$

Example:



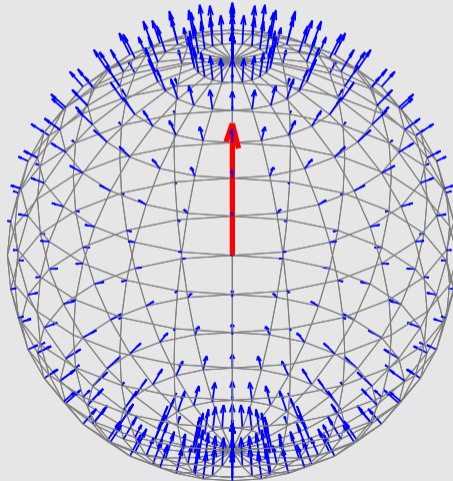
## The Point-Force Solution

Spatial pattern of the first term,

$$\mathbf{P}\vec{F},$$

for

$$\vec{F} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



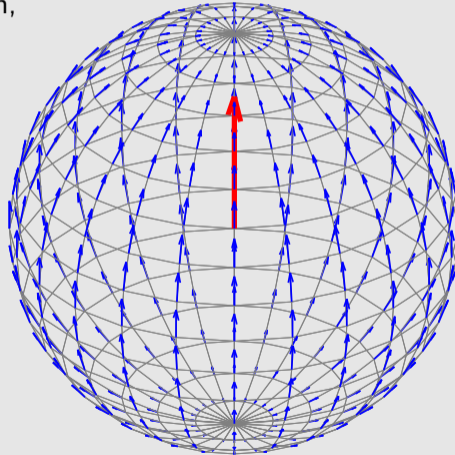
## The Point-Force Solution

Spatial pattern of the second term,

$$(\mathbf{1} - \mathbf{P}) \vec{F},$$

for

$$\vec{F} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



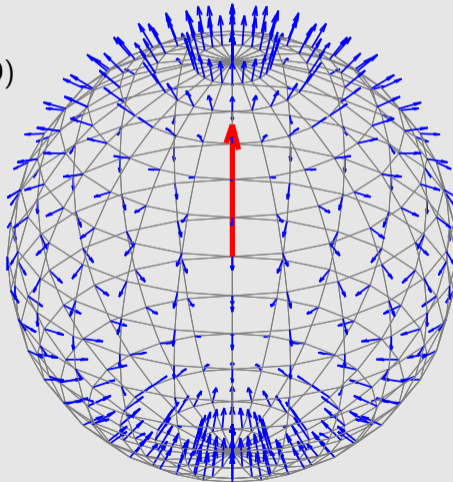
## The Point-Force Solution

Spatial pattern of the third term,

$$(3\mathbf{P} - \mathbf{1})\vec{F}, \quad (29)$$

for

$$\vec{F} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



## Force Couples

Solution for a single point force causes an overall displacement in direction of the force.



not possible



Consider a couple of opposite forces  $\vec{F}$  and  $-\vec{F}$  displaced by a small vector  $\vec{a}$  (at  $\frac{\vec{a}}{2}$  and  $-\frac{\vec{a}}{2}$ ):

$$\vec{u}(\vec{x}, t) = \vec{u}_f(\vec{x} - \frac{\vec{a}}{2}, t) - \vec{u}_f(\vec{x} + \frac{\vec{a}}{2}, t) \approx \boxed{\phantom{\vec{u}(\vec{x}, t)}} \quad (30)$$

## Force Couples



$$\begin{aligned} \vec{u}_f(\vec{x}, t) = & \frac{s_p^2}{4\pi\rho r} \mathbf{P} \vec{F}(t - s_p r) + \frac{s_s^2}{4\pi\rho r} (\mathbf{1} - \mathbf{P}) \vec{F}(t - s_s r) \\ & + \frac{1}{4\pi\rho r^3} (3\mathbf{P} - \mathbf{1}) \int_{s_p r}^{s_s r} \tau \vec{F}(t - \tau) d\tau \end{aligned} \quad (31)$$



$$\begin{aligned} -\nabla \vec{u}_f(\vec{x}, t) = & \frac{s_p^2}{4\pi\rho r} \mathbf{P} \quad \square + \frac{s_s^2}{4\pi\rho r} (\mathbf{1} - \mathbf{P}) \quad \square \\ & + \text{terms with } \frac{1}{r^2}, \frac{1}{r^3}, \dots \end{aligned} \quad (32)$$

where  $\vec{e} = \frac{\vec{x}}{r}$  = unit vector in radial direction.

## The Seismic Moment Tensor



$$\vec{u}(\vec{x}, t) = -(\nabla \vec{u}_f(\vec{x}, t)) \vec{a} \quad (33)$$

$$= \frac{s_p^3}{4\pi\rho r} \mathbf{P} \quad \square \quad + \frac{s_s^3}{4\pi\rho r} (\mathbf{1} - \mathbf{P}) \quad \square$$

+ terms with  $\frac{1}{r^2}, \frac{1}{r^3}, \dots$  (34)

$$= \frac{s_p^3}{4\pi\rho r} \mathbf{P} \quad \square \quad + \frac{s_s^3}{4\pi\rho r} (\mathbf{1} - \mathbf{P}) \quad \square$$

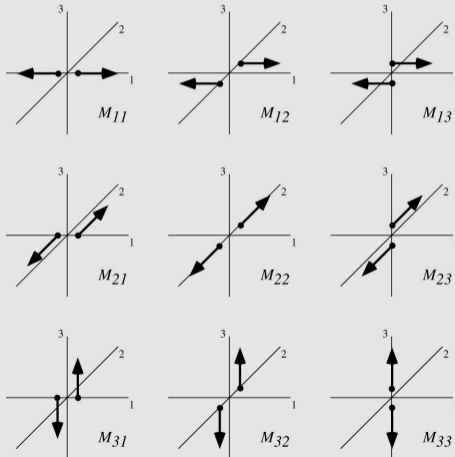
+ terms with  $\frac{1}{r^2}, \frac{1}{r^3}, \dots$  (35)

with the seismic moment tensor (centroid moment tensor, CMT)

$$\mathbf{M}(t) = \vec{F}(t) \vec{a}^T \quad [Nm] \quad (36)$$



## Components of the Seismic Moment Tensor



Source: Shearer, Introduction to Seismology

## Seismic Waves vs. Permanent Displacement

Terms with the longest range ( $\sim \frac{1}{r}$ ) depend on  $\dot{\mathbf{M}}$  and not on  $\mathbf{M}$ .



Vanish some time after the earthquake has terminated.



Transient displacement = seismic waves

$$\vec{u}(\vec{x}, t) = \frac{s_p^3}{4\pi\rho r} \mathbf{P} \dot{\mathbf{M}}(t - s_p r) \vec{e} + \frac{s_s^3}{4\pi\rho r} (\mathbf{1} - \mathbf{P}) \dot{\mathbf{M}}(t - s_s r) \vec{e} \quad (37)$$

↑  
P-wave

↑  
S-wave

with  $\vec{e} = \frac{\vec{x}}{r}$  and  $\mathbf{P} = \vec{e} \vec{e}^T$

## Seismic Waves vs. Permanent Displacement

Terms  $\sim \frac{1}{r^2}$  also contain components with **M**.



Persist after the earthquake has terminated (**M** = const.)



Permanent displacement

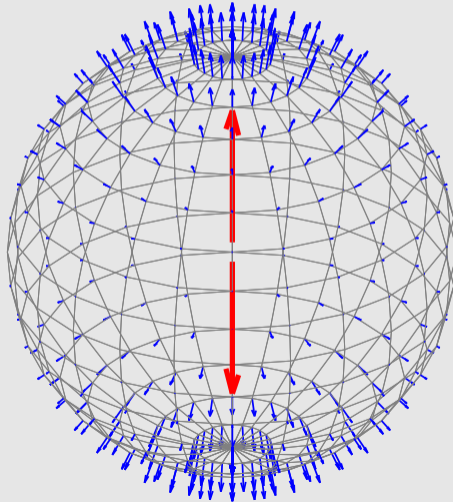
## Seismic Wave Patterns

P-wave radiation pattern

$$\mathbf{P}\dot{\mathbf{M}}\vec{e}$$

for

$$\dot{\mathbf{M}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



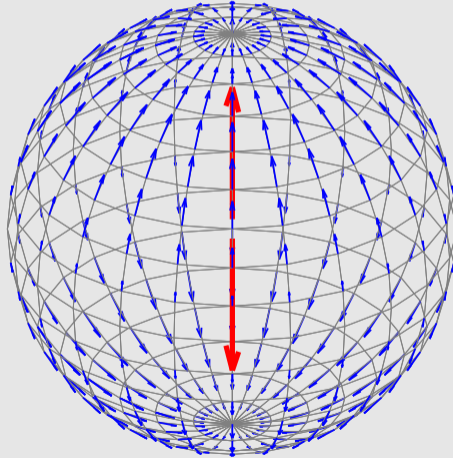
## Seismic Wave Patterns

S-wave radiation pattern

$$(\mathbf{1} - \mathbf{P}) \dot{\mathbf{M}} \vec{e}$$

for

$$\dot{\mathbf{M}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



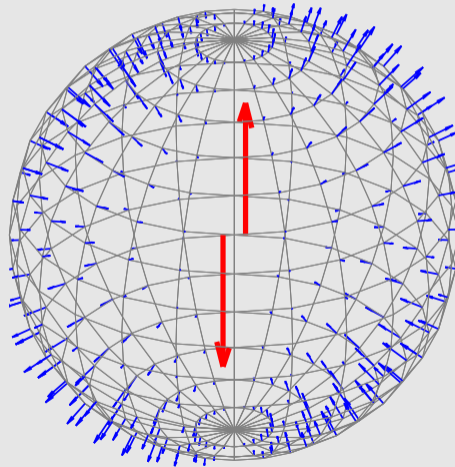
## Seismic Wave Patterns

P-wave radiation pattern

$$P\dot{\mathbf{M}}\vec{e}$$

for

$$\dot{\mathbf{M}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$



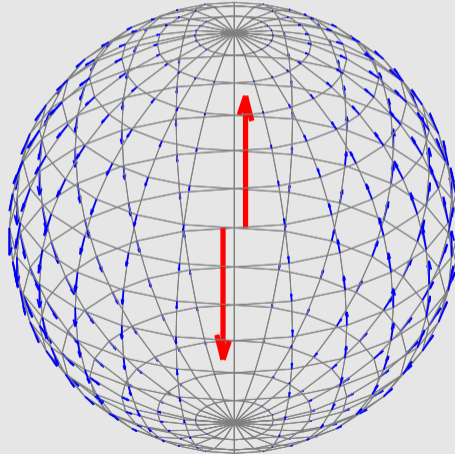
## Seismic Wave Patterns

S-wave radiation pattern

$$(\mathbf{1} - \mathbf{P}) \dot{\mathbf{M}} \vec{e}$$

for

$$\dot{\mathbf{M}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$



## Symmetry of the Seismic Moment Tensor

Nondiagonal components of  $\mathbf{M}$  cause an overall rotation.



not possible



$\mathbf{M}$  must be symmetric:  $M_{ji} = M_{ij}$ .



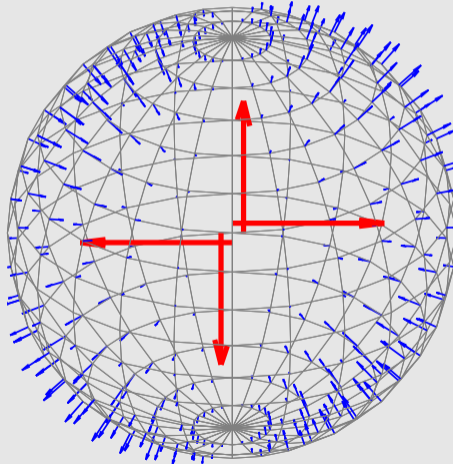
## Seismic Wave Patterns

P-wave radiation pattern

$$\mathbf{P}\dot{\mathbf{M}}\vec{e}$$

for

$$\dot{\mathbf{M}} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$



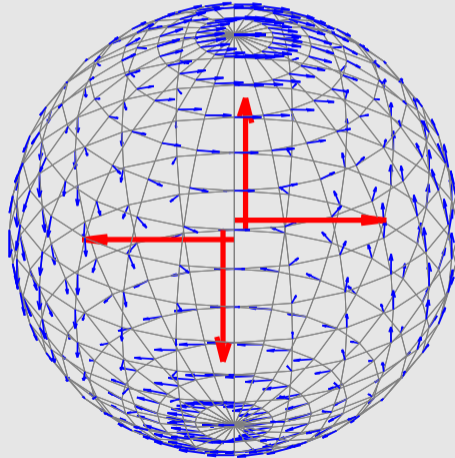
## Seismic Wave Patterns

S-wave radiation pattern

$$(\mathbf{1} - \mathbf{P}) \dot{\mathbf{M}} \vec{e}$$

for

$$\dot{\mathbf{M}} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$



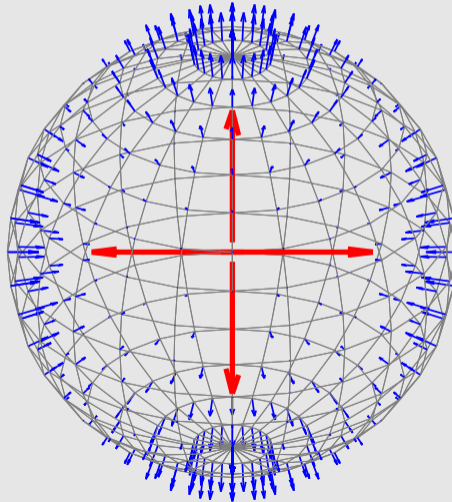
## Seismic Wave Patterns

P-wave radiation pattern

$$\mathbf{P}\dot{\mathbf{M}}\vec{e}$$

for

$$\dot{\mathbf{M}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



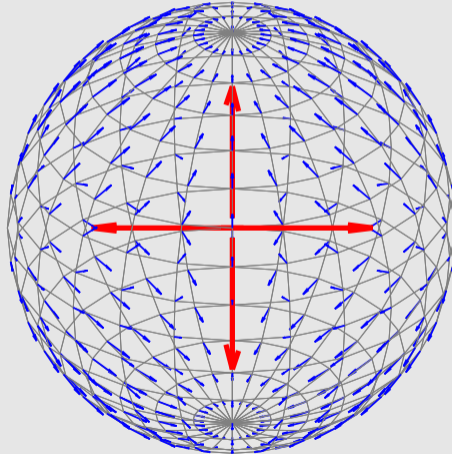
## Seismic Wave Patterns

S-wave radiation pattern

$$(\mathbf{1} - \mathbf{P}) \dot{\mathbf{M}} \vec{e}$$

for

$$\dot{\mathbf{M}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



## The Scalar Seismic Moment

If

$$\mathbf{M} = \begin{pmatrix} 0 & 0 & M_0 \\ 0 & 0 & 0 \\ M_0 & 0 & 0 \end{pmatrix} \quad \text{or} \quad \mathbf{M} = \begin{pmatrix} -M_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M_0 \end{pmatrix} \quad (38)$$

(or similar),  $M_0$  is called (scalar) seismic moment.

In general:

$$M_0 = \frac{M_1 - M_3}{2} \quad (39)$$

where  $M_1$ ,  $M_2$ , and  $M_3$  are the eigenvalues of  $\mathbf{M}$  in descending order.

## The Scalar Seismic Moment

Alternative interpretation of the seismic moment:

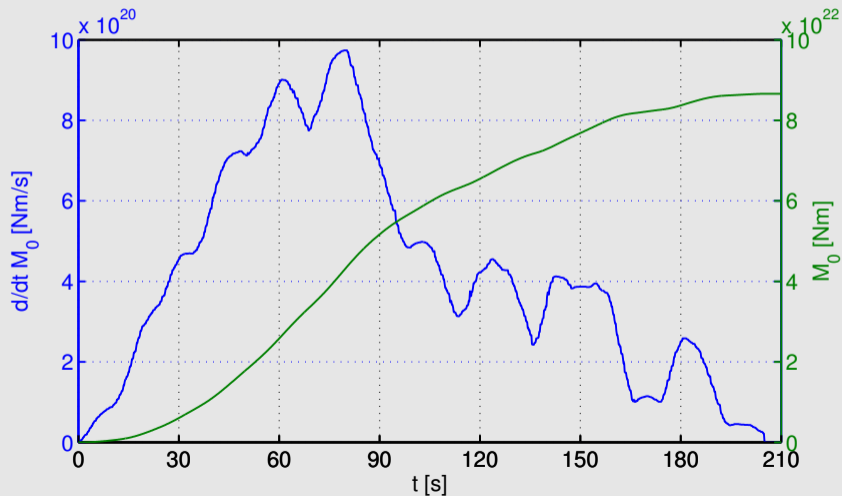
$$M_0 = \mu A \bar{u} \quad (40)$$

where

$A$  = size of the rupture area [ $\text{m}^2$ ]

$\bar{u}$  = mean displacement along the rupture area [m]

## Seismic Moment vs. Moment Rate for the Alaska 1964 Earthquake



## Amplitudes of Body Waves

P-wave displacement:

$$\vec{u}_p(\vec{x}, t) = \frac{s_p^3}{4\pi\rho r} \mathbf{P} \dot{\mathbf{M}}(t - s_p r) \vec{e} \quad (41)$$

Maximum displacement occurs in the directions of the first and third principal axes of  $\dot{\mathbf{M}}$ :

$$|\vec{u}_p|_{\max} = \frac{s_p^3}{4\pi\rho r} |\dot{M}_0|_{\max} \quad (42)$$



## Amplitudes of Body Waves

S-wave displacement:

$$\vec{u}_s(\vec{x}, t) = \frac{s_s^3}{4\pi\rho r} (\mathbf{1} - \mathbf{P}) \dot{\mathbf{M}}(t - s_s r) \vec{e} \quad (43)$$

Maximum displacement occurs in the directions  $45^\circ$  between the first and third principal axis of  $\dot{\mathbf{M}}$ :

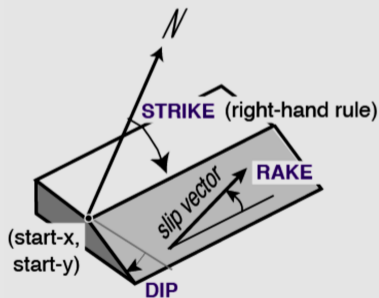
$$|\vec{u}_s|_{\max} = \frac{s_s^3}{4\pi\rho r} |\dot{M}_0|_{\max} \quad (44)$$



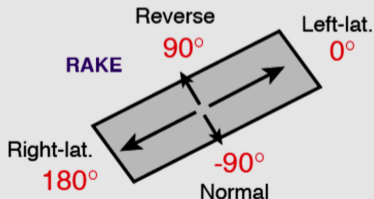
$$\frac{|\vec{u}_s|_{\max}}{|\vec{u}_p|_{\max}} = \frac{s_s^3}{s_p^3} = \left(\frac{v_p}{v_s}\right)^3 \approx 5 \quad (45)$$

## The Seismic Moment Tensor for a Double Force Couple

Definition of strike  $\phi$ , dip  $\delta$ , and rake  $\lambda$  according to Aki and Richards (1980)



$$0 \leq \text{STRIKE} < 360^\circ (2\pi)$$
$$0 < \text{DIP} \leq 90^\circ (\pi/2)$$
$$-180^\circ (-\pi) \leq \text{RAKE} \leq 180^\circ (\pi)$$



Source: Toda et al., Coulomb 3.3 User Guide

## The Seismic Moment Tensor for a Double Force Couple

**Step 1:** Start with a single force couple in  $x_2$  direction displaced in  $x_3$  direction:

$$\vec{F} = \begin{pmatrix} 0 \\ F \\ 0 \end{pmatrix}, \quad \vec{a} = \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix} \quad (46)$$

## The Seismic Moment Tensor for a Double Force Couple

**Step 2:** Rotate  $\vec{F}$  and  $\vec{a}$  counterclockwise by the rake angle  $\lambda$  in the  $x_1$ - $x_2$  plane:

$$\vec{F}_\lambda = \mathbf{R}_\lambda \vec{F} \quad (47)$$

$$\vec{a}_\lambda = \mathbf{R}_\lambda \vec{a} \quad (48)$$

with

$$\mathbf{R}_\lambda = \begin{pmatrix} \cos \lambda & -\sin \lambda & 0 \\ \sin \lambda & \cos \lambda & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (49)$$

## The Seismic Moment Tensor for a Double Force Couple

**Step 3:** Rotate  $\vec{F}_\lambda$  and  $\vec{a}_\lambda$  clockwise by the dip angle  $\delta$  in the  $x_1$ - $x_3$  plane:

$$\vec{F}_{\lambda\delta} = \mathbf{R}_\delta \vec{F}_\lambda = \mathbf{R}_\delta \mathbf{R}_\lambda \vec{F} \quad (50)$$

$$\vec{a}_{\lambda\delta} = \mathbf{R}_\delta \vec{a}_\lambda = \mathbf{R}_\delta \mathbf{R}_\lambda \vec{a} \quad (51)$$

with

$$\mathbf{R}_\delta = \begin{pmatrix} \cos \delta & 0 & \sin \delta \\ 0 & 1 & 0 \\ -\sin \delta & 0 & \cos \delta \end{pmatrix} \quad (52)$$

## The Seismic Moment Tensor for a Double Force Couple

**Step 4:** Rotate  $\vec{F}_{\lambda\delta}$  and  $\vec{a}_{\lambda\delta}$  clockwise by the strike angle  $\phi$  in the  $x_1$ - $x_2$  plane:

$$\vec{F}_{\lambda\delta\phi} = \mathbf{R}_\phi \vec{F}_{\lambda\delta} = \mathbf{R}_\phi \mathbf{R}_\delta \mathbf{R}_\lambda \vec{F} \quad (53)$$

$$\vec{a}_{\lambda\delta\phi} = \mathbf{R}_\phi \vec{a}_{\lambda\delta} = \mathbf{R}_\phi \mathbf{R}_\delta \mathbf{R}_\lambda \vec{a} \quad (54)$$

with

$$\mathbf{R}_\phi = \begin{pmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (55)$$

## The Seismic Moment Tensor for a Double Force Couple

$$\vec{F}_{\lambda\delta\phi} = \mathbf{R}\vec{F}, \quad \vec{a}_{\lambda\delta\phi} = \mathbf{R}\vec{a} \quad \text{with} \quad \mathbf{R} = \mathbf{R}_\phi\mathbf{R}_\delta\mathbf{R}_\lambda \quad (56)$$



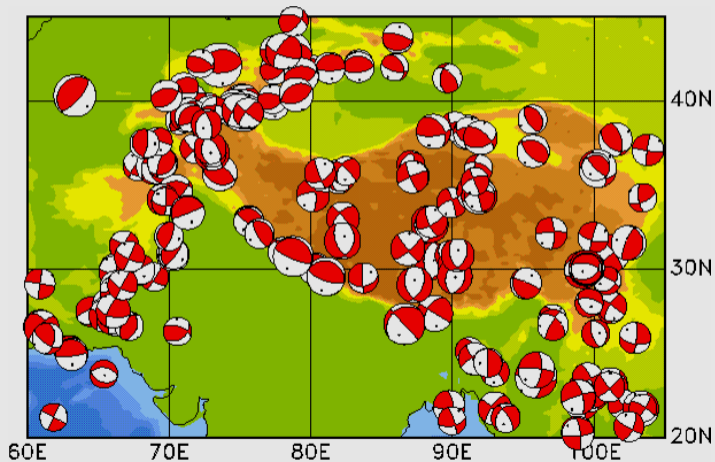
$$\mathbf{M} = \vec{F}_{\lambda\delta\phi} \vec{a}_{\lambda\delta\phi}^T + \vec{a}_{\lambda\delta\phi} \vec{F}_{\lambda\delta\phi}^T \quad (57)$$

$$= (\mathbf{R}\vec{F}) (\mathbf{R}\vec{a})^T + (\mathbf{R}\vec{a}) (\mathbf{R}\vec{F})^T \quad (58)$$

$$= M_0 \mathbf{R} \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix} \mathbf{R}^T \quad (59)$$

with  $M_0 = Fa$

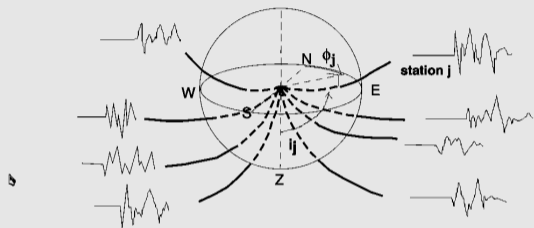
## Beachball Plots



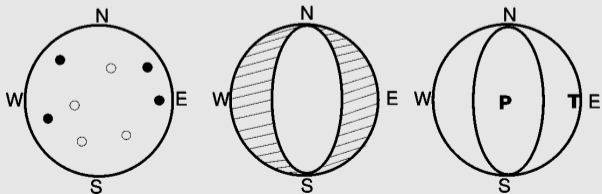
Source: Earthquake-Report.com



## Beachball Plots



(I)



Source: Ph.D. thesis A. Belmonte-Pool, FU Berlin

## Beachball Plots

Basic assumption:  $\dot{\mathbf{M}}(t)$  has the same shape as  $\mathbf{M}(t)$ ,

$$\mathbf{M}(t) = f(t) \mathbf{M}, \quad \dot{\mathbf{M}}(t) = \dot{f}(t) \mathbf{M} \quad (60)$$

where  $\mathbf{M}$  is the total seismic moment, and  $f(t)$  increases from 0 to 1.



P-wave arrives with

compression

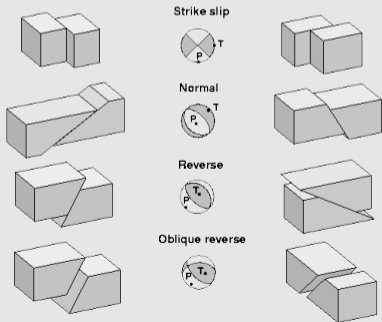
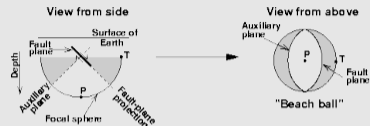
dilatation

first if

$$(\mathbf{M}\vec{e}) \cdot \vec{e} > 0$$

$$(\mathbf{M}\vec{e}) \cdot \vec{e} < 0$$

## Beachball Plots



Source: US Geological Survey

## Examples of Beachball Plots

- normal fault ( $\lambda = -90^\circ$ ) for different dip angles  $\delta$
- reverse fault ( $\lambda = 90^\circ$ ) for different dip angles  $\delta$
- transform fault ( $\lambda = 0^\circ$ ) for different dip angles  $\delta$
- fault dipping at  $\delta = 45^\circ$  for different rake angles  $\lambda$
- fault dipping at  $\delta = 45^\circ$  for different rake angles  $\lambda$  with additional isotropic expansion

## Intensity and Magnitude

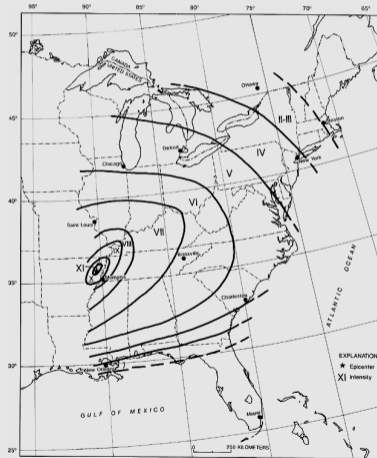
**Intensity** describes the severity of an earthquake in terms of its effects on the Earth's surface and on humans and their structures.

- Usually written as a Roman numeral.
- Goes back to a 12 level scale (originally 10) from I (not felt) to XII (total destruction) named after G. Mercalli (1850–1914).
- Several extensions / refinements: MCS (Mercalli-Cancani-Sieberg) scale, MWN (Mercalli-Wood-Neumann) scale, MSK scale (Medvedev, Sponheuer & Karnik, 1964), EMS-98 scale (European Macroseismic Scale, 2000).

**Magnitude** characterizes the size of an earthquake using measured values.

- Usually written as an Arabic numeral with one decimal digit.
- Several different magnitude definitions.
- Logarithmic scale.

## Example of an Iseismic Map of Earthquake Intensity



Source: USGS

## General Definition of Earthquake Magnitude

If  $X$  is any physically measured property of an earthquake, e. g.

- total seismic moment  $M_0$  or
- maximum ground displacement  $|\vec{u}|_{\max}$ ,

the corresponding earthquake magnitude is defined by

$$M_X = e \log_{10} \left( \frac{X}{X_0} \right) \quad (61)$$

where

$X_0$  = measured value for an earthquake of  $M_X = 0$  under the same conditions

$e$  = factor used for making different magnitude definitions consistent (mostly  $e = 1$ )

## General Definition of Earthquake Magnitude

If  $X$  is a property related to any point different from the earthquake focus,  $X_0$  is a function of distance  $\Delta$  and depth  $h$  (and other properties).



$$M_X = e \log_{10} \left( \frac{X}{X_0(\Delta, h)} \right) = e \log_{10} X + \sigma(\Delta, h) \quad (62)$$

with the distance-depth correction function

$$\sigma(\Delta, h) = -e \log_{10} X_0(\Delta, h) \quad (63)$$

This only makes sense if the distance-depth dependence of  $X$  is independent of  $X$  itself.

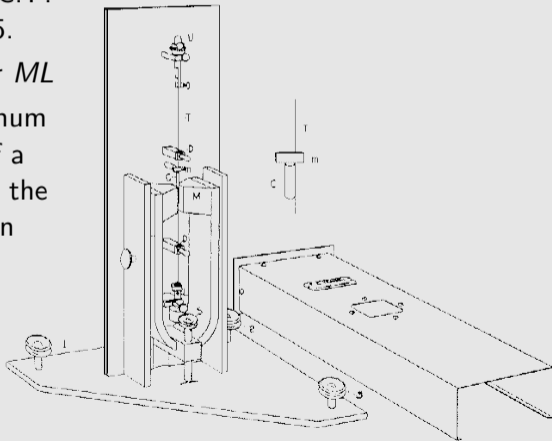


## Upper und Lower Limits of Magnitude Scales

- All magnitude scales are from their definition open and both ends.
- Upper limits on Earth are introduced by geological constraints and by the process of wave propagation.
- Negative magnitudes are possible. The definition of zero magnitude is arbitrary and corresponds to what was detectable when the first magnitude definition (C. F. Richter, 1935) was introduced.

## The Local Magnitude (Richter Scale)

- Introduced by C. F. Richter in 1935.
- Symbol:  $M_L$  or  $ML$
- $X$  is the maximum amplitude  $A$  of a specific device, the Wood-Anderson seismometer.



Source: Southern California Earthquake Data Center

## The Wood-Anderson Seismometer

- Oscillation by torsion of a wire
- Electromagnetic damping
- Natural period of  $\approx 0.8$  s (frequency  $f_0 = 1.25$  Hz); close to the natural period of many building structures.



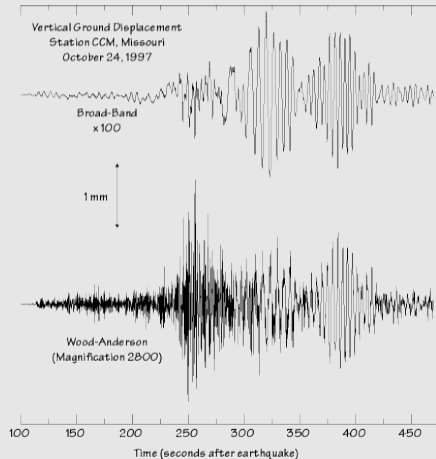
Relevant for earthquake hazard.

- Maximum magnification (record vs. ground displacement) of  $\approx 2080$  at  $f_0$ ; sometimes a wrong value of 2800 was assumed.



Local magnitudes derived from synthesized seismograms were too high for some time.

## The Wood-Anderson Seismometer



Source: C. J. Ammon, Pennsylvania State University

## The Local Magnitude (Richter Scale)

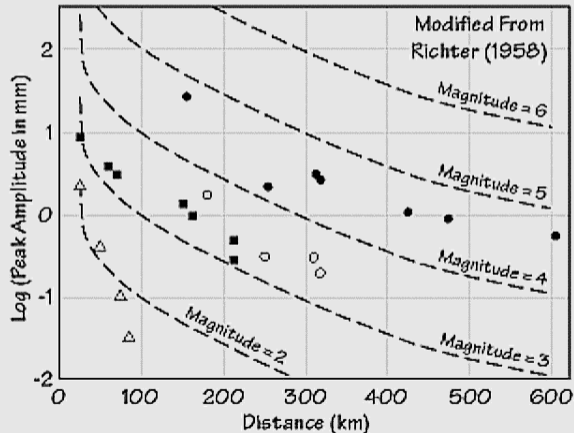
- The local magnitude was originally defined as

$$M_L = \log_{10} A \quad (64)$$

where the maximum amplitude  $A$  of the Wood-Anderson seismometer is measured in  $\mu\text{m}$  at 100 km distance from the epicenter.

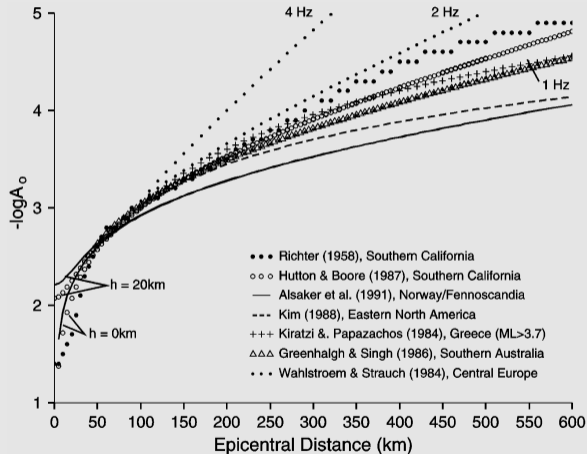
- $e = 1 \rightarrow$  1 unit increase in magnitude corresponds to an increase in the instrument's amplitude by a factor 10.
- Originally only a distance correction  $\sigma(\Delta) = -\log_{10} A_0(\Delta)$  for shallow earthquakes ( $h \leq 15$  km) in California was provided.

## Richter's Original Distance Correction



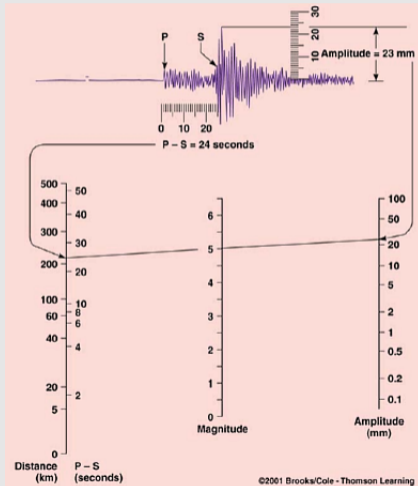
Source: C. J. Ammon, Pennsylvania State University

## Distance Corrections for Different Regions and Depths



Source: Bormann (ed), New Manual of Seismological Observatory Practice

## Determining the Local Magnitude of an Earthquake





## The Surface-Wave Magnitude

- Symbol:  $M_S$  or  $MS$
- Original definition by B. Gutenberg (1945):

$$M_S = \log_{10} u_{h\max} + \sigma(\Delta) \quad (65)$$

where  $u_{h\max}$  is the maximum horizontal ground displacement at periods from  $T = 18$  s to 22 s.

- Widely used modified definition (Moscow-Prague formula, 1962):

$$M_S = \max \left\{ \log_{10} \frac{|\vec{u}|}{T} \right\} + 1.66 \log_{10} \Delta + 3.3 \quad (66)$$

for  $2^\circ \leq \Delta \leq 160^\circ$ . The maximum is taken over all periods of surface waves.

## Body-Wave Magnitudes

- Two significantly different definitions
- Symbols:  $m_B$ ,  $mB$ ,  $m_b$ ,  $mb$ ,
- Original definition by B. Gutenberg (1945):

$$m_B = \max \left\{ \log_{10} \frac{|\vec{u}|}{T} \right\} + \sigma(\Delta) \quad (67)$$

where  $|\vec{u}|$  is analyzed for different types of body waves separately (with different functions  $\sigma(\Delta)$  at periods from  $T = 0.5$  s to 12 s.

- Alternative definition ( $m_b$ ,  $mb$ ) refers to higher-frequency components of P-waves only.

## The Moment Magnitude

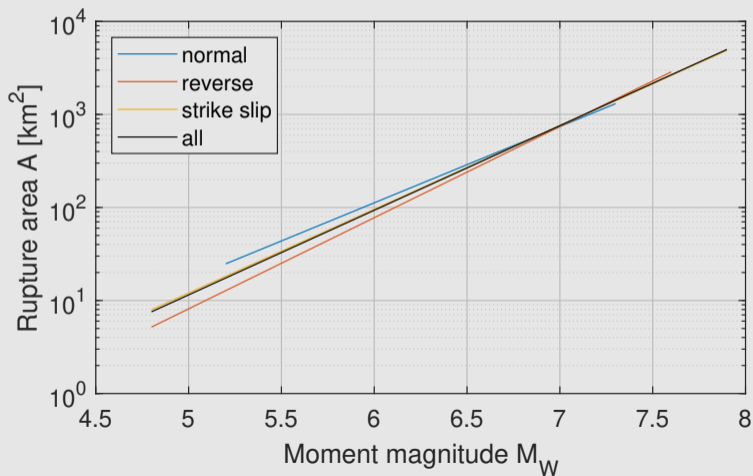
$$M_W = \frac{2}{3} \log_{10} M_0 - 6.1 \quad (68)$$

with  $M_0$  in Nm

- Introduced in 1977 by H. Kanamori in order to characterize large earthquakes.
- More closely related to the strength of earthquakes at the seismic focus than older magnitude definitions.
- Rather a tectonic than a seismological magnitude scale.

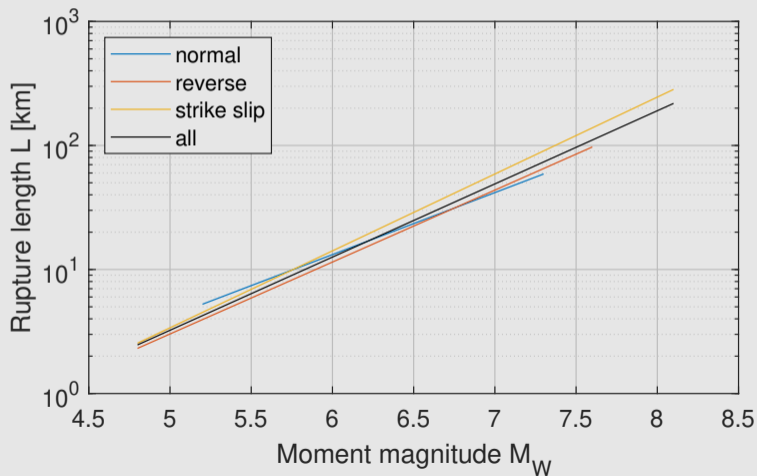
Why is  $e = \frac{2}{3}$  here while  $e = 1$  for other magnitude scales?

## Scaling Properties of Earthquakes



Data: Wells & Coppersmith, Bull. Seismol. Soc. Am., 1994

## Scaling Properties of Earthquakes



Data: Wells & Coppersmith, Bull. Seismol. Soc. Am., 1994

## Scaling Properties of Earthquakes

Alternative interpretation of the seismic moment (Eq. 40):

$$M_0 = \mu A \bar{u} \quad (69)$$

where

$A$  = size of the rupture area [ $\text{m}^2$ ]

$\bar{u}$  = mean displacement along the rupture area [m]

Simplest idea for the relation between  $A$ ,  $\bar{u}$ , and  $M_0$ :

$$\bar{u} \sim \square \sim \square \quad \text{and} \quad A \sim \square \quad (70)$$

## Scaling Properties of Earthquakes

Assume that rupture propagates along the rupture area at a given velocity.



Duration

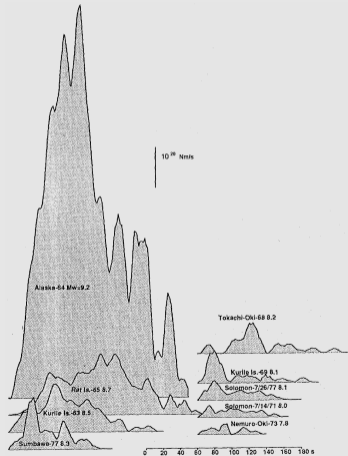
$$\tau \sim L \sim \boxed{\phantom{L}} \sim \boxed{\phantom{L}} \quad (71)$$



Wave amplitude

$$|\vec{u}| \sim \dot{M}_0 \sim \frac{M_0}{\tau} \sim \boxed{\phantom{M_0}} \quad (72)$$

## Scaling Properties of Earthquakes



Source: Bormann (ed), New Manual of Seismological Observatory Practice



## The Energy Magnitude

Particle velocity

$$|\vec{v}| \sim \frac{|\vec{u}|}{\tau} \sim \frac{\text{[ ]}}{\text{[ ]}} \sim \text{[ ]} \quad (73)$$



Total radiated kinetic energy

$$E_{\text{kin}} \sim |\vec{v}|^2 \tau \sim \text{[ ]} \quad (74)$$

Potential (elastic) energy equals kinetic energy in the mean.

## The Energy Magnitude

Theoretical relationship suggested by H. Kanamori (1977):

$$E \approx 5 \times 10^{-5} M_0 \quad (75)$$

Corresponding definition of the energy magnitude:

$$M_E = \frac{2}{3} \log_{10} \frac{E}{5 \times 10^{-5}} - 6.1 = \quad \square \quad \log_{10} E - \quad \square \quad (76)$$

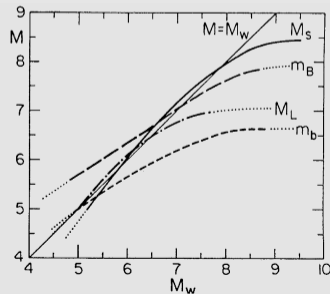
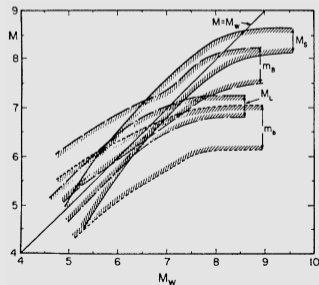
Up to one order of magnitude deviation from Kanamori's relationship was found for individual earthquakes.



Differences between  $M_E$  and  $M_W$  up to  $\square$  for individual earthquakes.

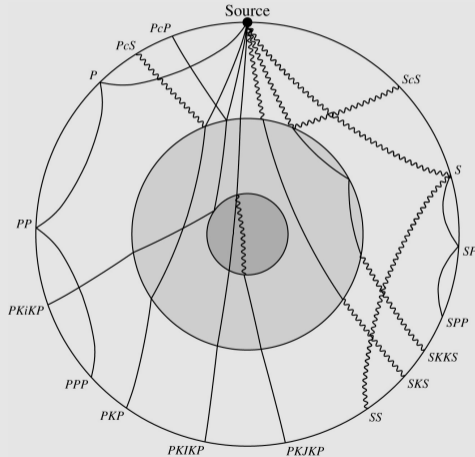
## Saturation of Magnitudes

- All magnitudes based on recording seismic waves fall below  $M_W$  for large earthquakes.
- Effect is stronger if short-term (high-frequency) components of the seismic waves are used.



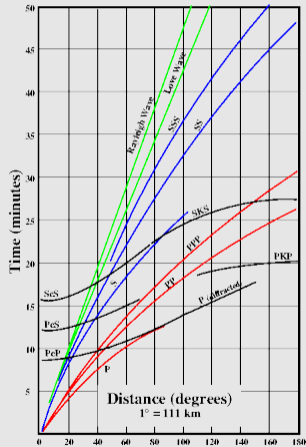
Source: Bormann (ed), New Manual of Seismological Observatory Practice

## Global Wave Propagation in the Earth's Interior



Source: Shearer, Introduction to Seismology

## Travel Time Curves

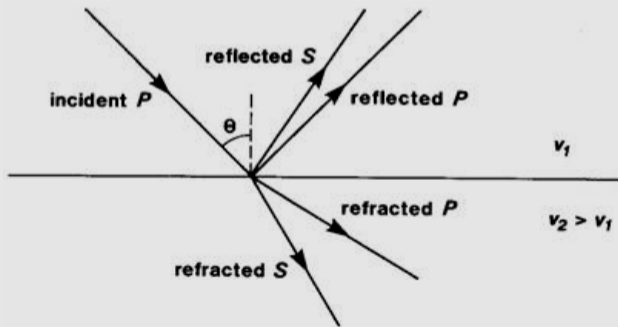


Source: Southern Arizona Seismic Observatory

## Reflection and Refraction



Simplest case: two homogeneous, isotropic halfspaces with different properties ( $\lambda$ ,  $\mu$ ,  $\rho$ ) and plane waves in each of them; horizontal interface at  $x_3 = 0$ .



Source: University College London

## Reflection and Refraction



Each wave is considered as a plane wave

$$\vec{u}(\vec{x}, t) = f(t - \vec{s} \cdot \vec{x}) \vec{a} \quad (77)$$

with individual values  $\vec{s}$  and  $\vec{a}$  where either

$$\vec{a} \parallel \vec{s} \quad \text{and} \quad |\vec{s}| = s_p = \sqrt{\frac{\rho}{\lambda + 2\mu}} \quad (78)$$

or

$$\vec{a} \perp \vec{s} \quad \text{and} \quad |\vec{s}| = s_s = \sqrt{\frac{\rho}{\mu}} \quad (79)$$

## Reflection and Refraction



Displacement must be continuous at  $x_3 = 0$ .



$$\tau = t - \vec{s} \cdot \vec{x} = t - s_1 x_1 - s_2 x_2 \quad (80)$$

must be the same for all waves at the interface.



Components of  $\vec{s}$  parallel to the interface ( $s_1, s_2$ ) must be the same for all waves.



Horizontal slowness remains constant in reflection and refraction.



## Reflection and Refraction

Second condition at the interface:

Stress acting on the interface, given by

$$\vec{\sigma}_{\text{int}} = \sigma|_{x_3=0} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (81)$$

must be continuous.

Continuity of displacement and stress at the interface



Linear equation system for the amplitude vectors  $\vec{a}$  of the 5 involved waves.

## Conversion of Waves in Reflection and Refraction

Align the coordinate system in such a way that all waves propagate in the  $x_1$ - $x_3$  plane ( $s_2 = 0$ , possible because  $s_1$  and  $s_2$  are the same for all involved waves).



Vertically polarized S-wave (SV-wave):

- $a_2 = 0 \rightarrow$  particle displacement in the  $x_1$ - $x_3$  plane (and normal to wave propagation)
- Converted to (and from) P and SV waves in reflection and refraction

Horizontally polarized S-wave (SH-wave):

- $a_1 = a_3 = 0 \rightarrow$  particle displacement in  $x_2$  direction
- Independent of P and SV waves

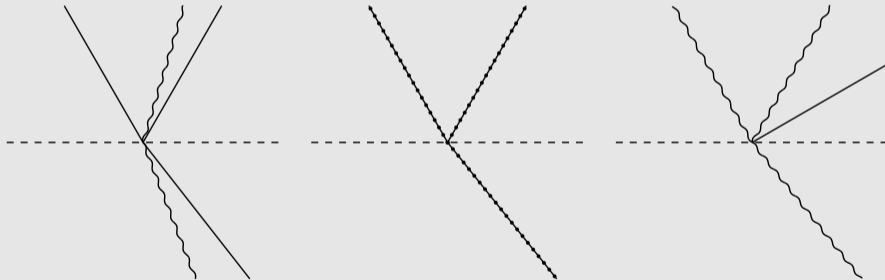
## Conversion of Waves in Reflection and Refraction



Incident P-wave

Incident SH-wave

Incident SV-wave



## Ray and Wave Front Approaches

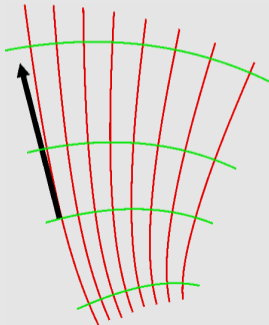
Plane wave approach and spherical wave solutions are only valid in a homogeneous medium.

Generalization:

**Wave fronts** are no longer planes, but arbitrary surfaces.

**Direction of propagation** is normal to the wave fronts.

**Ray paths** are lines following the local direction of propagation (no longer straight lines).



Source: Wikipedia, © ???

## Ray and Wave Front Approaches



Extensions towards the plane wave approach:

Replace

$$\vec{u}(\vec{x}, t) = f(t - \vec{s} \cdot \vec{x}) \vec{a} \quad (82)$$

by

$$\vec{u}(\vec{x}, t) = f(t - \psi(\vec{x})) \vec{a}(\vec{x}) \quad (83)$$

- Retarded time  $\tau = t - \psi(\vec{x})$  instead of  $\tau = t - \vec{s} \cdot \vec{x}$  with a general phase function  $\psi(\vec{x})$
- Spatially variable amplitude vector  $\vec{a}(\vec{x})$

## Ray and Wave Front Approaches

**Wave fronts:** surfaces where  $\psi(\vec{x})$  is constant

**Direction of propagation:** Wave propagates locally in direction of  $\nabla\psi(\vec{x})$ .

**Slowness:**  $\vec{s}(\vec{x}) = \nabla\psi(\vec{x})$  defines the local slowness vector.

**Ray paths:** lines following the direction of  $\vec{s} = \nabla\psi$

**Limitation:** Only valid in the limit of high frequencies.

## Ray and Wave Front Approaches



Main results:

**Types of waves:** two independent types of waves (P-wave and S-wave); amplitude vector  $\vec{a}$  is either parallel or normal to  $\vec{s} = \nabla\psi$  at each point; no reflection; no merging of both wave types

**Slowness/velocity:** same relationship as for plane and spherical waves (eikonal equation)

$$|\vec{s}|^2 = |\nabla\psi|^2 = \frac{\rho}{\lambda + 2\mu} \quad \text{or} \quad |\vec{s}|^2 = |\nabla\psi|^2 = \frac{\rho}{\mu} \quad (84)$$

## Ray and Wave Front Approaches

**Amplitudes:** Absolute values of the amplitudes are determined by the condition

$$\operatorname{div}(\vec{q}) = 0 \quad (85)$$

with the “energy flux density”

$$\vec{q} = \rho |\vec{a}|^2 \vec{v} = \epsilon \vec{v} \quad (86)$$

and the “energy density”  $\epsilon = \rho |\vec{a}|^2$ .



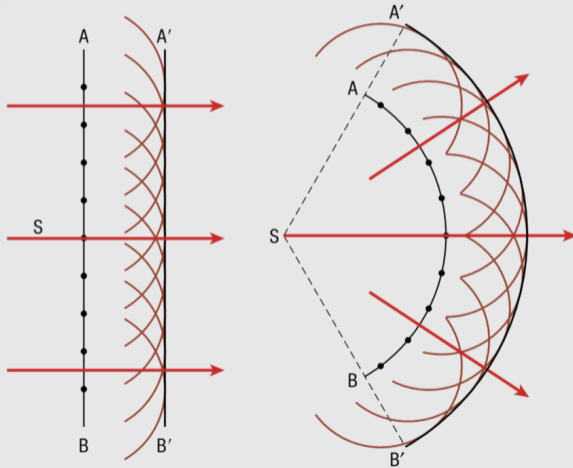
## Geometrical and Numerical Approaches

**Numerical solution of the eikonal equation:** one-sided difference quotients for  $\psi(\vec{x})$  from the considered points to neighbors where  $\psi$  is already known; similar to upstream scheme.

**Huygens' principle:** Construct wave fronts (planes where  $\psi(\vec{x})$  is constant) progressively for increasing values of  $\psi$ ; rather a geometrical approach than a numerical method.

**Ray tracing:** Compute individual rays (lines in direction of  $\nabla\psi(\vec{x})$ ).

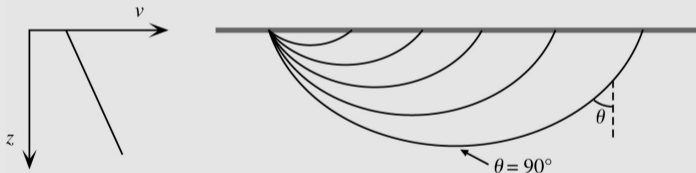
## Construction of Wave Fronts – Huygens' Principle



Source: WillowWood Lessons

## Construction of Ray Paths

- $\vec{s}$  changes along the ray path in direction of increasing slowness (decreasing velocity).
- Horizontal slowness remains constant.



Source: Shearer, Introduction to Seismology

Horizontal direction in seismology = planes where the properties of the material are constant.

## The Waves Disappearing in Reflection and Refraction

For all waves involved in reflection and refraction at a planar interface ( $s_2 = 0$ ):  $s_1$  given and

$$|\vec{s}'|^2 = s_1^2 + s_3^2 = \begin{cases} \frac{1}{v_p^2} = \frac{\rho}{\lambda + 2\mu} & \text{for P-waves} \\ \frac{1}{v_s^2} = \frac{\rho}{\mu} & \text{for S-waves} \end{cases} \quad (87)$$

given.



Waves cannot propagate if  $s_1 > |\vec{s}'|$ , but

$$s_3 = \pm \sqrt{|\vec{s}'|^2 - s_1^2} = \pm \square \sqrt{\square} \quad (88)$$

would be a formal solution.

## Harmonic Interface Waves



For a harmonic wave:

$$\vec{u}(\vec{x}, t) = e^{i\omega(t - \vec{s} \cdot \vec{x})} \vec{a} = e^{i\omega(t - s_1 x_1 \mp i \sqrt{s_1^2 - |\vec{s}'|^2} x_3)} \vec{a} \quad (89)$$

$$= e^{i\omega(t - s_1 x_1)} e^{\pm \text{[redacted]} x_3} \vec{a} \quad (90)$$

Can be considered as a wave propagating along the interface (here in  $x_1$  direction) with an amplitude depending on  $x_3$ :

$$\vec{u}(\vec{x}, t) = e^{i\omega(t - s_1 x_1)} \vec{a}_{\text{eff}} \quad (91)$$

with the effective amplitude

$$\vec{a}_{\text{eff}} = e^{\pm \text{[redacted]} x_3} \vec{a} \quad (92)$$

## The Depth of Penetration of Interface Waves

$$\vec{a}_{\text{eff}} = e^{\pm \frac{x_3}{d}} \vec{a} = e^{\pm \frac{x_3}{d}} \vec{a} \quad (93)$$

with the depth of penetration

$$d = \frac{1}{\frac{\omega}{s_1} \sqrt{\frac{\omega^2}{s_1^2} - \omega^2}} = \frac{L}{2\pi \sqrt{\frac{\omega^2}{s_1^2} - \omega^2}} \quad (94)$$

and the wavelength  $L = \frac{2\pi}{\omega s_1}$ .

- $d \rightarrow \frac{1}{\omega s_1}$  if the wave is only slightly too slow for the medium ( $s_1 \rightarrow |\vec{s}'|$ ).
- $d \rightarrow \frac{1}{\omega s_1}$  if the wave is much too slow for the medium ( $s_1 \gg |\vec{s}'|$ ).

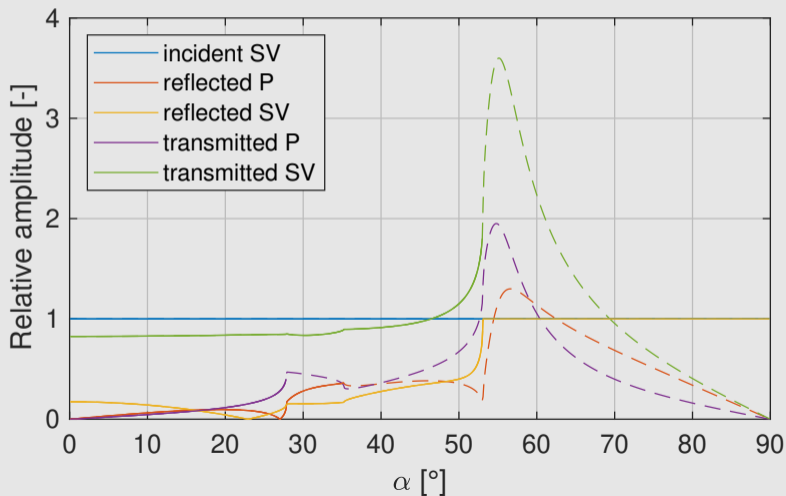
## Particle Orbits of P and SV Interface Waves

Examples of particle orbits for an incident SV wave at the crust-mantle boundary:  $\alpha = 20^\circ$ ,  $\alpha = 30^\circ$ ,  $\alpha = 40^\circ$ ,  $\alpha = 70^\circ$

General properties:

- Particles move on elliptical orbits.
- Horizontal ellipses for P interface waves, vertical ellipses for S interface waves.
- Aspect ratio of the ellipses =  $\frac{1}{\sqrt{1 - \frac{|\bar{s}|^2}{s_1^2}}}$ .
- Prograde rotation in the lower halfspace; retrograde rotation in the upper halfspace.

## Amplitudes at the Crust-Mantle Boundary





## Surface Waves at a Free Surface

**Interface waves:** driven by plane waves (incident, reflected, refracted)

**Surface waves:** living on their own at a free surface (interface to air);  $\vec{\sigma}_{\text{surf}} = \vec{0}$

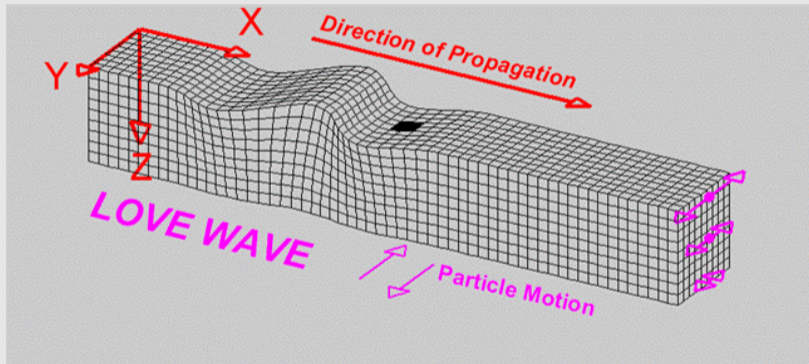


Two fundamental types of surface waves in a semi-infinite halfspace:

**Rayleigh wave**, named after J. W. Strutt (later 3. Lord Rayleigh)

**Love wave**, named after A. E. H. Love

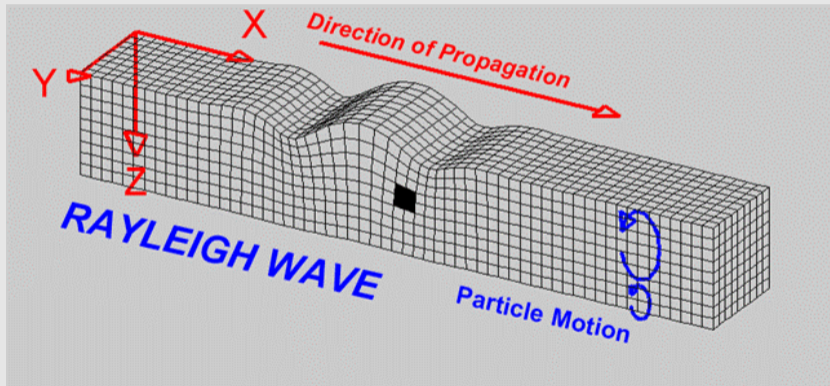
## The Love Wave



Source: L. Braille, Purdue University

The same as SH interface wave; not possible in a homogeneous halfspace because  $\vec{\sigma}_{\text{surf}} \neq \vec{0}$  for  $a_2 \neq 0$ .

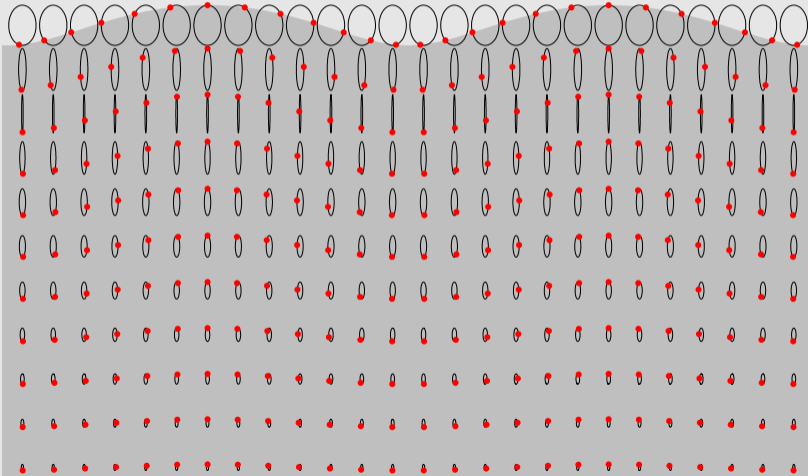
## The Rayleigh Wave



Source: L. Braille, Purdue University

Specific superposition of P and SV interface wave so that  $\vec{\sigma}_{\text{surf}} = \vec{0}$

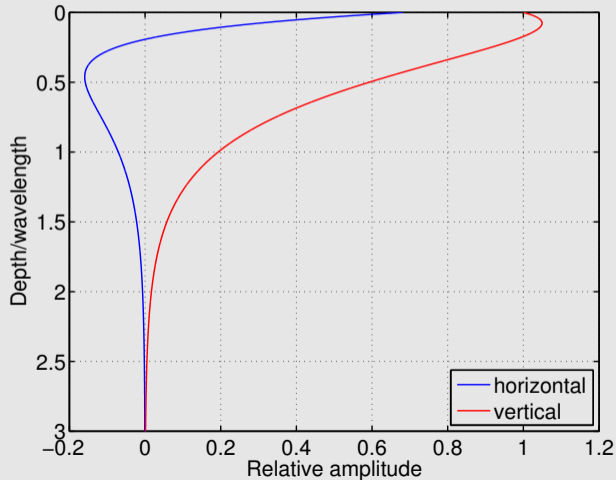
## The Rayleigh Wave in a Homogeneous Poisson Solid ( $\lambda = \mu$ )



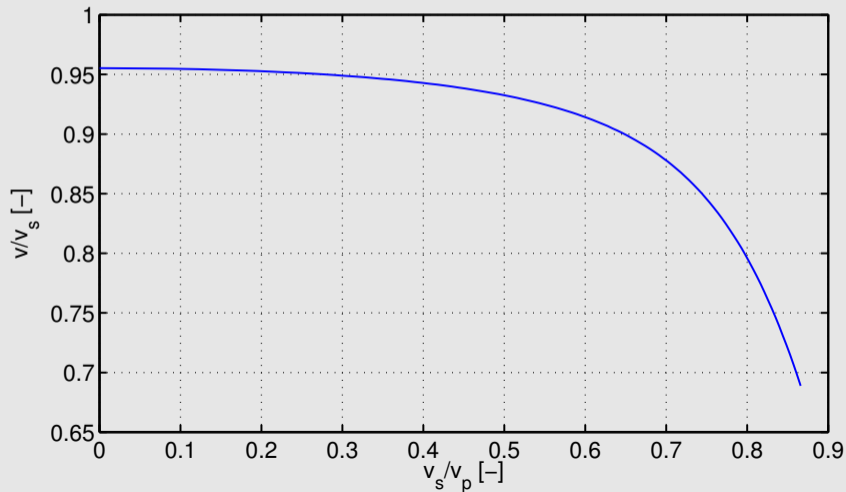
## The Rayleigh Wave in a Homogeneous Poisson Solid ( $\lambda = \mu$ )

- Retrograde particle motion on elliptical orbits at the surface.
- Prograde particle motion on elliptical orbits at greater depth.
- Velocity  $v \approx 0.92 v_s$ .

## The Rayleigh Wave in a Homogeneous Poisson Solid ( $\lambda = \mu$ )



## The Rayleigh Wave in a Homogeneous Halfspace



## Main Differences Between Body Waves and Surface Waves

- Decrease with the distance from the hypocenter/epicenter  $r$ :

	Energy flux density	Amplitude
body waves	$\propto \frac{1}{r^2}$	$\propto \frac{1}{r}$
surface waves	$\propto \frac{1}{r}$	$\propto \frac{1}{\sqrt{r}}$



Surface waves have a longer range than body waves.

- Velocity of harmonic surface waves depends on the wavelength (dispersion).



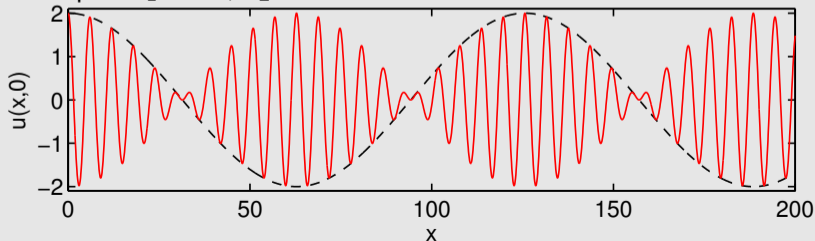
## Dispersion

Simplest situation: superposition of two harmonic waves with the same amplitude (= 1), but different frequencies in 1D:

$$u(x, t) = e^{i\omega_1(t-s_1x)} + e^{i\omega_2(t-s_2x)} = e^{i(\omega_1t-k_1x)} + e^{i(\omega_2t-k_2x)} \quad (95)$$

with  $k_1 = \omega_1s_1$ ,  $k_2 = \omega_2s_2$ .

Example:  $k_1 = 10$ ,  $k_2 = 11$



## Dispersion

$$u(x, t) = e^{i\omega_1(t-s_1x)} + e^{i\omega_2(t-s_2x)} = e^{i(\omega_1t-k_1x)} + e^{i(\omega_2t-k_2x)} \quad (96)$$

$$= e^{i\left(\bar{\omega}t + \frac{\omega_1-\omega_2}{2}t - \bar{k}x - \frac{k_1-k_2}{2}x\right)} + e^{i\left(\bar{\omega}t - \frac{\omega_1-\omega_2}{2}t - \bar{k}x + \frac{k_1-k_2}{2}x\right)} \quad (97)$$

$$= e^{i(\bar{\omega}t - \bar{k}x)} \left( e^{i\left(\frac{\omega_1-\omega_2}{2}t - \frac{k_1-k_2}{2}x\right)} + e^{i\left(-\frac{\omega_1-\omega_2}{2}t + \frac{k_1-k_2}{2}x\right)} \right) \quad (98)$$

$$= e^{i(\bar{\omega}t - \bar{k}x)} 2 \cos\left(\frac{\omega_1-\omega_2}{2}t - \frac{k_1-k_2}{2}x\right) \quad (99)$$

$$= e^{i\bar{\omega}\left(t - \frac{\bar{k}}{\bar{\omega}}x\right)} 2 \cos\left(\frac{\omega_1-\omega_2}{2}\left(t - \frac{k_1-k_2}{\omega_1-\omega_2}x\right)\right) \quad (100)$$

with  $\bar{\omega} = \frac{\omega_1+\omega_2}{2}$ ,  $\bar{k} = \frac{k_1+k_2}{2}$ .

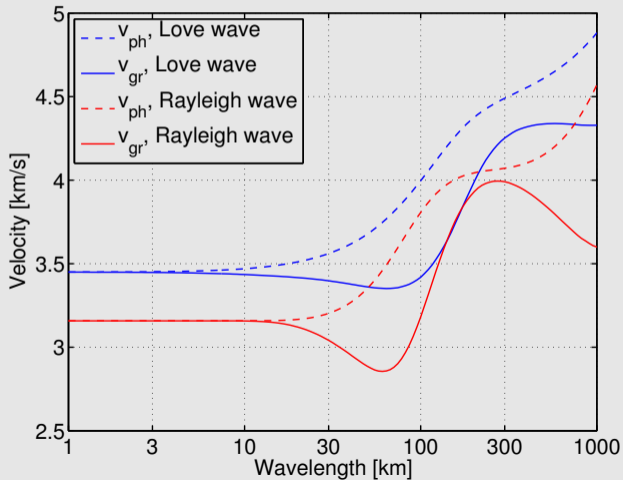
## Dispersion

For  $\omega_1 \approx \omega_2$ ,  $k_1 \approx k_2$ :

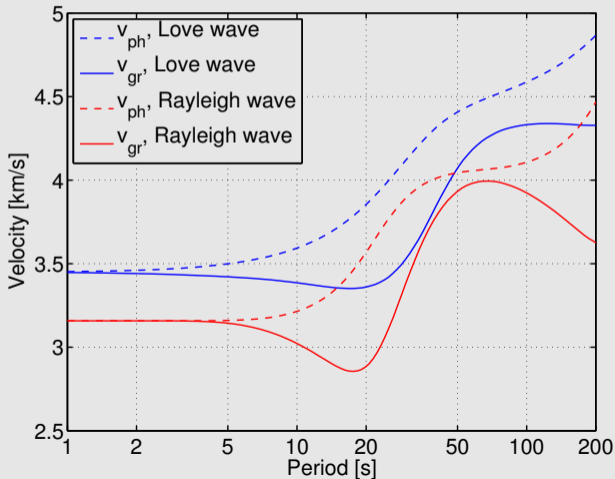
- High-frequency oscillation with an angular frequency  $\bar{\omega}$  propagating with the **phase slowness**  $s_{\text{ph}} = \frac{\bar{k}}{\bar{\omega}}$
- Low-frequency oscillation of the amplitude with an angular frequency  $\frac{\omega_1 - \omega_2}{2}$  propagating with the **group slowness**

$$s_{\text{gr}} = \frac{k_1 - k_2}{\omega_1 - \omega_2} \rightarrow \frac{dk}{d\omega} \quad \text{for} \quad \omega_1 - \omega_2 \rightarrow 0 \quad (101)$$

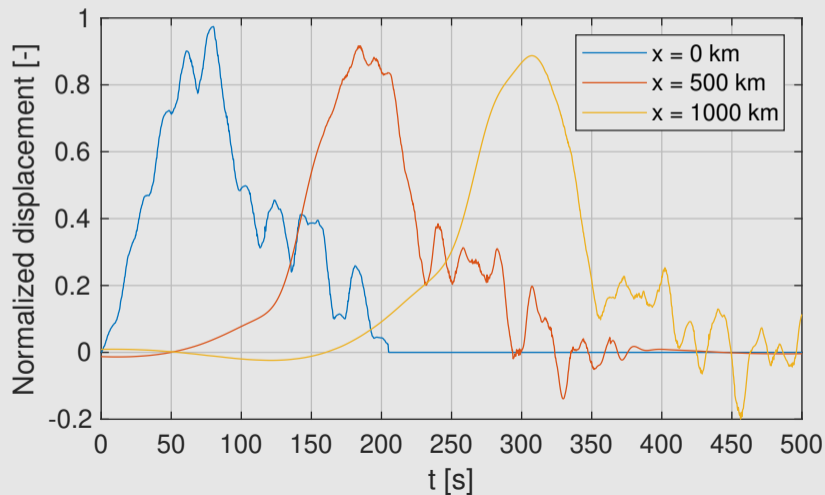
## Velocities in Typical Continental Subsurface (PEM)



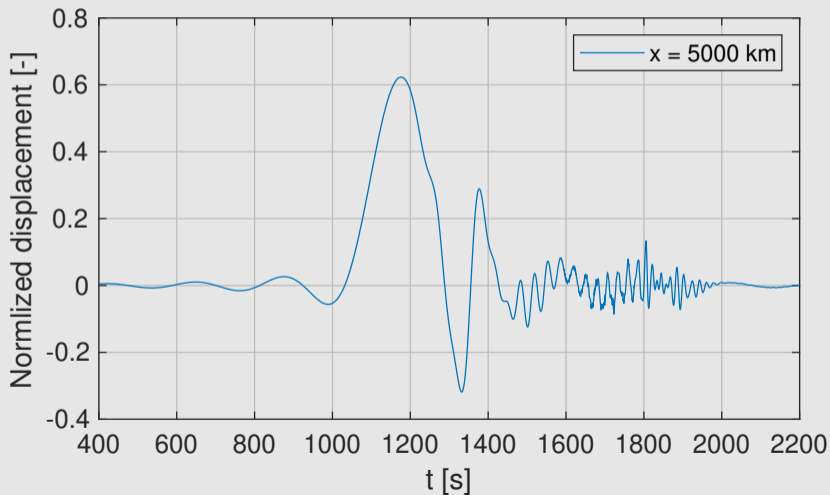
## Velocities in Typical Continental Subsurface (PEM)



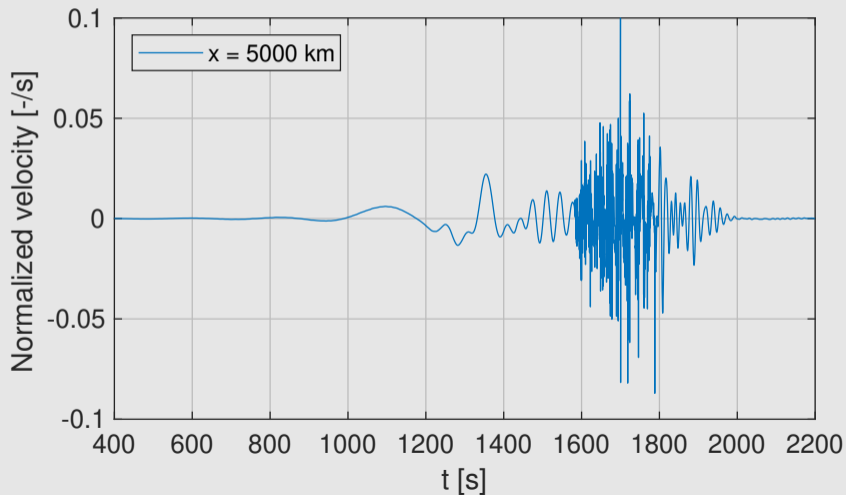
## Dispersion of Rayleigh Waves in Typical Continental Subsurface



## Dispersion of Rayleigh Waves in Typical Continental Subsurface

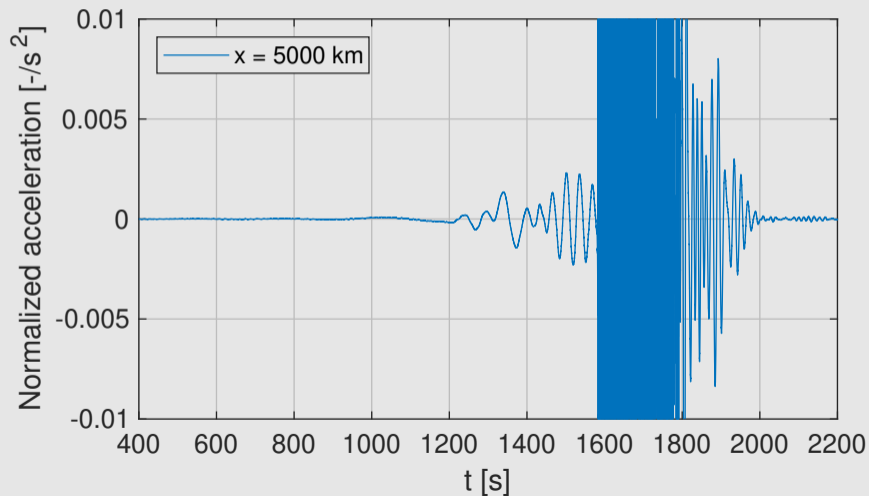


## Dispersion of Rayleigh Waves in Typical Continental Subsurface

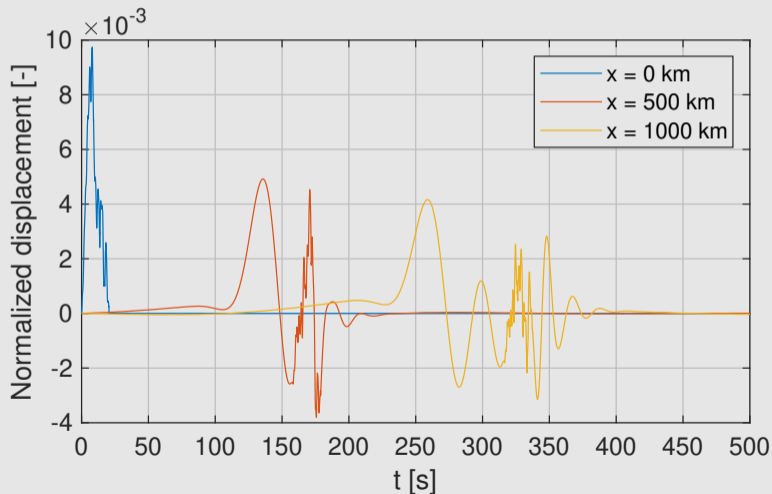




## Dispersion of Rayleigh Waves in Typical Continental Subsurface



## Dispersion of Rayleigh Waves in Typical Continental Subsurface



## Dispersion of Rayleigh Waves in Typical Continental Subsurface

