Tsunamis

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Main Properties of Tsunamis

- Gravity waves with periods between $\approx 100\,\mathrm{s}$ and $10{,}000\,\mathrm{s}$
- Propagate at high velocities in deep water
- Mainly horizontal particle motion of the entire water column down to the ocean floor

$\Downarrow$

Rather small dissipation of energy

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Travel over large distances

- Wave height increases with decreasing ocean depth

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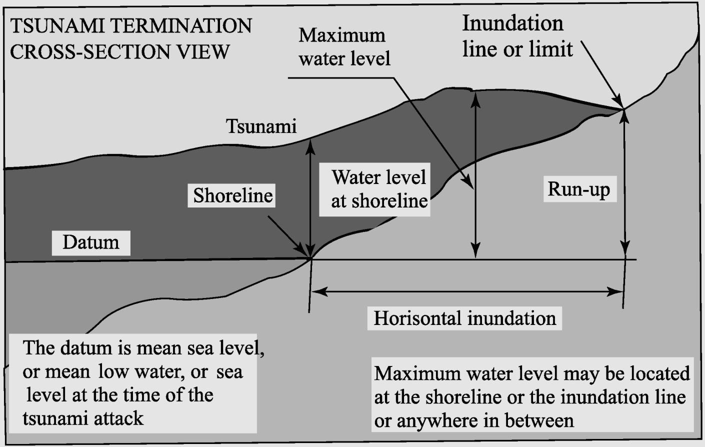
Large wave heights at the coast

Basic Terms

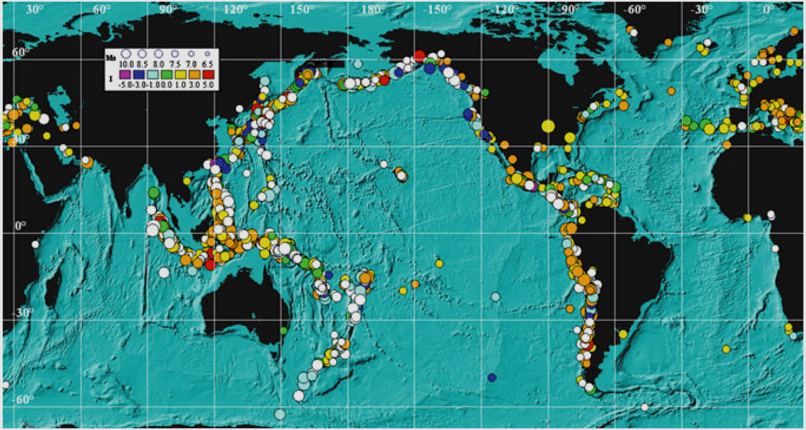


Source: Levin & Nosov, Physics of Tsunamis

Main Sources of Tsunamis

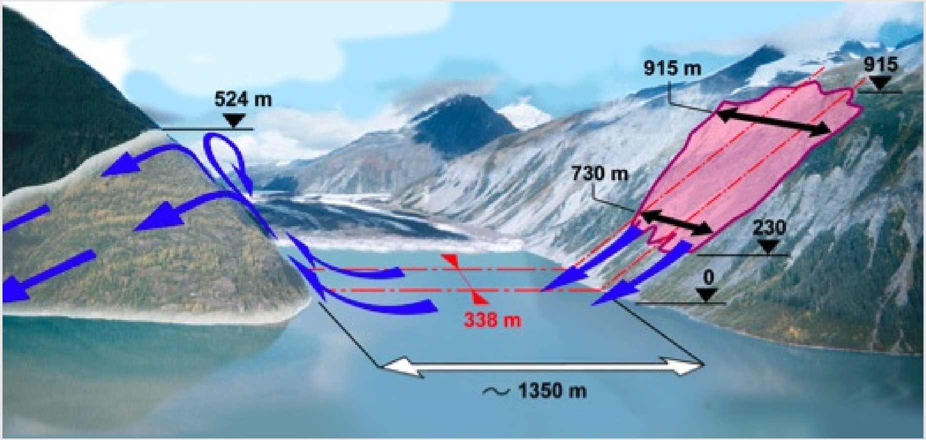
- Earthquakes (more than 90 % of all tsunamis)
- Landslides
- Volcanic eruptions
- Meteorite impact (rare)

Known Tsunami Sources from 2000 B.C. to 2014



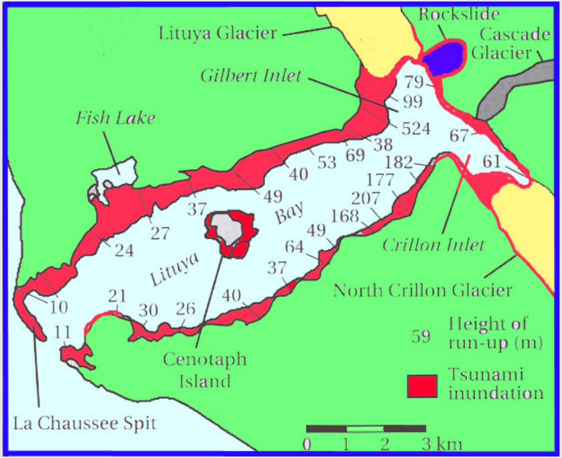
Source: Levin & Nosov, Physics of Tsunamis

The Tallest Tsunami Known so far: Lituya Bay, 1958



Source: Pararas-Carayannis, The Mega-Tsunami of July 9, 1958 in Lituya Bay, Alaska

The Tallest Tsunami Known so far: Lituya Bay, 1958



Source: Pararas-Carayannis, The Mega-Tsunami of July 9, 1958 in Lituya Bay, Alaska

The Tallest Tsunamis 2000–2014

Date	Location	M_W	H_{max} [m]	Death toll
11.03.2011	Japan	9.0	56	18,482
24.12.2004	Indonesia, Sumatra	9.1	51	227,899
27.02.2010	Chile	8.8	29	156
29.09.2009	Samoa	8.1	22	192
15.11.2006	Russia, Kuril Islands	8.3	22	0
17.07.2006	Indonesia, South of Java	7.7	21	802
25.10.2010	Indonesia, Sumatra	7.8	17	431

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Types of Intensity and Magnitude Scales

Type	Adressed property	Examples
intensity	effect on humans and infrastructure	Sieberg-Ambraseys scale Papadopoulos-Imamura scale
	wave height at the coast	Imamura-Iida scale Soloviev-Imamura scale
magnitude	strength at the source of the tsunami	Abe-Hatori scale Murty-Loomis scale

The Sieberg-Ambraseys Scale

- Originally introduced by A. H. Sieberg (1927)
- Modified by N. N. Ambraseys (1962)
- Six-point scale from 1 = very light to 6 = disastrous

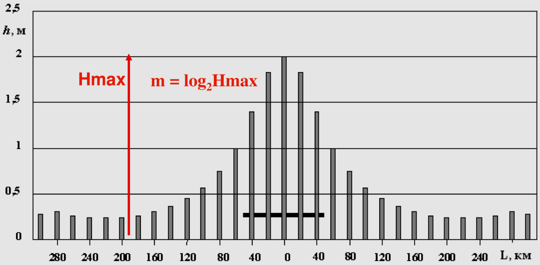
The Papadopoulos-Imamura Scale

- Introduced by G. A. Papadopoulos and F. Imamura (2001)
- 12-point scale similar to the Mercalli scale for earthquakes from I = not felt to XII = destructive

The Imamura-Iida Scale

- Introduced by A. Imamura (1942); modified by K. Iida (1956)
- Defined as

$$m = \log_2 H_{max} \quad (1)$$



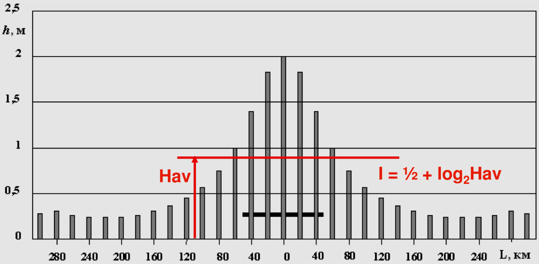
Source: Gusiakov, Tsunami Quantification

- Originally termed magnitude

The Soloviev-Imamura Scale

- Modification of the Imamura-Iida scale by S. Soloviev (1972)
- Defined as

$$I = \frac{1}{2} + \log_2 H_{av} \quad (2)$$

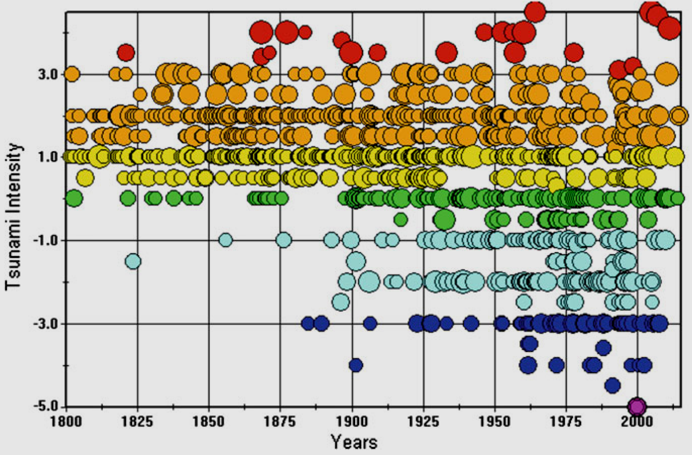


Source: Gusiakov, Tsunami Quantification

- Widely used in tsunami catalogs

Intensity and Magnitude

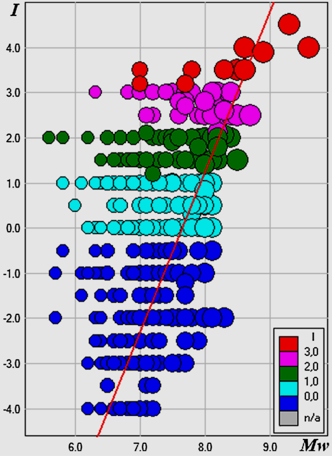
The Soloviev-Imamura Scale



Source: Levin & Nosov, Physics of Tsunamis

Intensity and Magnitude

The Soloviev-Imamura Scale



Source: Gusiakov, Pure Appl. Geophys., 2015

The Abe-Hatori Scale

- Introduced in 1979 by K. Abe
- First attempt to define a tsunami magnitude taking into account the distance from the source:

$$M_t = a \log_{10} H_{max} + b \log_{10} \Delta + D \qquad (3)$$

where

$$H_{max} = \text{maximum wave amplitude at the coast}$$
$$\Delta = \text{distance}$$
$$a, b, D = \text{constants}$$

The Murty-Loomis Scale

- Introduced in 1980 by T. S. Murty and H. G. Loomis.
- Based on the total potential energy E (in J here, originally in ergs):

$$ML = 2\,(\log_{10} E - 12) \tag{4}$$

- Well-defined and theoretically a good measure of the strength of a tsunami, but suffers from the problem of determining the total potential energy.

Theory of Ocean Waves Without Fluid Dynamics

Starting Point

Elastic medium with $\mu = 0$:

$$\boldsymbol{\sigma} \;=\; \lambda\,\epsilon_v\,\mathbf{1} \;=\; -\,p\,\mathbf{1} \tag{5}$$

where

$$p \;=\; -\,\lambda\,\epsilon_v \;=\; -\,\lambda\,\mathrm{div}(\vec{u}) \tag{6}$$

is called pressure.

- Mechanically equivalent to a compressible, inviscid fluid.
- Theory is only valid for small displacement.

↓

Kinematically not appropriate for describing fluids in general, but for waves with small amplitudes.

Types of Waves in Fluids

P body waves: sound wave; slowness

$$s = \frac{1}{v_p} = \sqrt{\frac{\rho}{\lambda}} \qquad (7)$$

P interface waves:

- Prograde particle movement on horizontal elliptical orbits
- Amplitude decreases with depth; depth of penetration

$$d = \frac{1}{\omega\sqrt{s_1^2 - |\vec{s}|^2}} = \frac{L}{2\pi\sqrt{1 - \frac{|\vec{s}|^2}{s_1^2}}} \qquad (8)$$

with $L = \frac{2\pi}{\omega s_1}$

P Interface Waves in Fluids

Particle displacement:

$$\vec{u}(\vec{x}, t) = e^{i\omega(t - \vec{s}\cdot\vec{x})}\, \vec{a} \tag{9}$$

$$= e^{i\omega(t - s_1 x_1)}\, e^{\pm\frac{x_3}{d}}\, \vec{a} \tag{10}$$

with

$$\frac{u_3(\vec{x}, t)}{u_1(\vec{x}, t)} = \frac{a_3}{a_1} = \frac{s_3}{s_1} \tag{11}$$

$$= \pm i\sqrt{1 - \frac{|\vec{s}|^2}{s_1^2}} = \pm\frac{i}{\omega s_1 d} \tag{12}$$

where only the $+$ sign makes sense in the lower halfspace

P Interface Waves in Fluids

Pressure:

$$p(\vec{x}, t) = -\lambda \operatorname{div}(\vec{u}(\vec{x}, t)) = \qquad e^{i\omega(t-\vec{s}\cdot\vec{x})} \quad (13)$$

From Eq. 12:

$$\vec{s} \cdot \vec{a} = s_1 a_1 + s_3 a_3 = s_1 a_1 \left(1 + \frac{s_3}{s_1} \frac{a_3}{a_1}\right) \quad (14)$$

$$= \frac{a_1 |\vec{s}|^2}{s_1} = a_1 \frac{\rho}{\lambda s_1} = \pm a_3 \frac{\rho \omega d}{i\lambda} \quad (15)$$

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$$p(\vec{x}, t) = \pm \qquad e^{i\omega(t-\vec{s}\cdot\vec{x})} = \pm \qquad u_3(\vec{x}, t) \quad (16)$$

P Surface Waves in Fluids

No surface with $p(\vec{x}, t) = 0$ (or constant) at any time

$$\Downarrow$$

P interface wave cannot be a surface wave.

P Surface Waves in Fluids With Gravity

Gravity causes additional hydrostatic pressure

$$p_{\text{hy}}(\vec{x}, t) = -\rho g(x_3 + u_3(\vec{x}, t)) \tag{17}$$

$$\Downarrow$$

$$p(\vec{x}, t) = \pm \rho\omega^2 d\, u_3(\vec{x}, t) - \rho g(x_3 + u_3(\vec{x}, t)) \tag{18}$$

Ocean Waves at Infinite Ocean Depth

P Surface Waves in Fluids With Gravity

Free surface at $x_3 = 0$ ($p(x_1, x_2, 0, t) = 0$) is possible if

$$\omega^2 = \frac{g}{d} = \omega g \sqrt{s_1^2 - |\vec{s}|^2} \tag{19}$$

The Velocity of Propagation

Slowness:

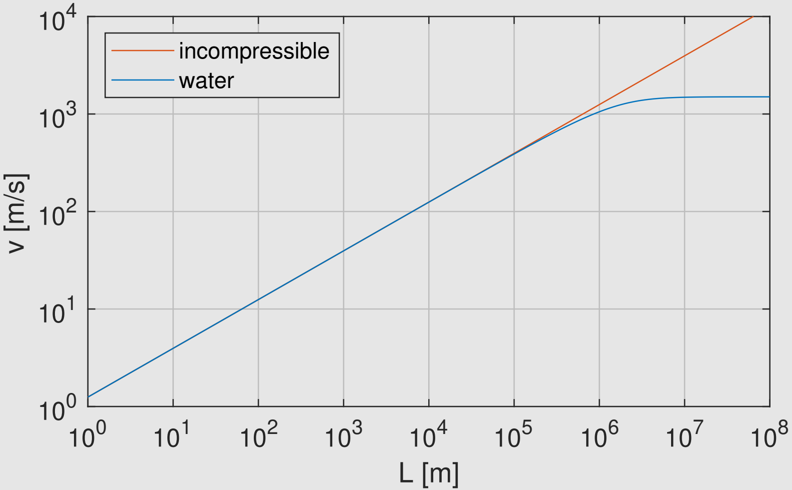
$$s_1^2 = |\vec{s}|^2 + \left(\frac{\omega}{g}\right)^2 \tag{20}$$

Expressed in terms of the wavelength $L = \frac{2\pi}{\omega s_1}$:

$$s_1^2 = \frac{|\vec{s}|^2}{2} + \sqrt{\left(\frac{|\vec{s}|^2}{2}\right)^2 + \left(\frac{2\pi}{gL}\right)^2} \tag{21}$$

Ocean Waves at Infinite Ocean Depth

Velocity of Propagation



Boundary Condition at the Ocean Floor

Consider domain $-H \leq x_3 \leq 0$ with a given ocean depth H.

$$\Downarrow$$

Solution must meet the condition $u_3(x_1, x_2, -H, t) = 0$.

Superposition of the solutions with $+$ and $-$ signs

$$u_3(\vec{x}, t) = \boxed{} \left(a_3 \, e^{i\omega(t - s_1 x_1)} \, e^{\frac{x_3}{d}} \right) + \boxed{} \left(a_3 \, e^{i\omega(t - s_1 x_1)} \, e^{-\frac{x_3}{d}} \right) \quad (22)$$

$$= a_3 \, e^{i\omega(t - s_1 x_1)} \left(\boxed{} \right) \quad (23)$$

$$= 2 a_3 \, e^{i\omega(t - s_1 x_1)} \boxed{} \left(\frac{x_3 + H}{d} \right) \quad (24)$$

satisfies the boundary condition at $x_3 = -H$.

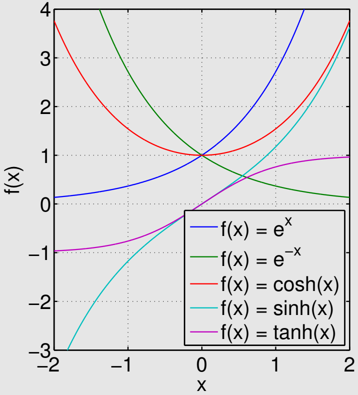
The Hyperbolic Cosine, Sine and Tangent Functions

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad (25)$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad (26)$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} \quad (27)$$

$$= \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad (28)$$



Ocean Waves at Finite Ocean Depth

Vertical Particle Displacement

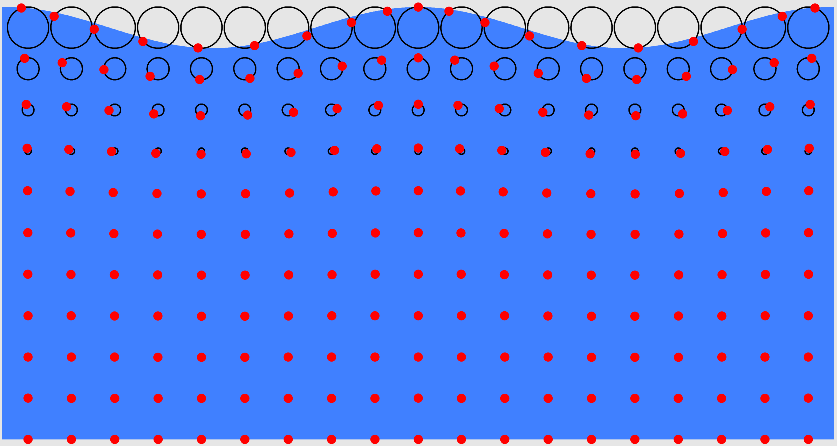
Wave height

$$h = u_3(\vec{0}, 0) = 2a_3 \sinh\left(\frac{H}{d}\right) \qquad (29)$$

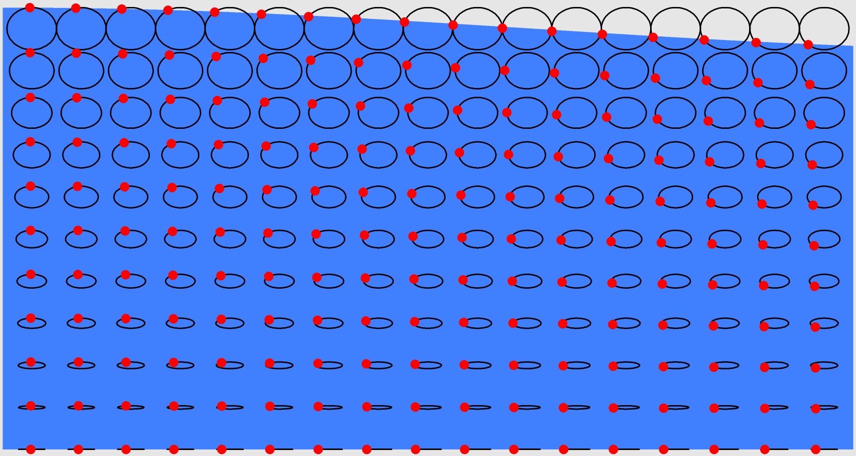
$$\Downarrow$$

$$u_3(\vec{x}, t) = h\, e^{i\omega(t - s_1 x_1)} \frac{\sinh\left(\frac{x_3 + H}{d}\right)}{\sinh\left(\frac{H}{d}\right)} \qquad (30)$$

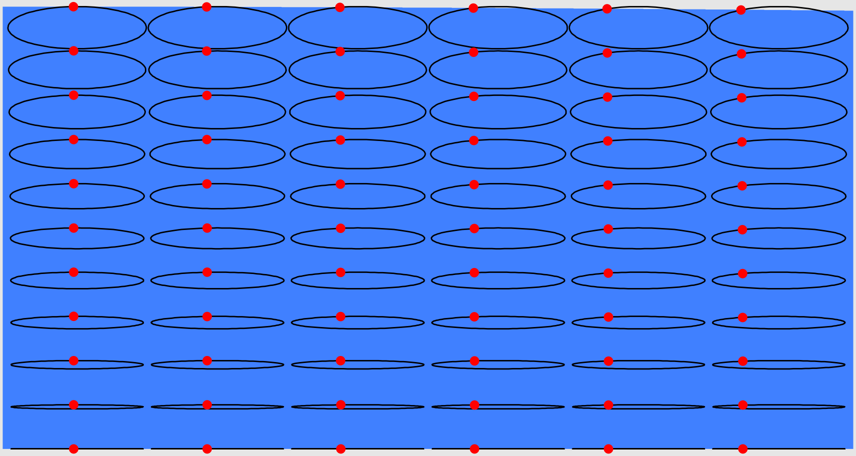
Particle Orbits for $L/H = 1$ (incompressible)



Particle Orbits for $L/H = 5$ (incompressible)



Particle Orbits for $L/H = 20$ (incompressible)



The Velocity of Propagation

Pressure (Eq. 18) for the superposed solution (Eq. 24):

$$
\begin{aligned}
p(\vec{x}, t) &= \rho\omega^2 d \left(a_3\, e^{i\omega(t - s_1 x_1)} \left(e^{\frac{x_3+H}{d}} + e^{-\frac{x_3+H}{d}} \right) \right) \\
&\quad -\rho g \left(x_3 + \left(a_3\, e^{i\omega(t - s_1 x_1)} \left(e^{\frac{x_3+H}{d}} - e^{-\frac{x_3+H}{d}} \right) \right) \right) \qquad (31) \\
&= 2 a_3 \rho\, e^{i\omega(t - s_1 x_1)} \left(\omega^2 d \cosh\left(\frac{x_3+H}{d} \right) - g \sinh\left(\frac{x_3+H}{d} \right) \right) \\
&\quad -\rho g\, x_3 \qquad (32)
\end{aligned}
$$

Free surface at $x_3 = 0$ ($p(x_1, x_2, 0, t) = 0$) is possible if

$$
\omega^2 = \frac{g}{d} \tanh\left(\frac{H}{d}\right) = \omega g \sqrt{s_1^2 - |\vec{s}|^2} \tanh\left(\frac{H}{d}\right) \qquad (33)
$$

The Velocity of Propagation

Generalization of Eq. 21:

$$s_1^2 = \frac{|\vec{s}|^2}{2} + \sqrt{\left(\frac{|\vec{s}|^2}{2}\right)^2 + \left(\frac{2\pi}{gL\tanh\left(\frac{H}{d}\right)}\right)^2} \tag{34}$$

For incompressible fluids ($|\vec{s}| = 0$, $d = \frac{L}{2\pi}$):

$$s_1^2 = \frac{2\pi}{gL\tanh\left(\frac{2\pi H}{L}\right)} \tag{35}$$

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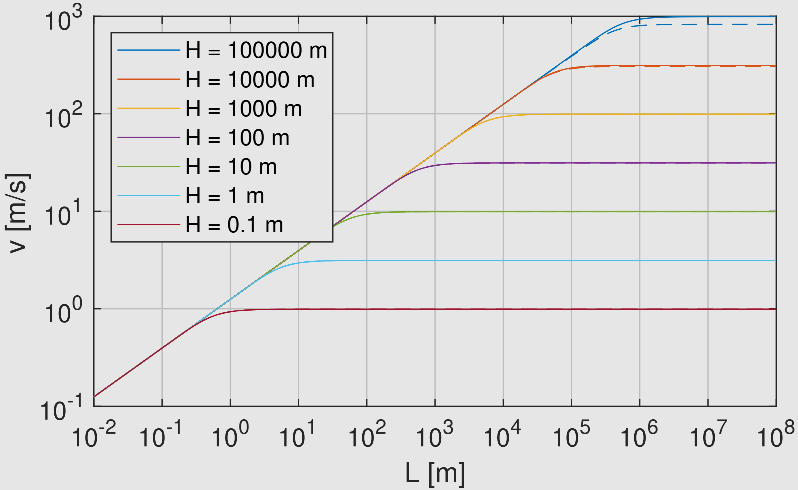
$$v = \sqrt{\frac{gL}{2\pi}\tanh\left(\frac{2\pi H}{L}\right)} \tag{36}$$

Ocean Waves at Finite Ocean Depth

The Velocity of Propagation



Ocean Waves at Finite Ocean Depth

Regimes of Ocean Wave Propagation

Deep water regime: $\frac{d}{H} \leq \frac{1}{\pi} \Leftrightarrow \frac{L}{H} \leq 2$

- Particles move on almost circular orbits.
- Particle movement is practically limited to a depth less than one wavelength.
- Horizontal particle displacement at the ocean floor is less than 10 % of the displacement at the surface.
- Velocity depends on the wavelength, but not on ocean depth:

$$v \approx \sqrt{\frac{gL}{2\pi}}, \tag{37}$$

- Strong dispersion

Regimes of Ocean Wave Propagation

Shallow water regime: $\frac{d}{H} \geq \frac{10}{\pi} \Leftrightarrow \frac{L}{H} \geq 20$

- Particles move on elliptical orbits.
- Horizontal particle movement persists down to the ocean floor; at the ocean floor more than 95 % of the displacement at the surface.
- Velocity only depends on ocean depth:

$$v \approx \sqrt{gH}$$

- No dispersion

Dispersion

Examples of tsunami wave dispersion in a 4000 m deep ocean (symmetric propagation to the left and to the right):

- bell-shaped (Gaussian) wave
- boxcar-shaped wave
- double boxcar-shaped wave
- step-like wave

The Fluid Pressure

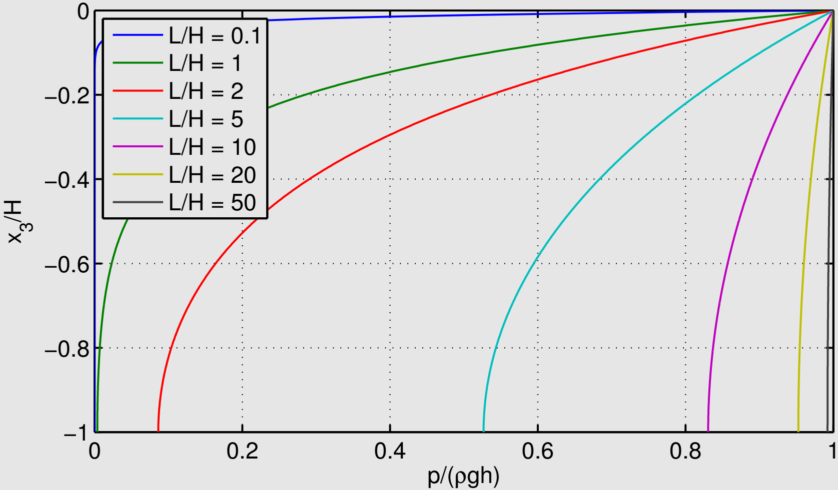
Variation in fluid pressure without hydrostatic pressure from Eqs. 32, 29, and 33:

$$p(\vec{x}, t) = 2a_3\rho\, e^{i\omega(t-s_1x_1)}\,\omega^2 d\ \cosh\left(\frac{x_3+H}{d}\right) \tag{38}$$

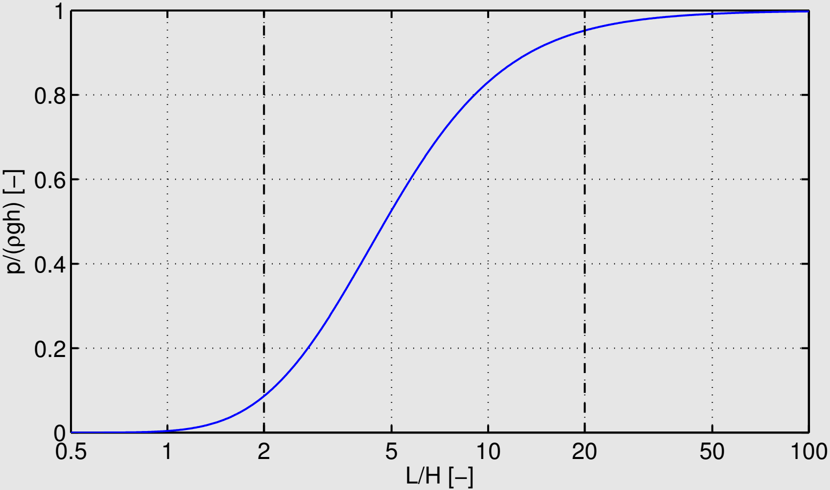
$$= \frac{h}{\sinh\left(\frac{H}{d}\right)}\rho\, e^{i\omega(t-s_1x_1)}\,\frac{g}{d}\ \tanh\left(\frac{H}{d}\right) d\ \cosh\left(\frac{x_3+H}{d}\right) \tag{39}$$

$$= \rho g h\, e^{i\omega(t-s_1x_1)}\frac{\cosh\left(\frac{x_3+H}{d}\right)}{\cosh\left(\frac{H}{d}\right)} \tag{40}$$

The Fluid Pressure



Variation in Pressure at the Ocean Floor



Variation in Pressure at the Ocean Floor

Deep water regime ($\frac{L}{H} \leq 2$): $< 10\,\%$ of the near-surface variation at the ocean floor.

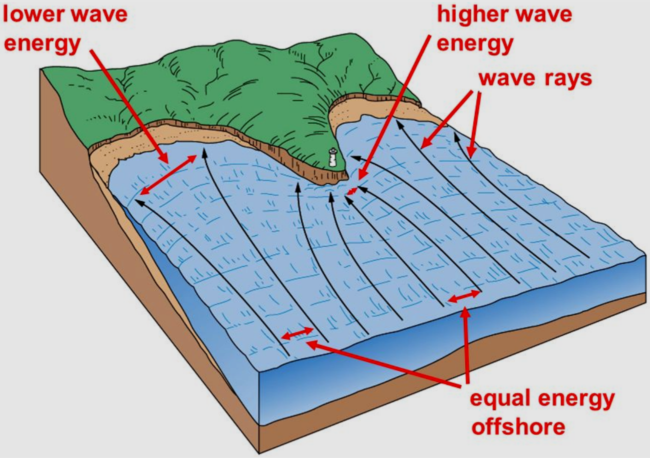
Shallow water regime ($\frac{L}{H} \geq 20$): $> 95\,\%$ of the near-surface variation at the ocean floor.

$\Downarrow$

Most important component of tsunami warning systems beyond earthquake registration.

Wave Propagation at Non-Constant Ocean Depth

Wave Shoaling



Source: Carpenter, Ocean Waves

Ray Theory

Extensions towards the harmonic plane wave approach:

- Retarded time $\tau = t - \psi(x_1, x_2)$ instead of $\tau = t - s_1 x_1$ with a general phase function $\psi(x_1, x_2)$

  $\Downarrow$

  Propagation in direction of $\nabla\psi(x_1, x_2)$ with local slowness $|\nabla\psi(x_1, x_2)|$
- Spatially variable wave height $h(x_1, x_2)$
- Vertical particle displacement in analogy to Eq. 30,

$$u_3(\vec{x}, t) = h\, e^{i\omega(t - \psi(x_1, x_2))} \frac{\sinh\left(\frac{x_3 + H}{d}\right)}{\sinh\left(\frac{H}{d}\right)} \tag{41}$$

so that $u_3 = 0$ at the ocean floor.

Ray Theory

Calculations in analogy the eikonal equation for seismic waves.

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Terms $\sim \omega^2$:

- Horizontal particle displacement only in direction of propagation
- Velocity of propagation according to Eq. 36

Terms $\sim \omega$:

$$\mathrm{div}(\vec{q}) = 0 \tag{42}$$

with the energy flux density

$$\vec{q} = \frac{1}{2}\rho g\, h^2\, \vec{v} \tag{43}$$