Near-Surface Geophysics

Seismology and Seismics

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Introduction

Seismology

- Comprises all about earthquakes and the propagation of seismic waves in the Earth.
- One of the main fields of solid-earth geophysics.
- Has provided the majority of our knowledge on Earth‘s interior.

Seismics

- Exploration of the deep and shallow subsurface with the help of artificial seismic waves.
- The perhaps most important field of applied geophysics.

https://www.youtube.com/watch?v=Tm9kqImgsxQ

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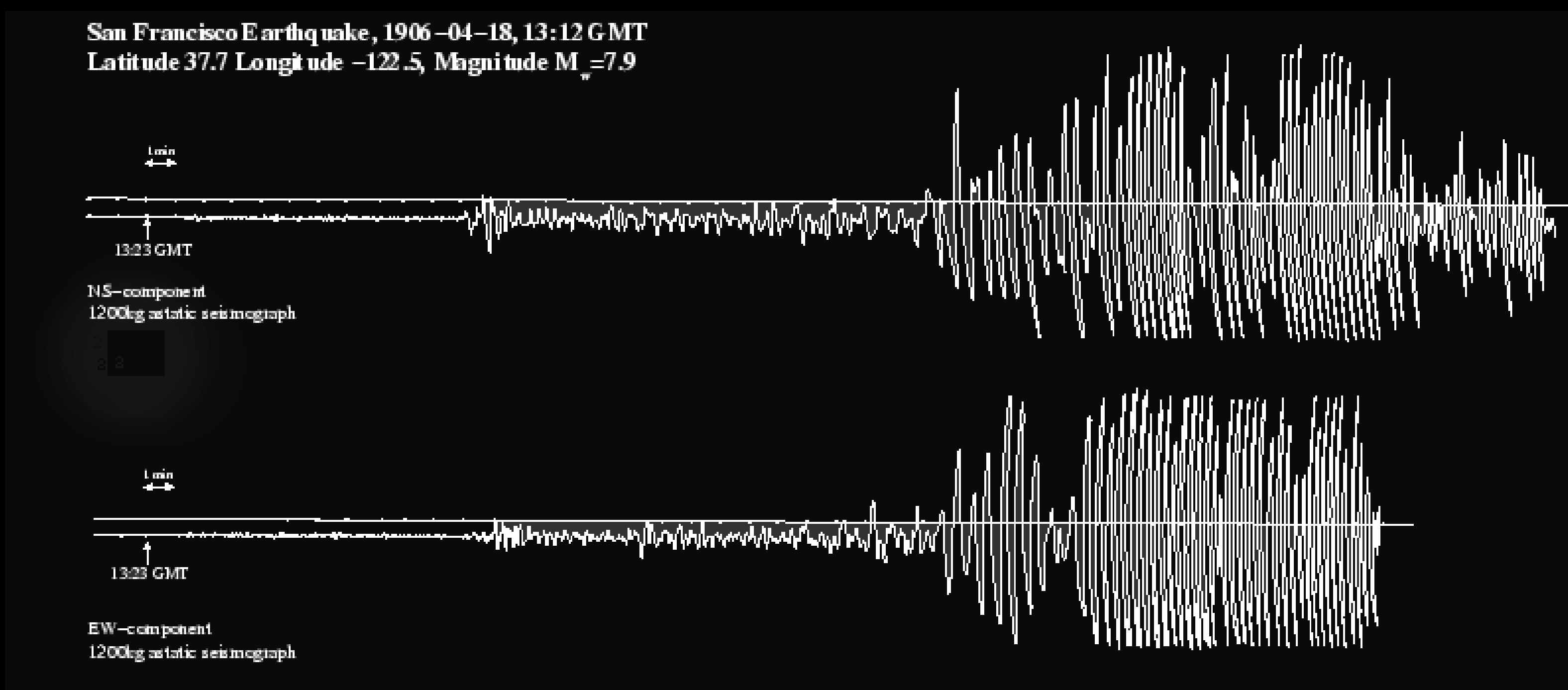
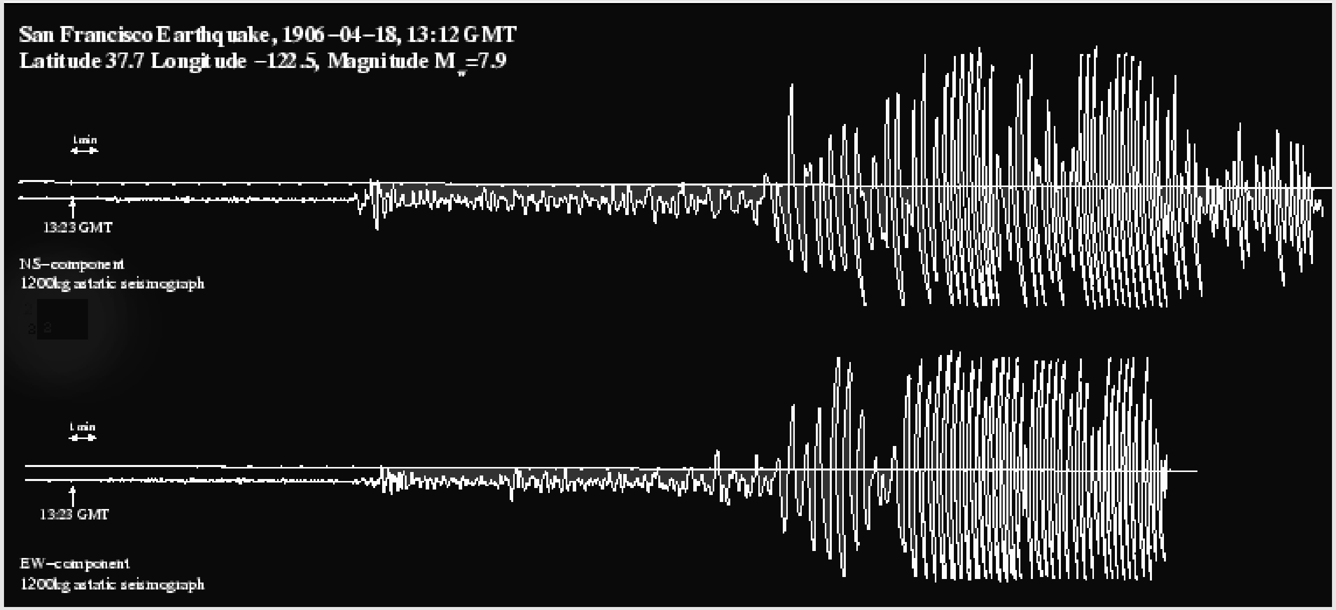


Introduction

History of Seismology

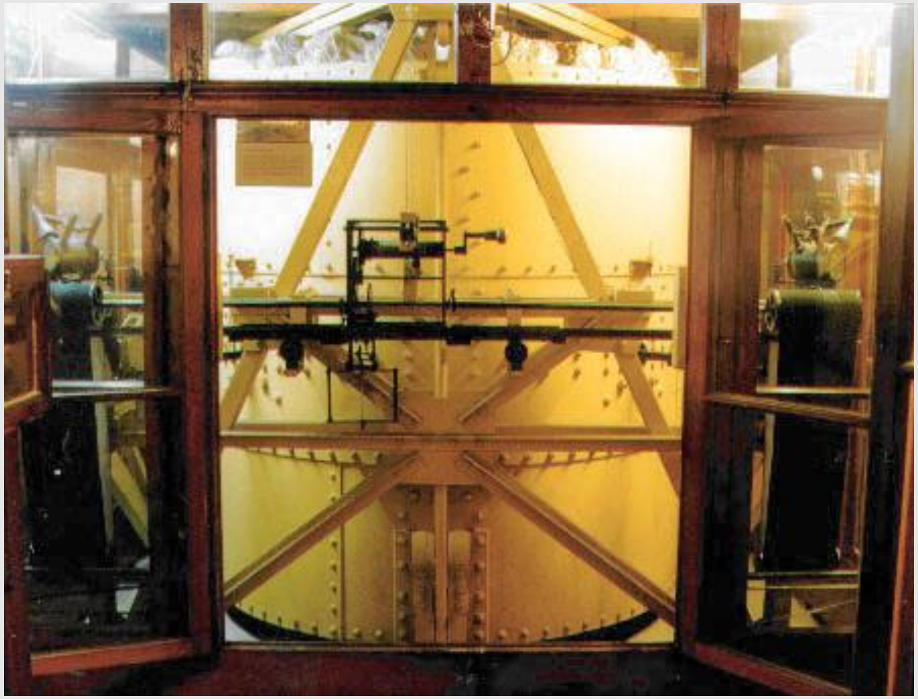


Recording of San Francisco earth quake 1906 registered in Göttingen, Germany

Introduction

History of Seismology

First “Seismometer”: China, Zhang Heng, 132 a.D.

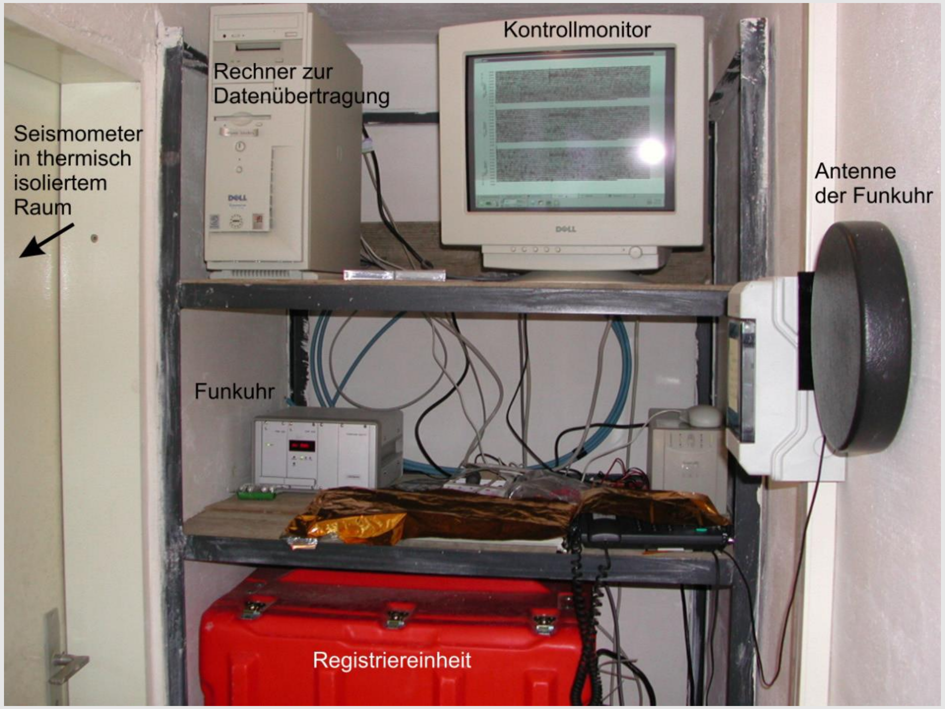


17 t Wiechert pendulum, Uni Göttingen

Introduction

History of Seismology

STS-2 Seismometer



Observatory Rüdersdorf:
http://www.fu-berlin.de/geophysik/

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History of Seismology

1660	basic law of elasticity	R. Hooke
1821–22	differential equations of elasticity	C. Navier A. L. Cauchy
1830	theory of two fundamental types of elastic waves (P- and S-wave)	S. D. Poisson
1875	First "serious" seismometer	F. Gecchi
1887	theory of the first type of surface waves	J. W. Strutt (3. Lord Rayleigh)
1889	first recording of a distant earthquake	
1892	first compact seismometer, used at about 40 stations	J. Milne
1894	statistics of aftershocks	F. Omori
1903	12 degree scale for the intensity of earthquakes based on the damage	G. Mercalli

History of Seismology

1906–1913	detection of the liquid core of the earth and determination of its size	R. D. Oldham, B. Gutenberg
1909	detection of the crust-mantle discontinuity	A. Mohorovičić
1911	theory of a second type of surface waves	A. E. H. Love
1935	local magnitude as an "objective" measure of earthquake intensity	C. F. Richter
1936	detection of the inner, solid core	I. Lehmann
1954	frequency-magnitude relation of earthquakes	B. Gutenberg, C. F. Richter
1975	first successful short-term prediction of a strong earthquake	
1977	moment magnitude as a measure of earthquake source strength	H. Kanamori

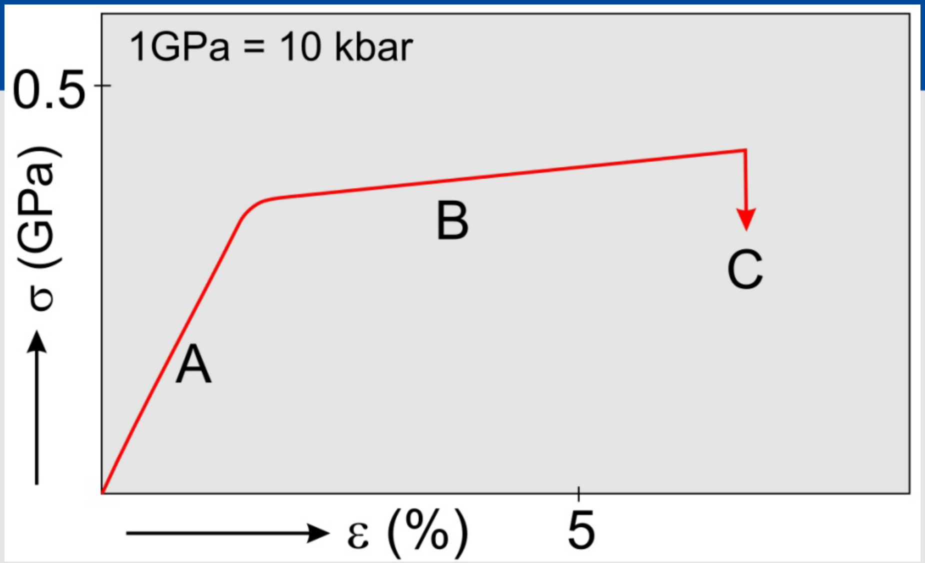
Seismic Waves

Waves in Elastic Media

- Propagating elastic deformation
- The same as sound waves in solids
- More complicated than sound waves in liquid and gases

Elastic modulus

Which section describes linear elasticity?



Elastic modulus

- Linear elasticity is the mathematical study of how solid objects deform and become internally stressed due to prescribed loading conditions.

↓

- The fundamental assumptions of linear elasticity are: infinitesimal strains or small deformations and linear relationships between the components of stress and strain.

- In continuum mechanics, the Lamé parameters are two material-dependent quantities denoted by λ and μ that arise in strain-stress relationships.

  λ: Lamé's first parameter

  μ: shear modulus (ratio of shear stress to the shear strain)

Seismic Waves

Elastic modulus

Hooke's Law

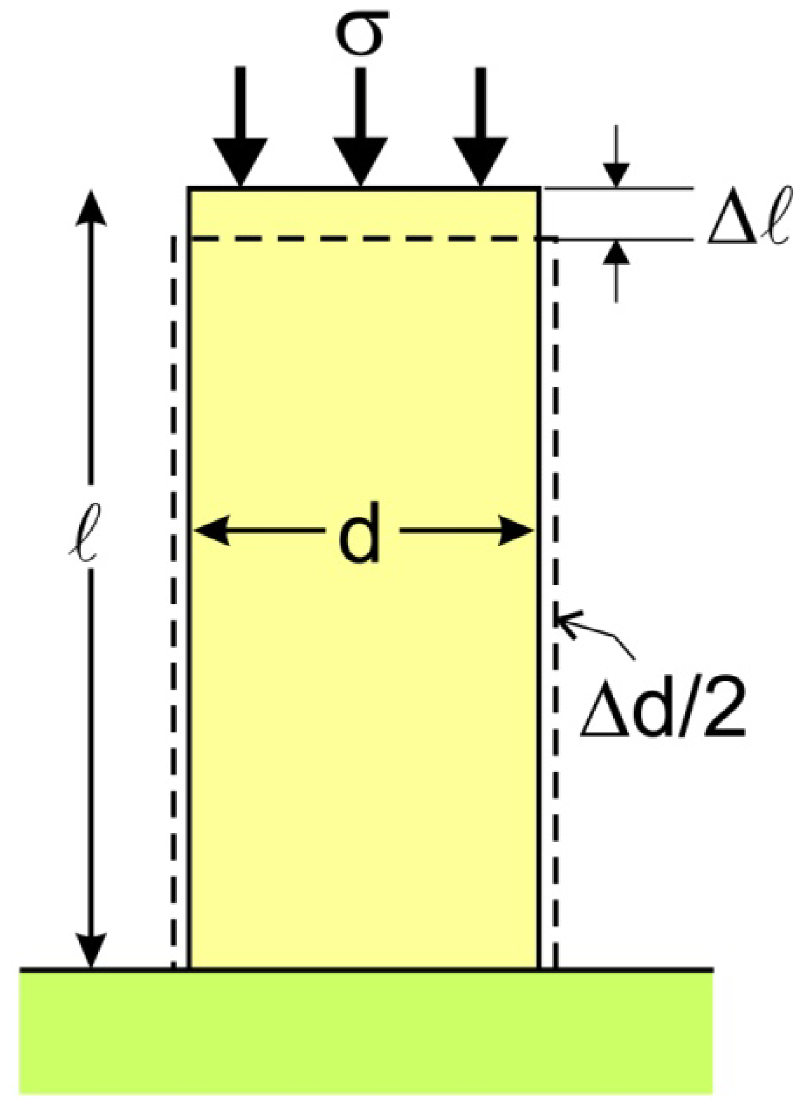
$$\varepsilon = \frac{1}{E}\sigma$$

Poisson's ratio

$$\nu = \frac{\Delta d/d}{\Delta \ell/\ell}$$

- Elastic modulus measures an object or substance's resistance to being deformed elastically
- The elastic modulus of an object is defined as the slope of its stress-strain curve in the elastic deformation region:

$$\lambda = \frac{stress}{strain}$$



Seismic Waves

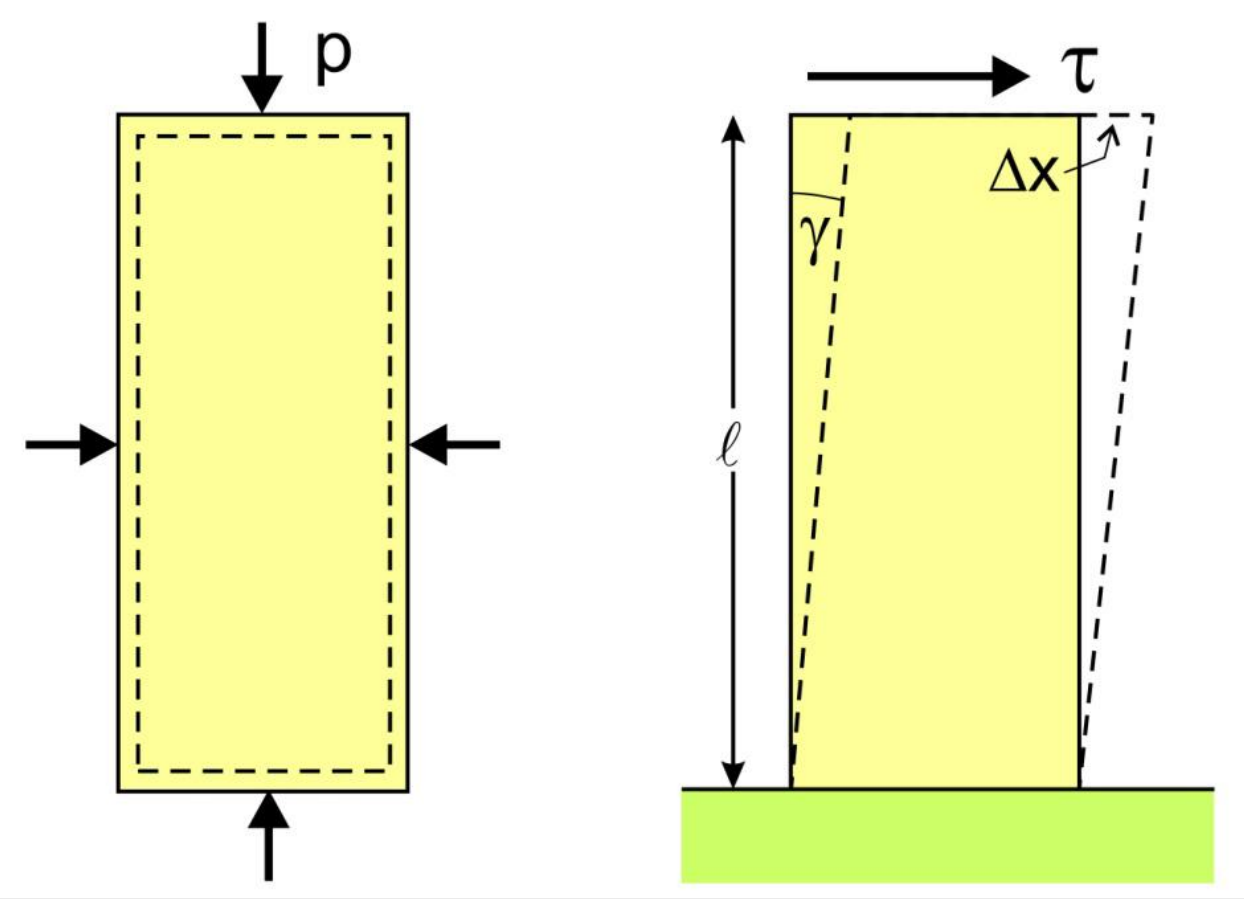
Elastic modulus

Bulk modulus

$$\frac{\Delta V}{V} = \frac{1}{K} p$$

Shear modulus

$$\gamma = \frac{1}{G} \tau$$



Seismic Waves

Elastic modulus

Rock Type	Density r	Young's Modulus E	Poisson's Ratio m	Vp (m/s)	Vs (m/s)	Vp/Vs	Vs as %Vp
Shale	2,67	0,120	0,040	2124	1470	1,44	69,22%
Siltstone	2,50	0,130	0,120	2319	1524	1,52	65,71%
Limestone	2,71	0,337	0,156	3633	2319	1,57	63,84%
Quartzite	2,66	0,636	0,115	4965	3274	1,52	65,96%
Sandstone	2,28	0,140	0,060	2488	1702	1,46	68,42%
Slate	2,67	0,487	0,115	4336	2860	1,52	65,96%
Schist	2,70	0,544	0,181	4680	2921	1,60	62,41%
Gneiss	2,64	0,255	0,146	3189	2053	1,55	64,38%
Marble	2,87	0,717	0,270	5587	3136	1,78	56,13%
Marble	2,71	0,343	0,141	3643	2355	1,55	64,65%
Granite	2,67	0,605	0,259	5260	3000	1,75	57,03%
Gabbro	3,05	0,727	0,162	5043	3203	1,57	63,51%
Diabase	2,96	1,020	0,271	6569	3682	1,78	56,05%
Basalt	2,74	0,630	0,220	5124	3070	1,67	59,91%
Andesite	2,57	0,540	0,180	4776	2984	1,60	62,47%
Tuff	1,45	0,014	0,110	996	659	1,51	66,20%

Units for Young's modulus are (N/m2) x 1011. Velocities computed from r, E, and m. Values selected from Press (1966, p. 97-173).

Theory of Wave Propagation

Navier-Cauchy equations for the displacement $\vec{u}(\vec{x}, t)$ in an elastic medium (http://hergarten.at/extra/continuummechanics.pdf):

$$\rho \frac{\partial^2}{\partial t^2} \vec{u} \;=\; \nabla \left(\lambda \, \mathrm{div}(\vec{u}) \right) + \mathrm{div} \left(\mu \left(\nabla \vec{u} + (\nabla \vec{u})^T \right) \right)$$

with

$$\begin{aligned} \rho(\vec{x}) &= \text{density} \\ \lambda(\vec{x}),\; \mu(\vec{x}) &= \text{Lamé's parameters of the medium} \end{aligned}$$

No general analytical solution for an inhomogeneous medium

If λ and μ are constant:

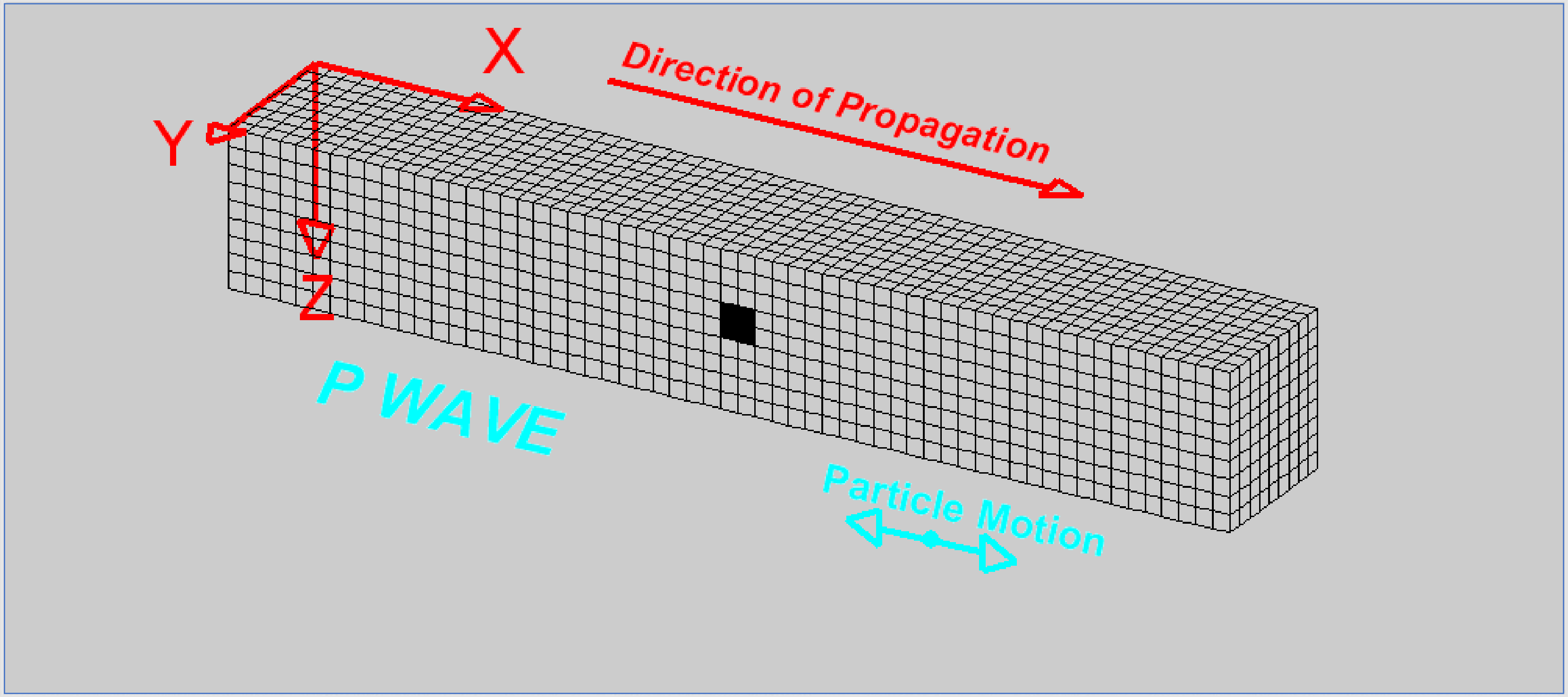
$$\rho \frac{\partial^2}{\partial t^2} \vec{u} \;=\; (\lambda + \mu) \, \nabla \mathrm{div}(\vec{u}) + \mu \, \Delta \vec{u}$$

Seismic Waves

Basic Types of Body Waves

‘Two types of independent plane waves in an infinite, homogeneous elastic medium:

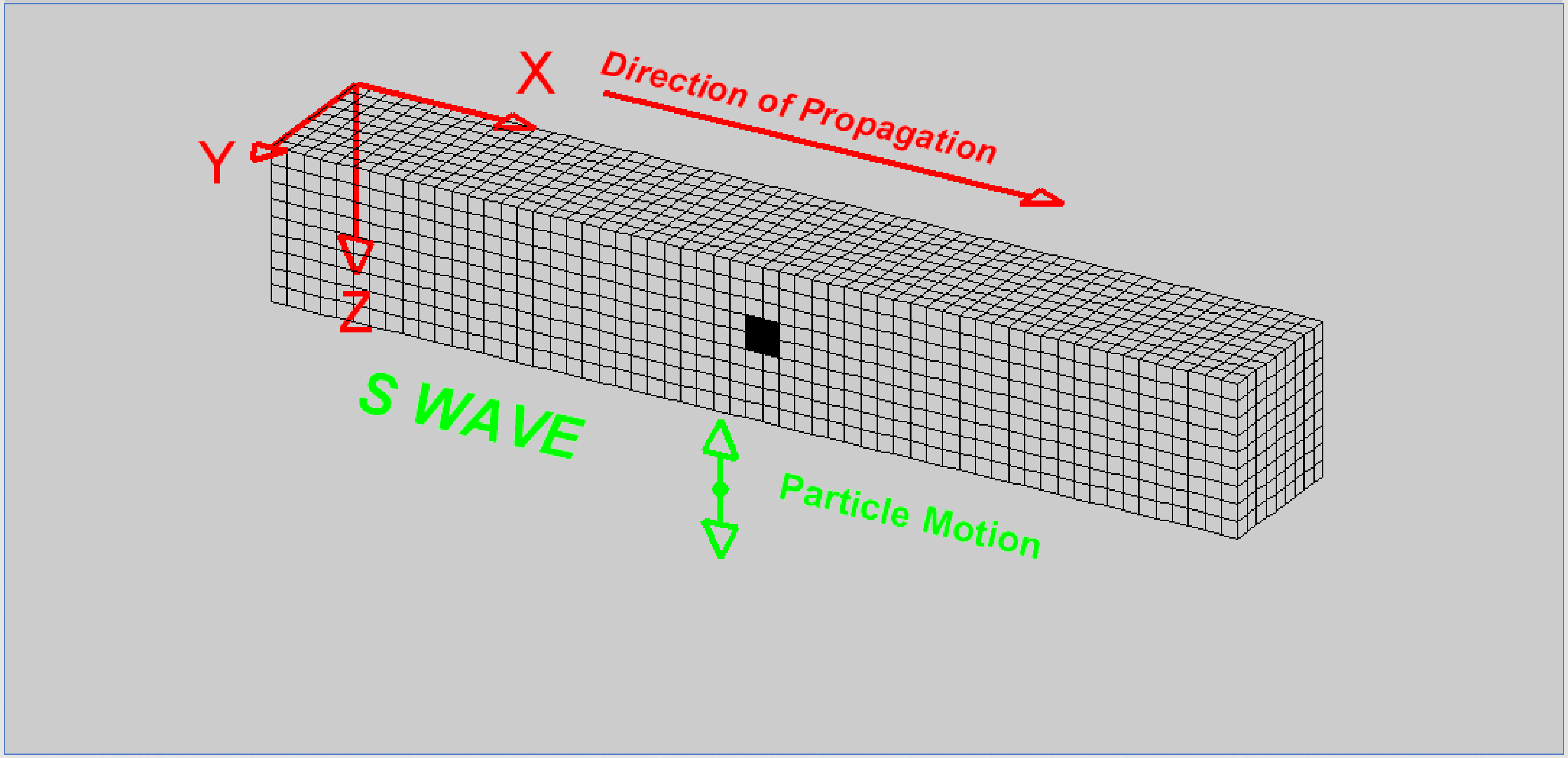
- Compressional wave (longitudinal wave, primary wave)



Source: L. Braile, Purdue University

Basic Types of Body Waves

- Shear wave (transverse wave, secondary wave)



Source: L. Braile, Purdue University

Seismic Waves

Comparison with Sound Waves in Liquids and Gases

The compressional wave is similar to sound waves in liquids and gases, while the shear wave has no counterpart in liquids and gases.

Seismic Velocities

Medium	Compressional wave [$\frac{km}{s}$]	Shear wave [$\frac{km}{s}$]
air	0.34	–
water	1.45	–
wood	about 3	about 1.8
Earth	5.8–13.7	3.2–7.3

Seismic Waves

Seismic Velocities

Velocity v_p of the compressional wave is always higher than the velocity v_s of the shear wave.

$\Downarrow$

Compressional wave always arrives prior to the shear wave.

$\Downarrow$

compressional wave = primary wave (P-wave)
shear wave = secondary wave (S-wave)

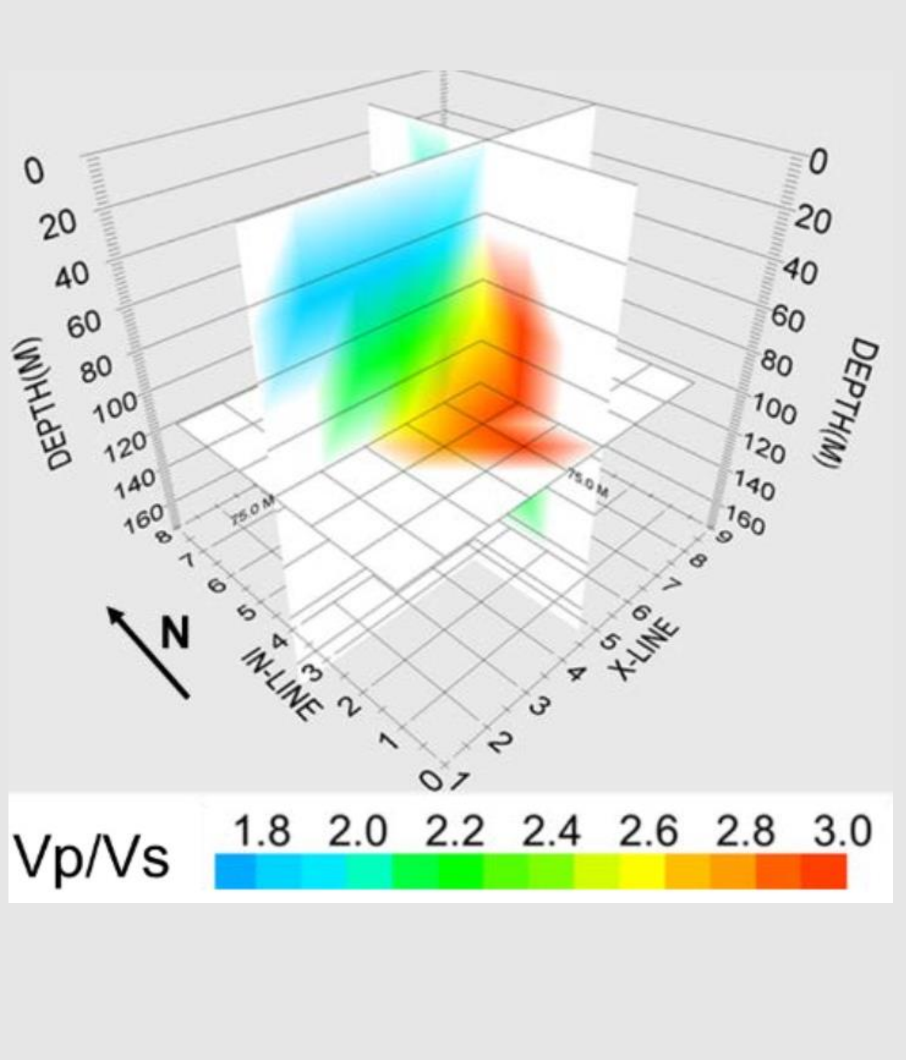
Rules of Thumb

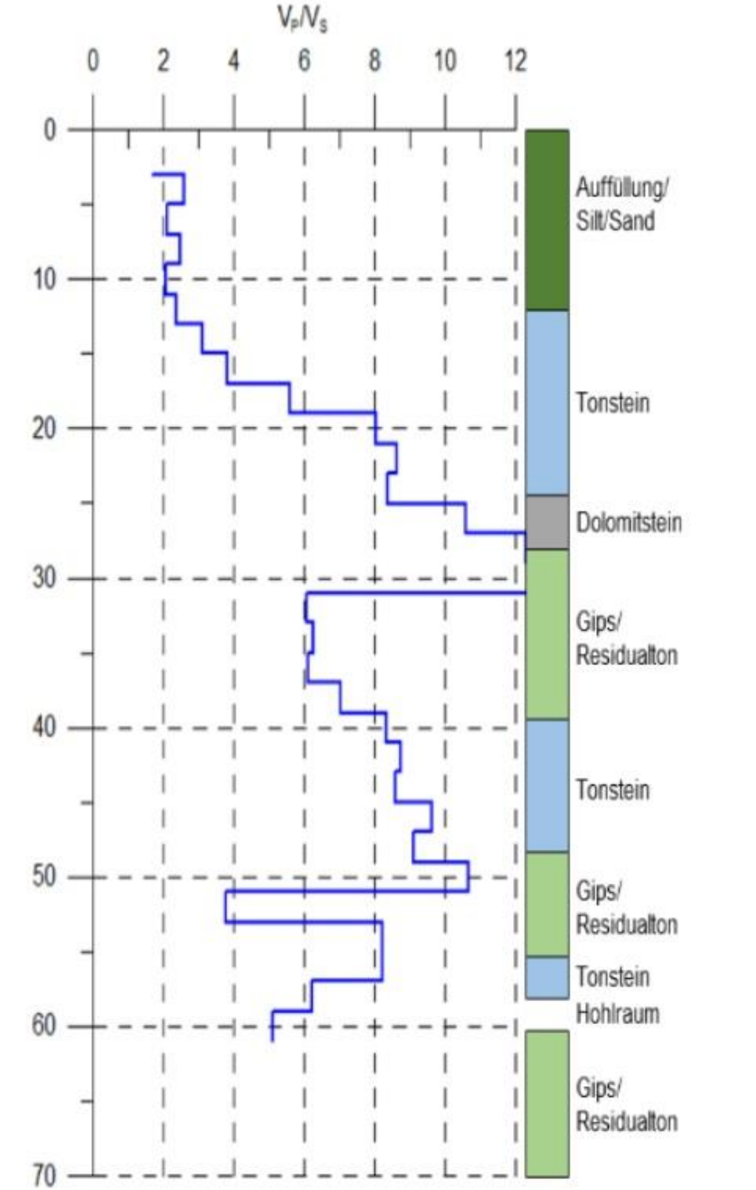
Solid rock: $v_s \approx 0.5\, v_p - 0.6\, v_p$

Soil / unconsolidated rock: $v_s \approx 0.4\, v_p$

Seismic Waves

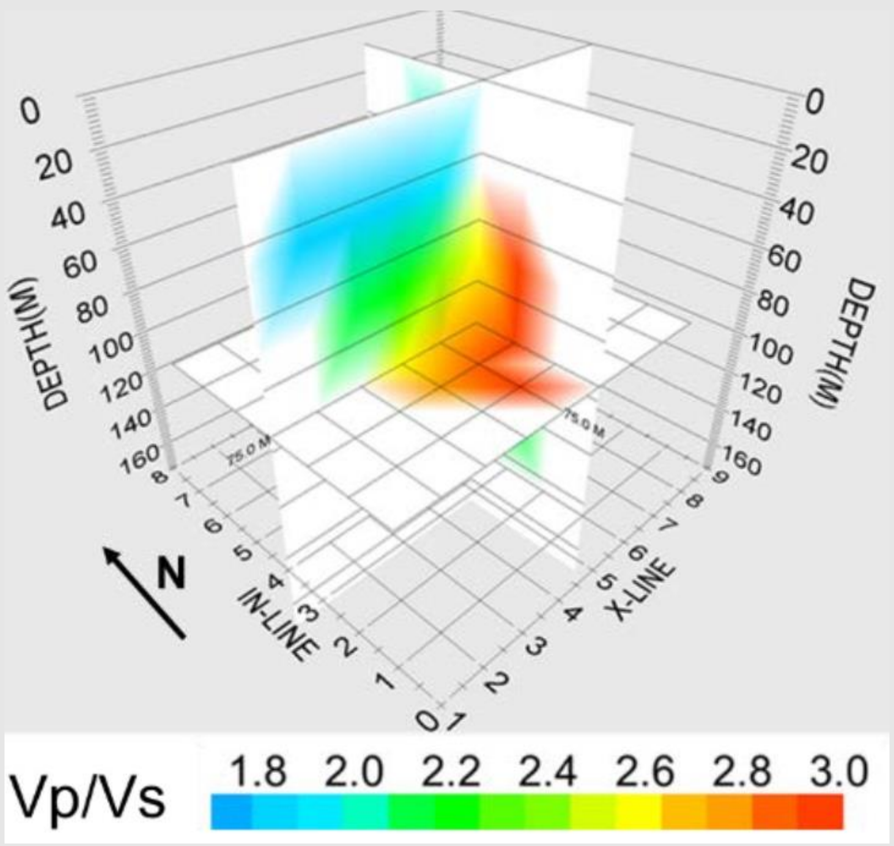
Seismic Velocities

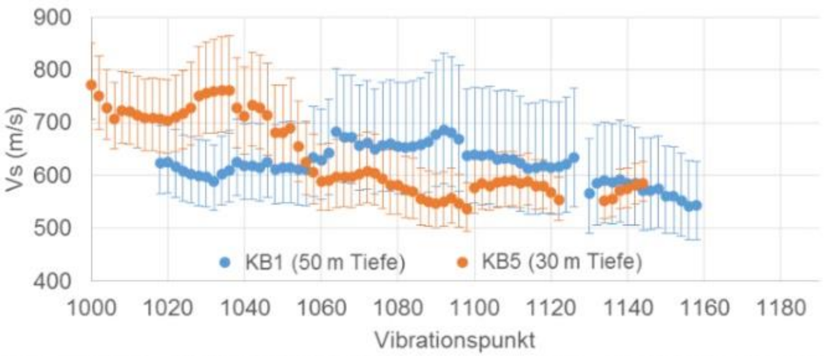


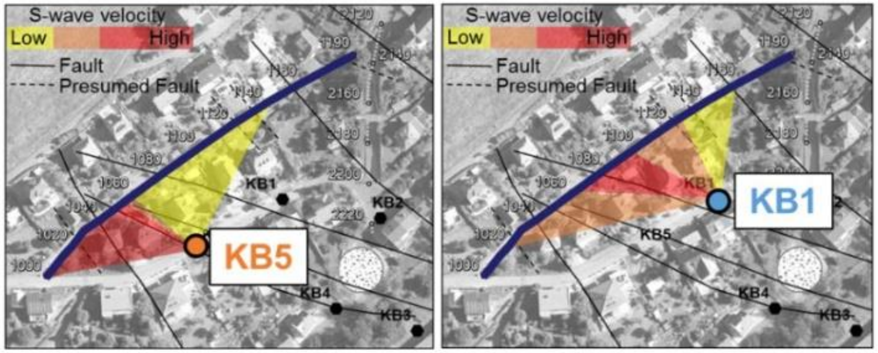


Seismic Waves

Seismic Velocities

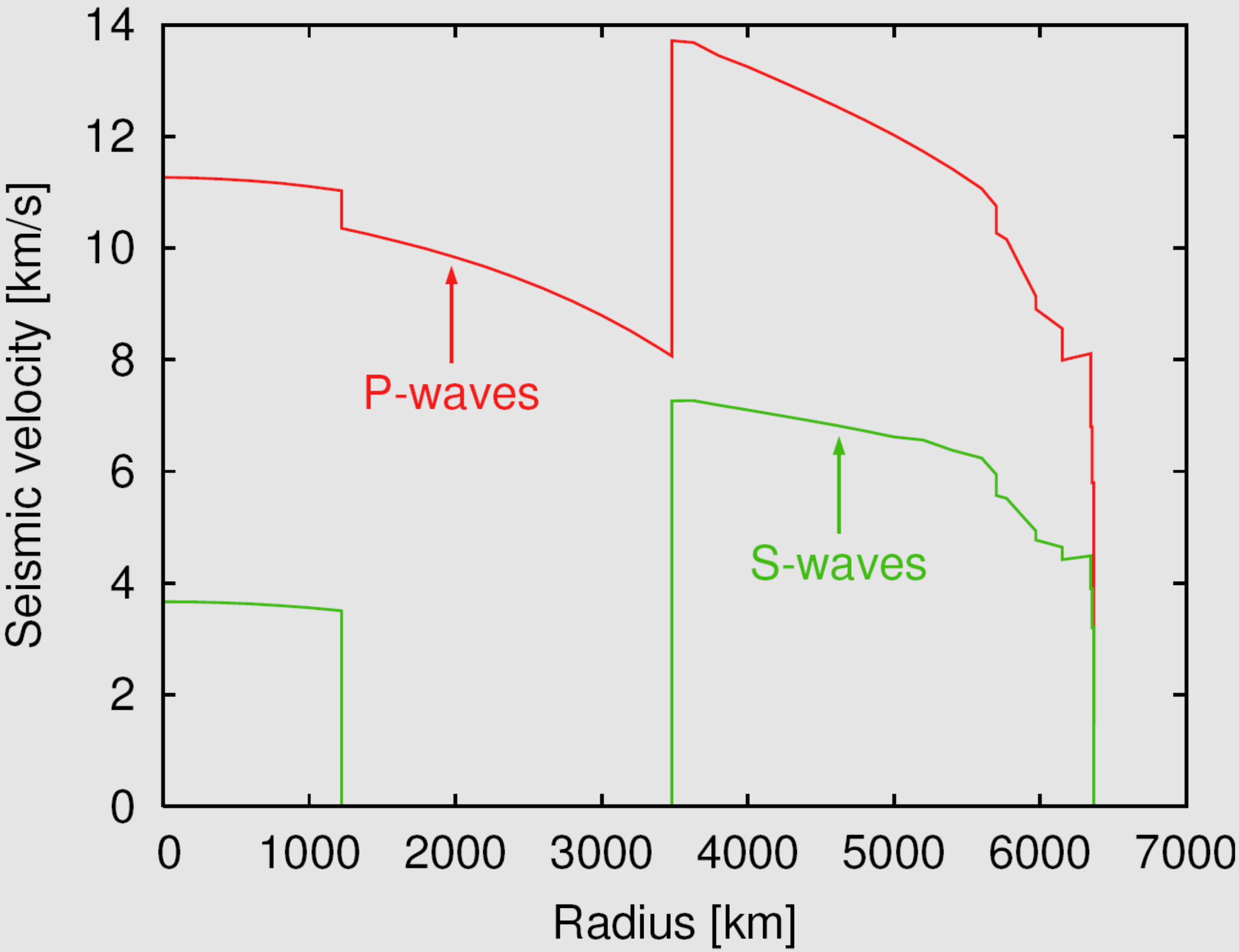






Seismic Waves

Seismic Velocities according to the Prelimenary Reference Earth Model



Seismic Waves

Typical P-wave Velocities in the Shallow Subsurface

$\vec{s}$ is the "slowness vector" of the wave ($|\vec{s}| = \frac{1}{v}$), so that

$$v_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{M}{\rho}} = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}}$$

$$v_s = \sqrt{\frac{\mu}{\rho}}$$

with

M = linear elastic modulus without lateral contraction

K = bulk modulus

μ = shear modulus

v_p und v_s are determined by the density and on the elastic modulus of the respective type of deformation.

Seismic Waves

Seismic Velocities and Elastic Properties – vertical slowness

Slowness (s) is a quantity introduced in Seismology which is the reciprocal of velocity. Thus travel time of a wave is the distance that the wave travels times the slowness of the medium.

Typical P-wave Velocities in the Shallow Subsurface

Medium	v_p [$\frac{km}{s}$]
weathering zone	0.1–0.5
dry sand	0.3–0.6
water-saturated sand	1.3–1.8
sandstone	1.8–4
pit coal	1.6–1.9

Medium	v_p [$\frac{km}{s}$]
clay	1.2–2.8
claystone	2.2–4.2
limestone	3–6
halite	4.5–6.5
granite	5–6.5

Propagation in Inhomogeneous Media

Two different approximations in analogy to optics:

- Computation of wavefronts (Huygens' principle, eikonal equation)
- Computation of ray paths normal to the wavefronts (ray optics)

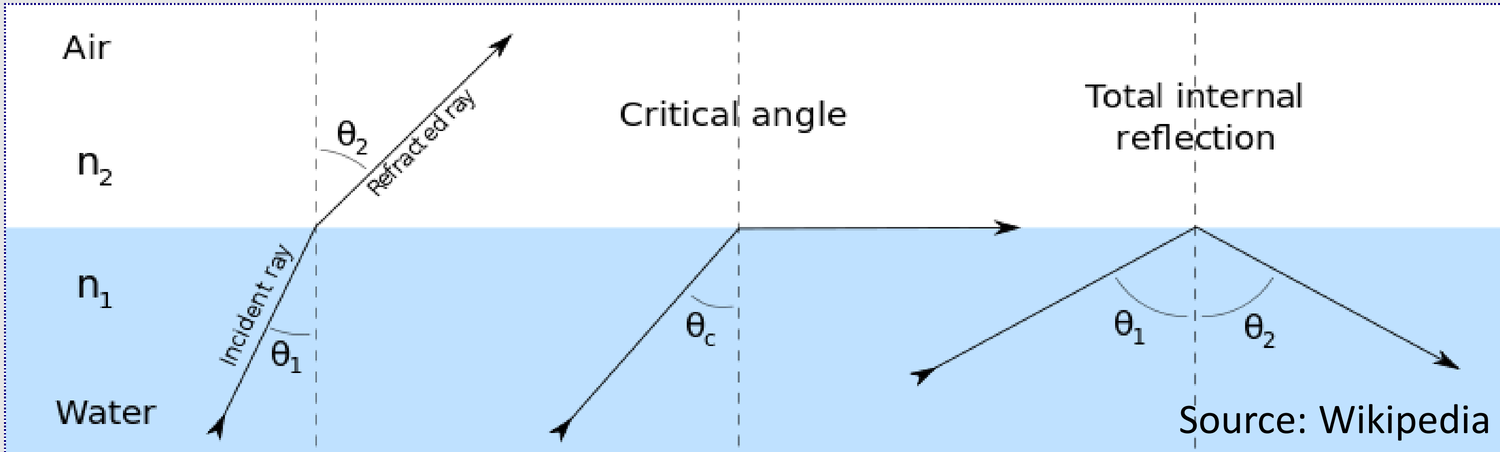
Ray Optics

Ray optics is rather simple in two limiting cases:

- Almost planar interface between two homogeneous media.
- Medium is almost homogeneous on the scale of the wavelength.

Seismic Waves

Reflection and Refraction



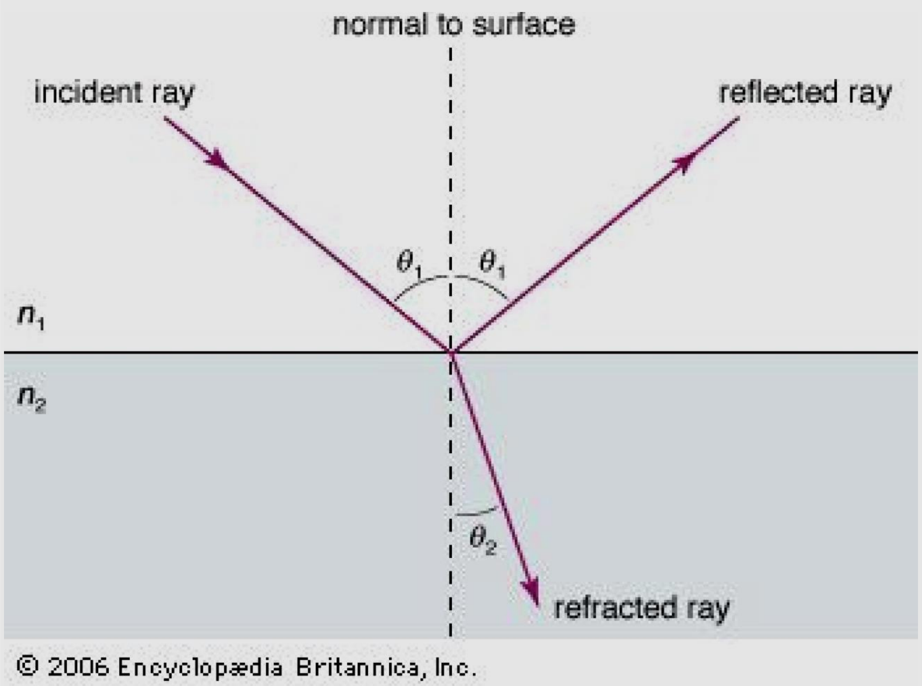
Snell's law

Simplest case: two homogeneous, isotropic halfspaces with different properties (λ, μ, ρ) and plane waves in each of them.

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \quad \Rightarrow \quad \begin{aligned} v_{refl} &= l - (2\cos\theta_1)n \\ v_{refr} &= \left(\frac{n_1}{n_2}\right) l + \left(\frac{n_1}{n_2}\cos\theta_1 \pm \cos\theta_2\right) n \end{aligned}$$

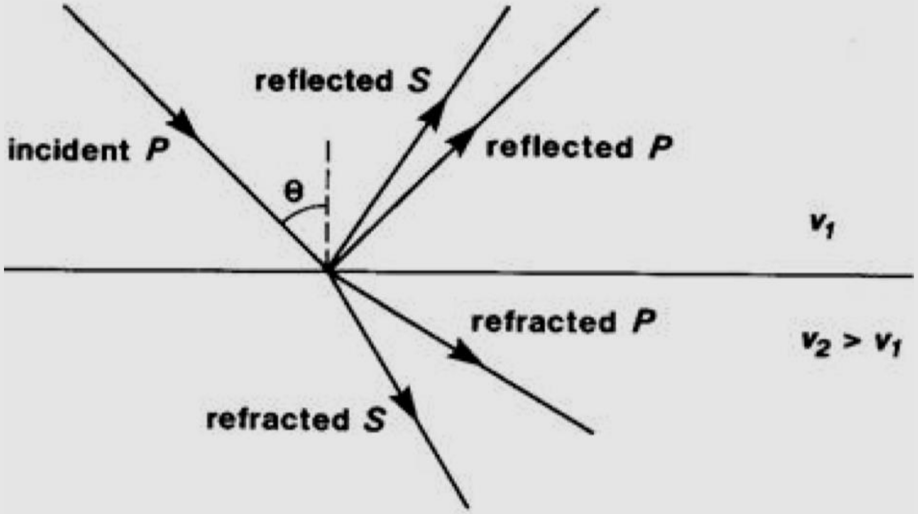
Seismic Waves

Reflection and Refraction of light



Source: Encyclopaedia Britannica

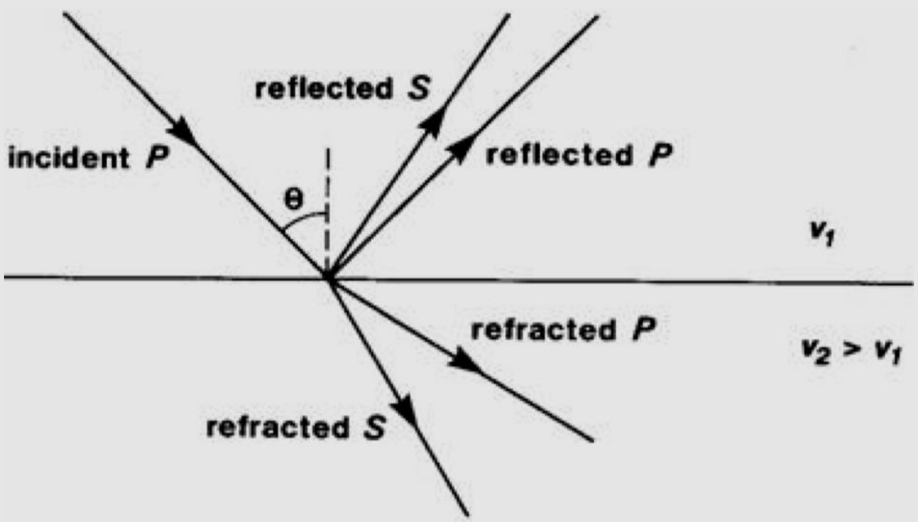
Reflection and Refraction of elastic waves



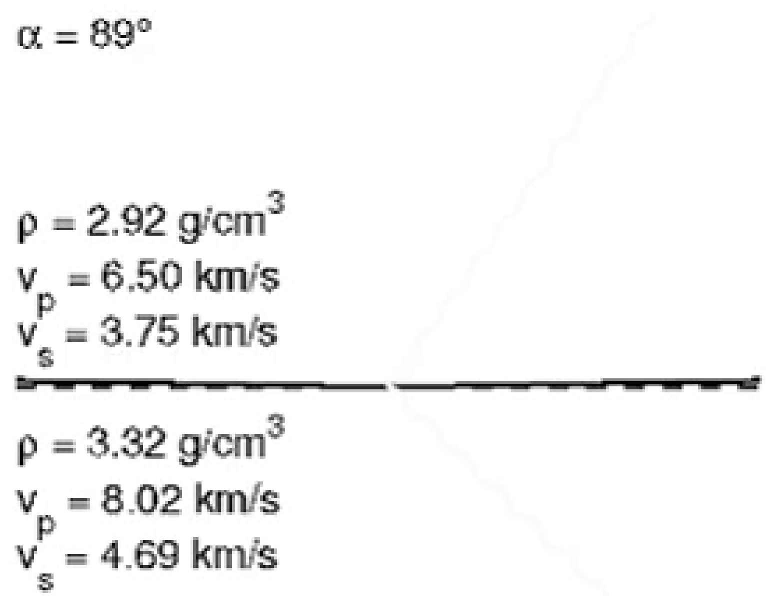
Source: University College London

Seismic Waves

Reflection and Refraction of elastic waves



Source: University College London



Conversion of Waves in Reflection and Refraction

Reflected and Refracted Waves

P- and S-waves are merged when reflected or refracted.

↓

Incoming P- or S-wave induces up to 4 reflected and refracted waves.

Snell's law holds for all involved pairs of waves

↓

Ray parameter

$$p = \frac{\sin \alpha}{v}$$

(also called horizontal slowness) is the same for all involved waves.

Reflected and Refracted Waves

General form of Snell's law:

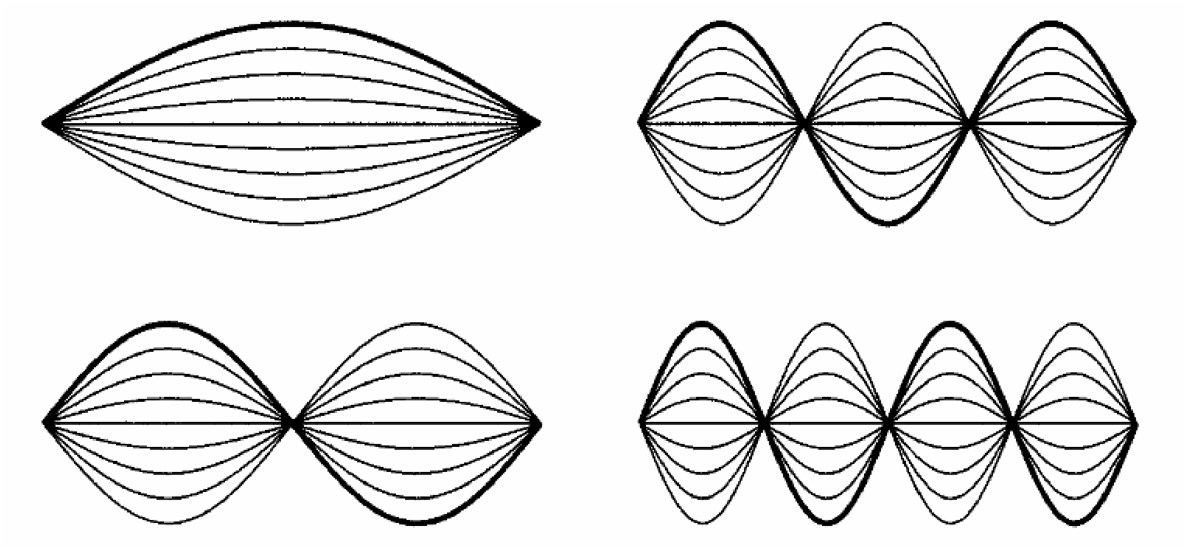
Horizontal slowness remains constant in reflection and refraction.

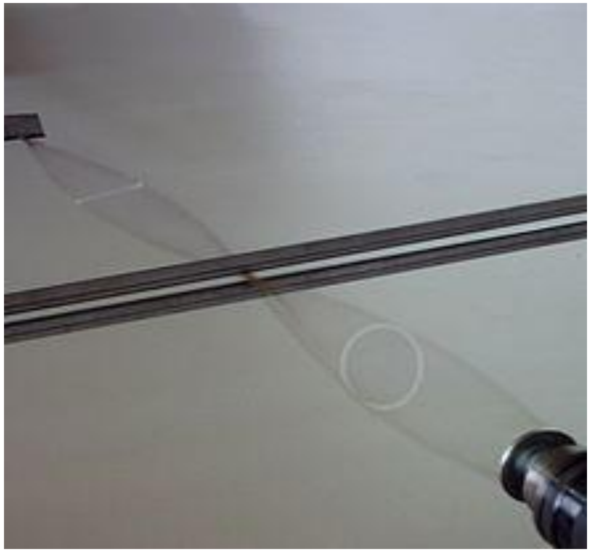
- Horizontal velocity is not constant!
- Conservation of horizontal slowness is the main reason why slowness is preferred to veloctiy in seismology.

Seismic Waves

Polarization (shear wave splitting)

- Parameter that specifies the geometrical orientation of the oscillations
- Horizontally and vertically polarized seismic waves (shear waves) are termed SH and SV, while waves with longitudinal polarization are termed P-waves.
- Differential propagation of the three polarizations through the earth is a crucial in the field of seismology.





Modes of vibration of a string between fixed endpoints, from Shearer 2010, Introduction to Seismology

Seismic Waves

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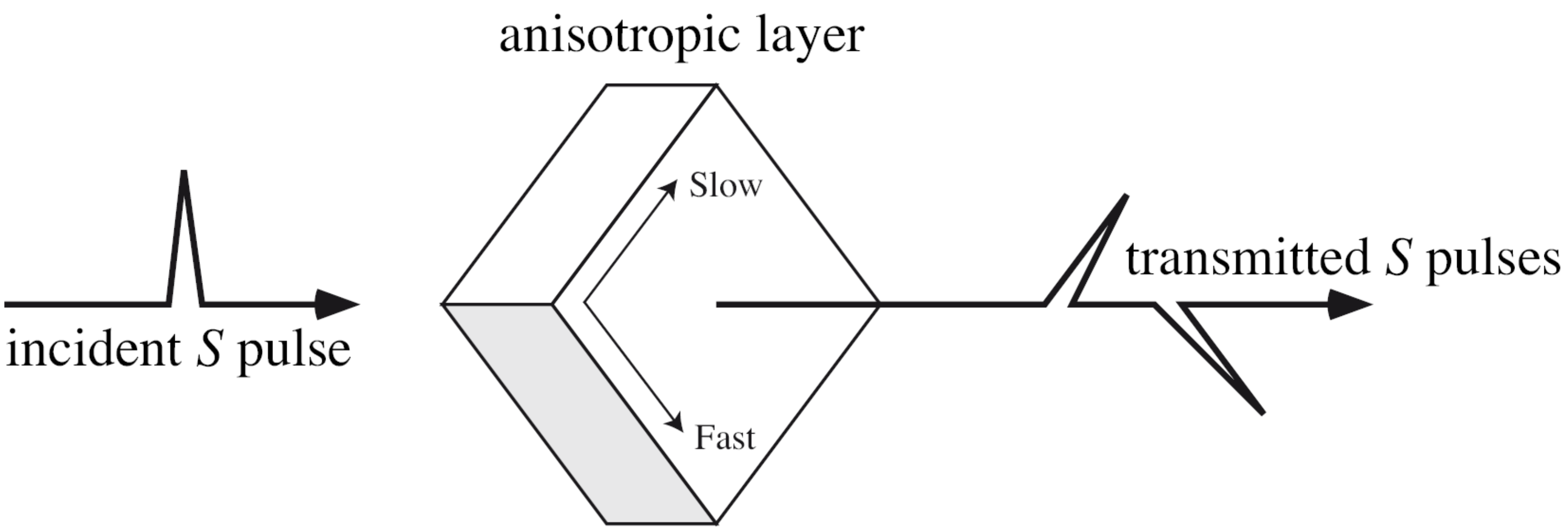

Shear-wave splitting (Long and Becker 2010, Mantle dynamics and seismic anisotropy)

Seismic Waves

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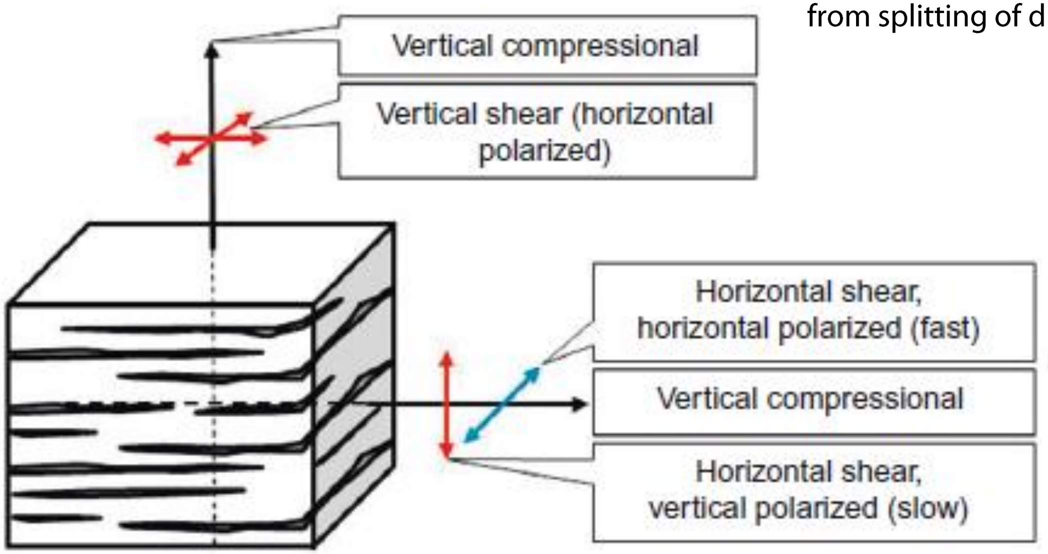
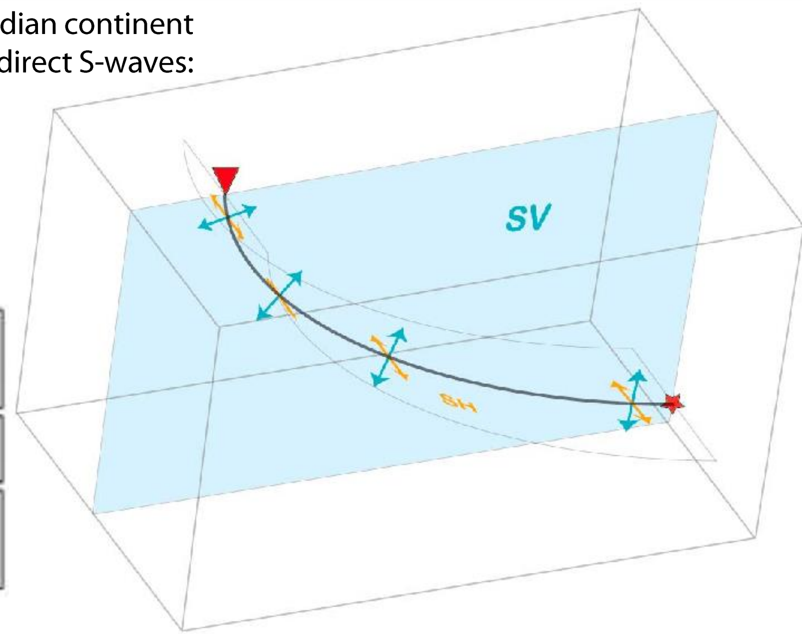
Dipankar et al. 2010, Seismic anisotropy beneath the Indian continent from splitting of direct S-waves:





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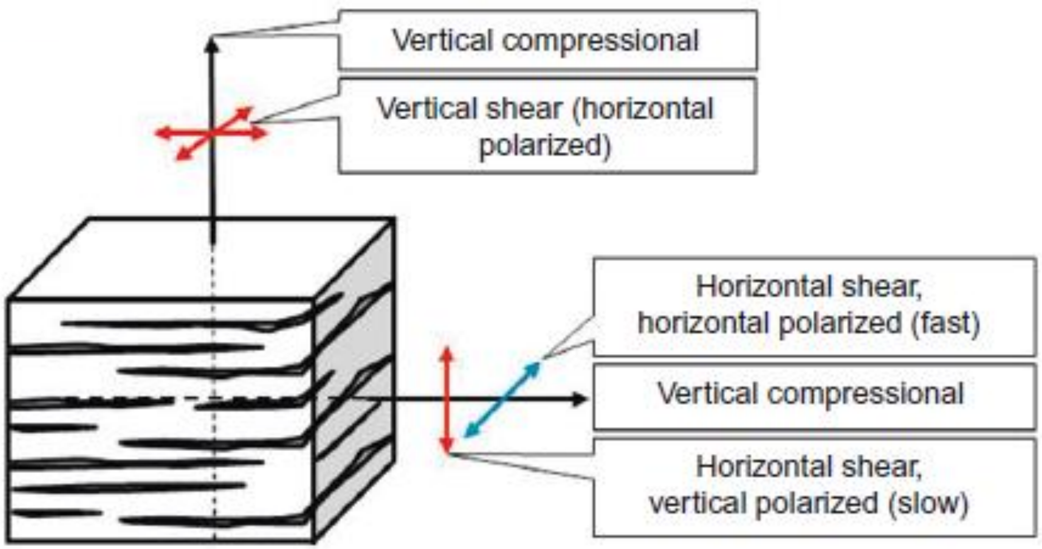
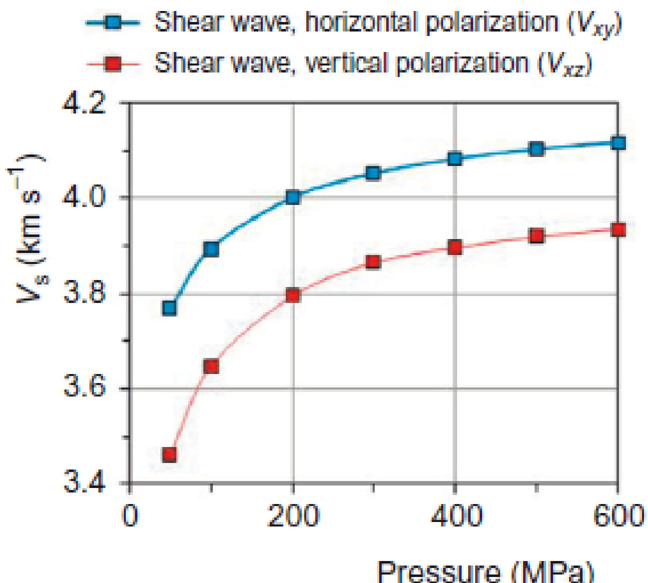




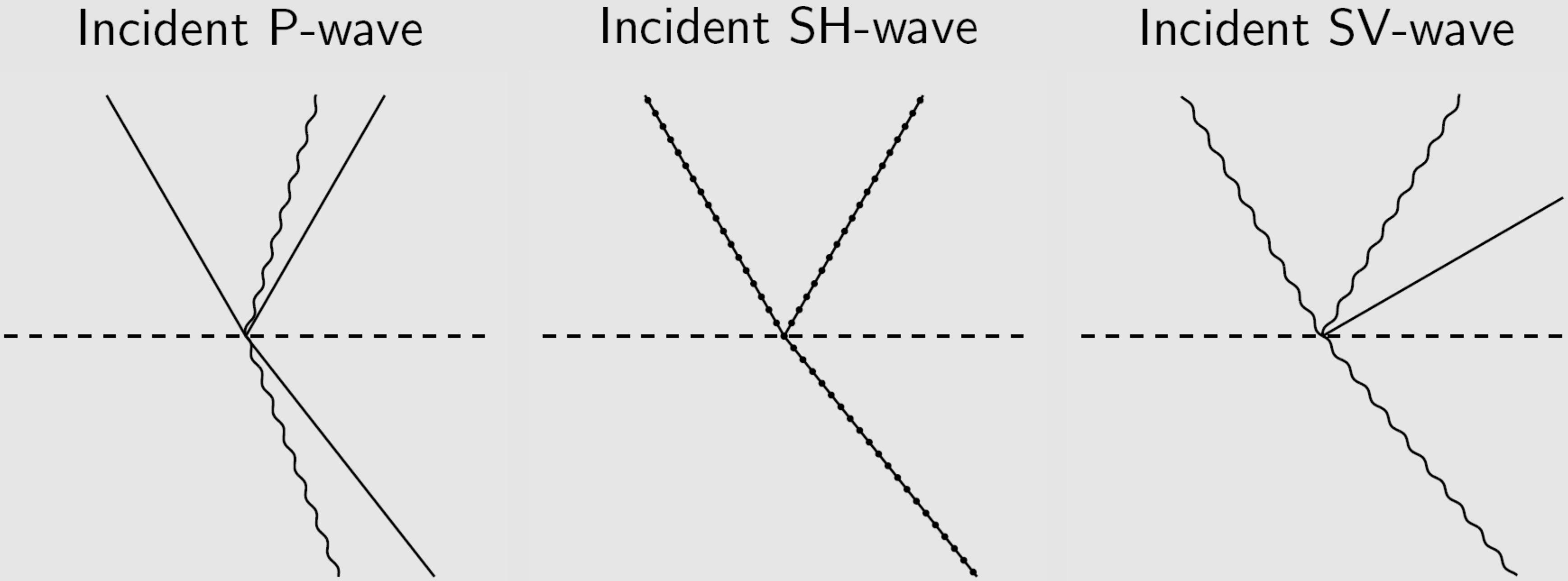
Siegesmund et al. 1993

Conversion of Waves in Reflection and Refraction

Conversion of waves depends on the polarization of the S-waves.

Vertically polarized S-wave (SV) merges with P-waves.

Horizontally polarized S-wave (SH) is independent of P-waves and SV-waves.



Seismic Waves

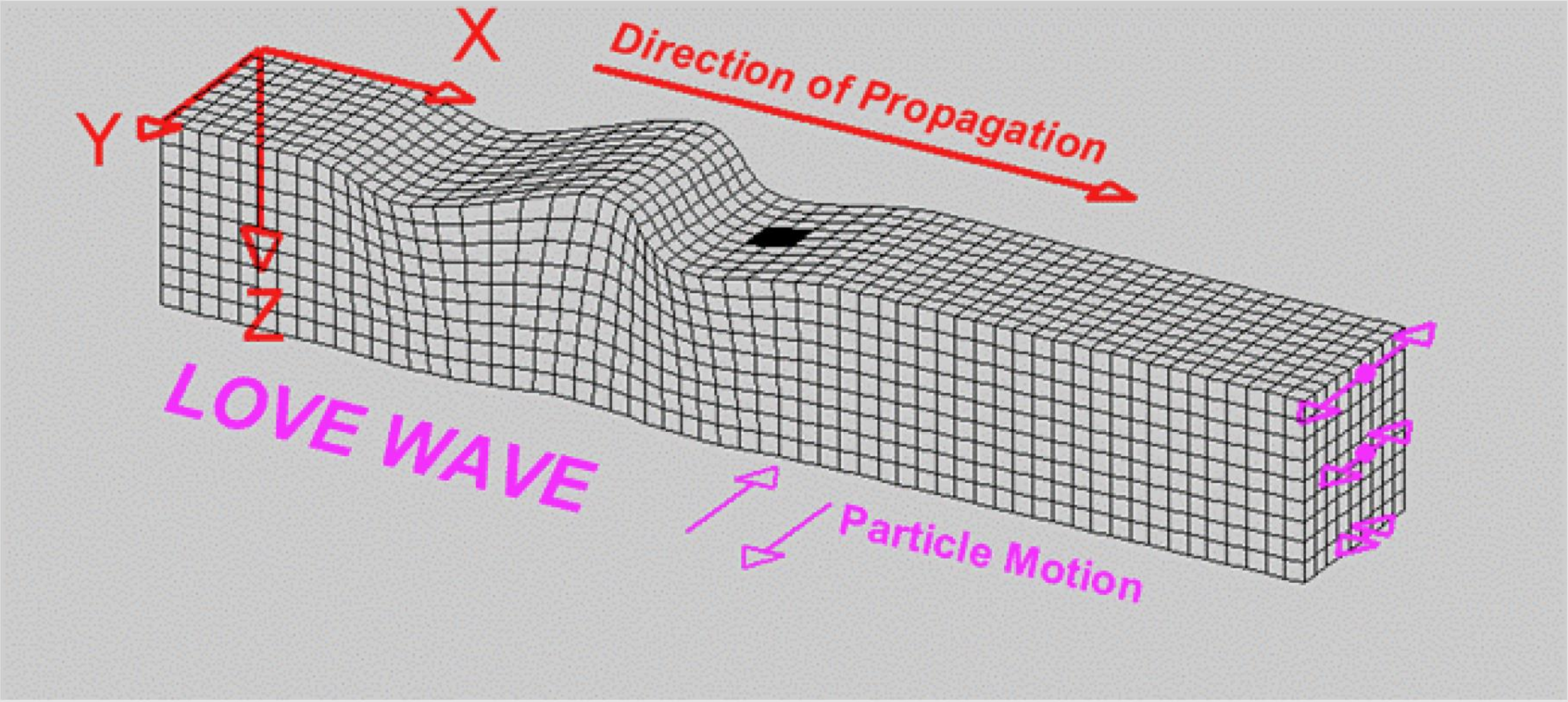
Surface Waves

Infinite domain in 3D → body waves (P- and S-wave)
Semi-infinite halfspace → surface waves (Love wave and Rayleigh wave)

The Love Wave

- Discovered quite late (1911), named after A. E. H. Love.
- In principle a S-wave with horizontal polarization.
- Amplitude decreases exponentially with depth.
- Exists only in inhomogeneous media where the seismic velocities increase with depth.

The Love Wave

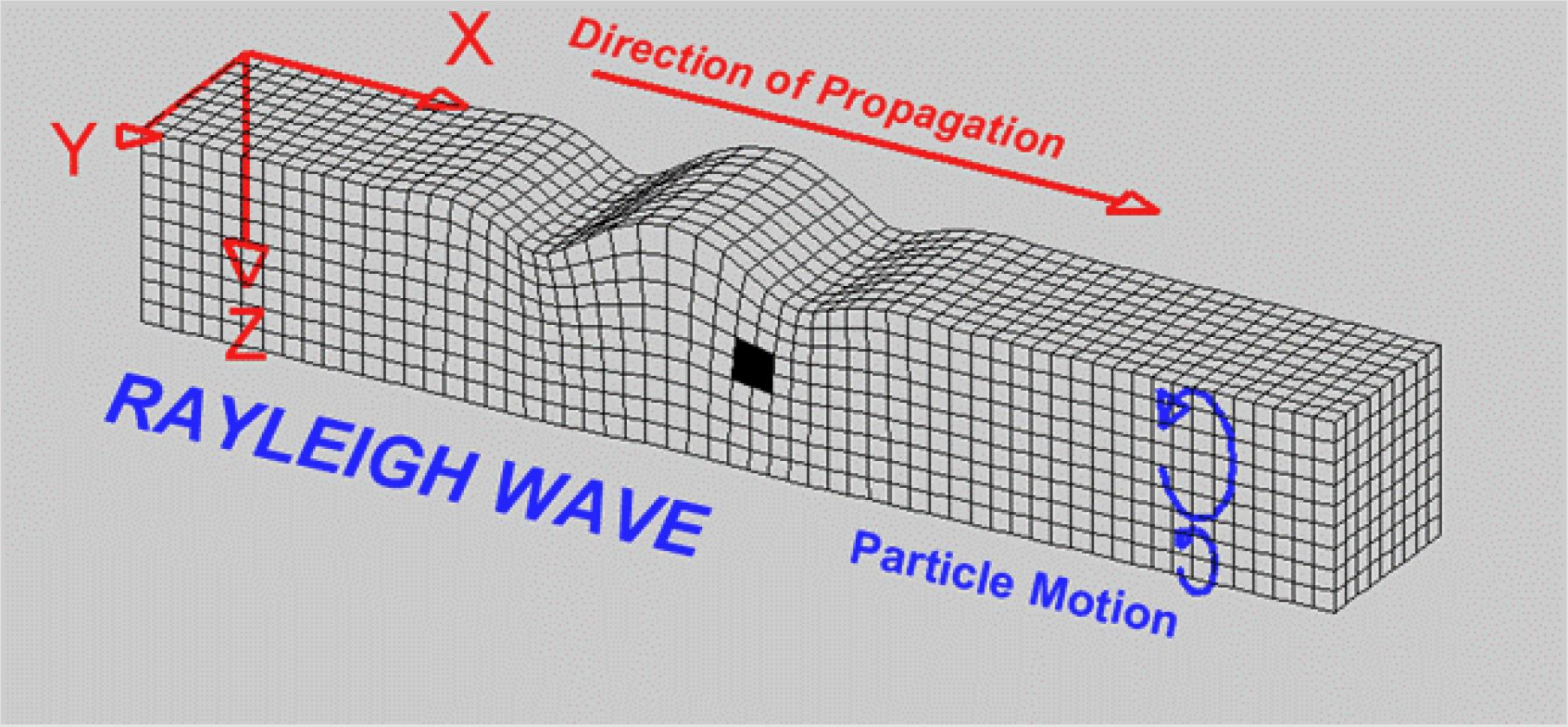


Source: L. Braile, Purdue University

The Rayleigh Wave

- Named after J. W. Strutt (later 3. Lord Rayleigh).
- In principle a mixture of P-wave and S-wave with vertical polarization.
- Particles rotate backwards on elliptic traces (ground roll).
- Causes most of the damage of earthquakes.
- Amplitude decreases exponentially with depth.
- Also exists in homogeneous media.
- Slightly slower than the S-wave.

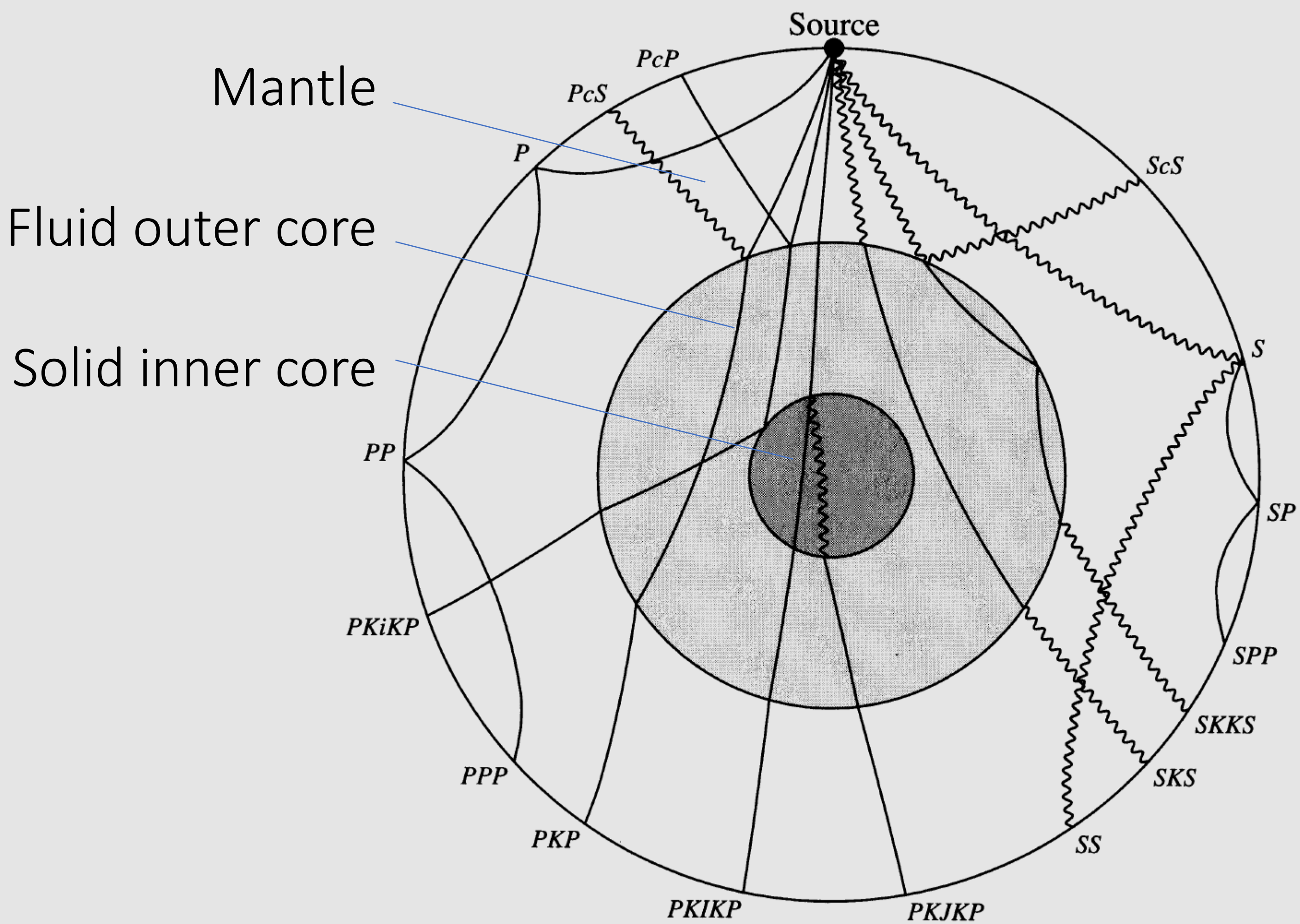
The Rayleigh Wave



Source: L. Braile, Purdue University

Seismic Waves

Global Wave Propagation in the Earth's Interior



Source: Shearer, Introduction to Seismology

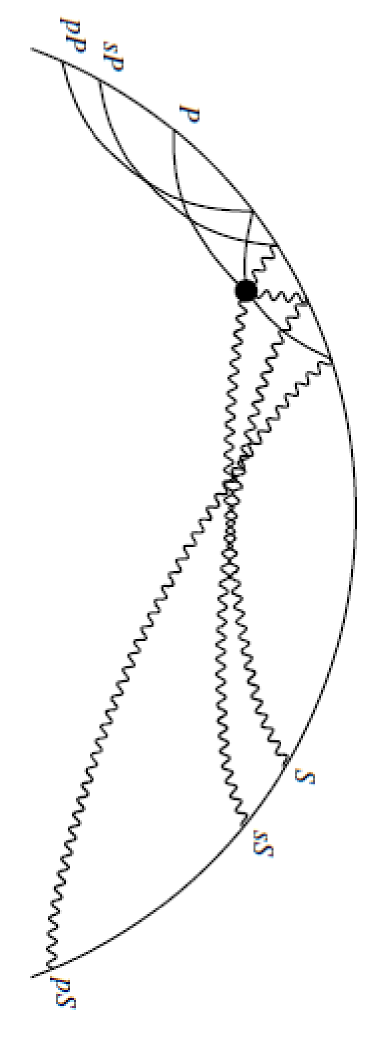
Seismic Waves

Global Wave Propagation in the Earth´s Interior

- P: P wave in the crust/mantle
- K: P wave in the outer core
- I: P wave in the inner core
- S: S wave in the crust/mantle
- J: S wave in the inner core
- c: reflection off the core–mantle boundary (CMB)
- i: reflection off the inner-core boundary (ICB)
- Combination of different letters indicate the seismic phases along the ray path travelling through the Earth.
- Repeating letters indicate reflection at the surface (PP, SS)

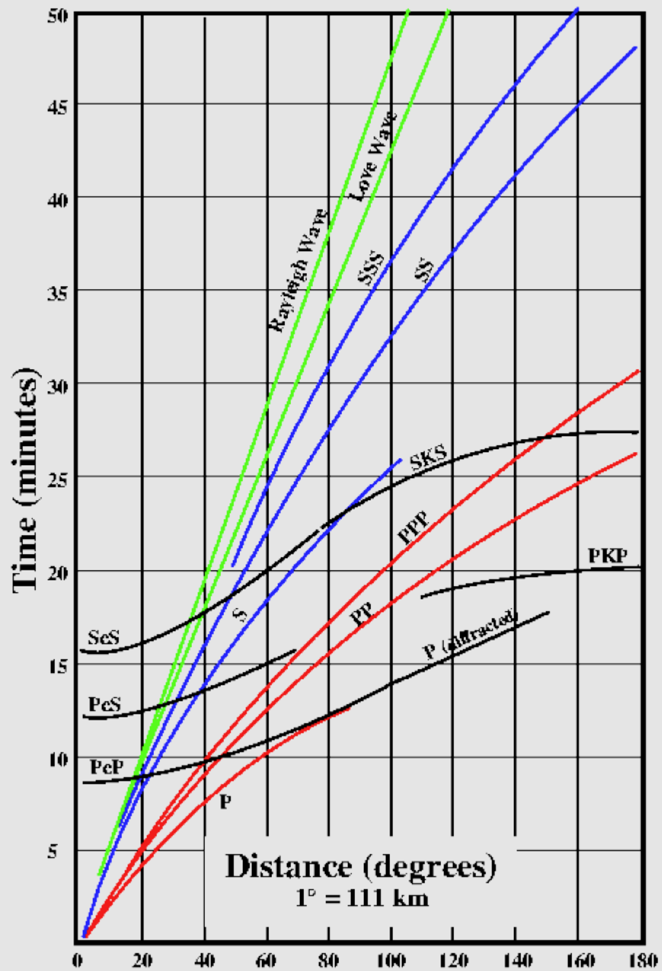
Seismic Waves

Global Wave Propagation in the Earth´s Interior



- Capital letters indicate the rays of depth phases, going downwards from the source, i.e. vertical angle lower than 90° (P,S)
- Lower case letters indicate the rays of depth phases, going upwards from the source, i.e. vertical angle greater than 90° (p,s)
- Hypocenter near reflexions at the surface (pP,pS,sP,sS)
- Conversion waves travelling upwards (Ps,Sp)

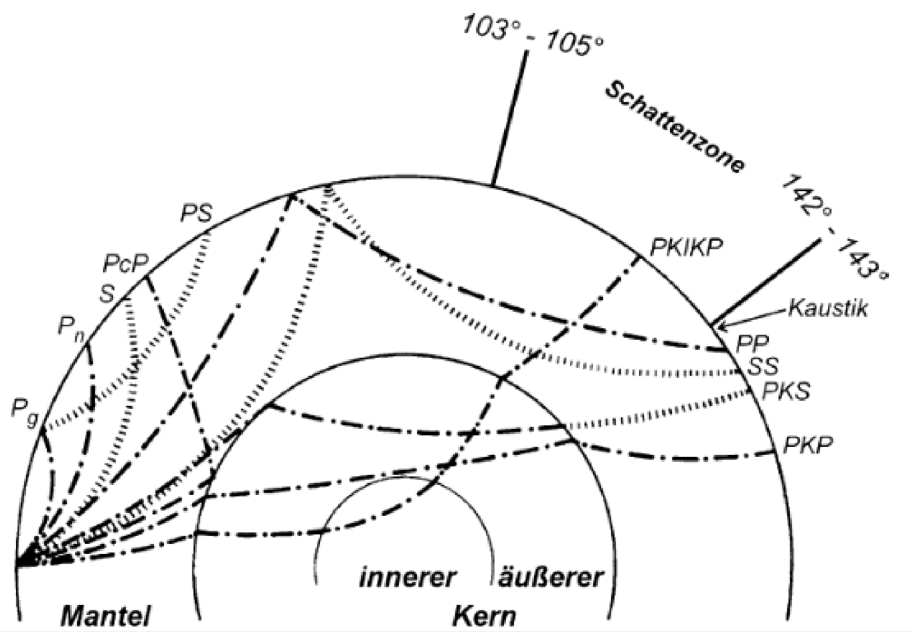
Localization of Earthquakes – Travel Time Curves



Source: Southern Arizona Seismic Observatory

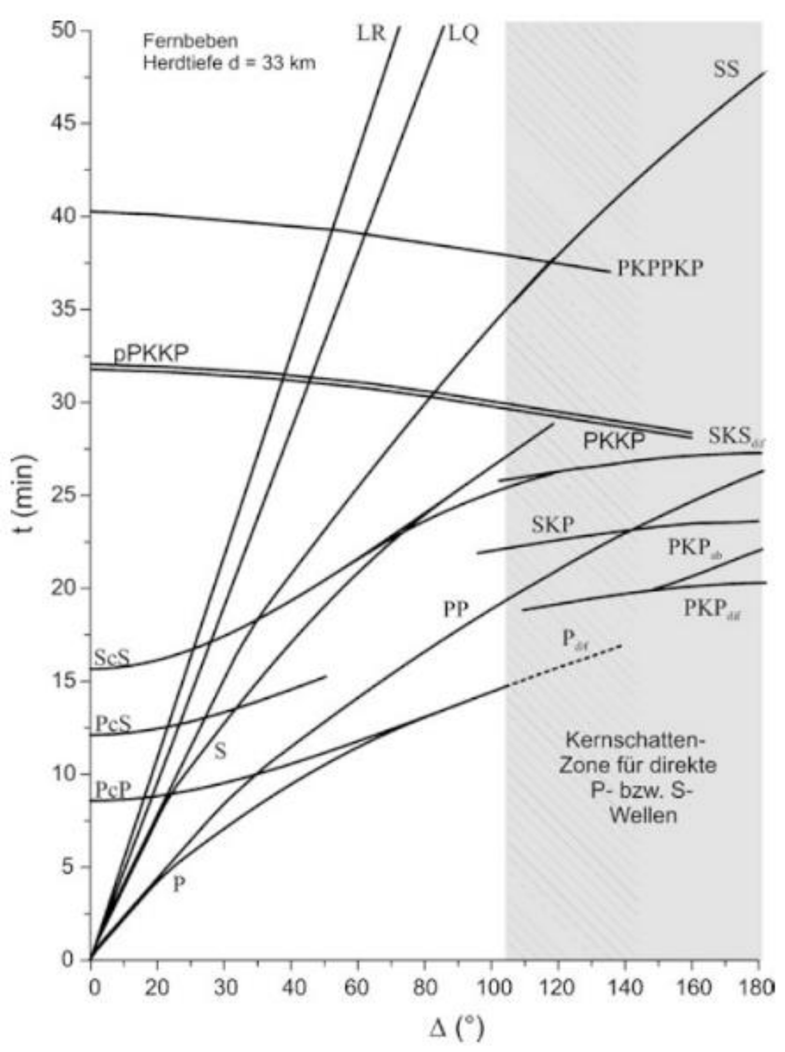
Seismic Waves

Localization of Earthquakes

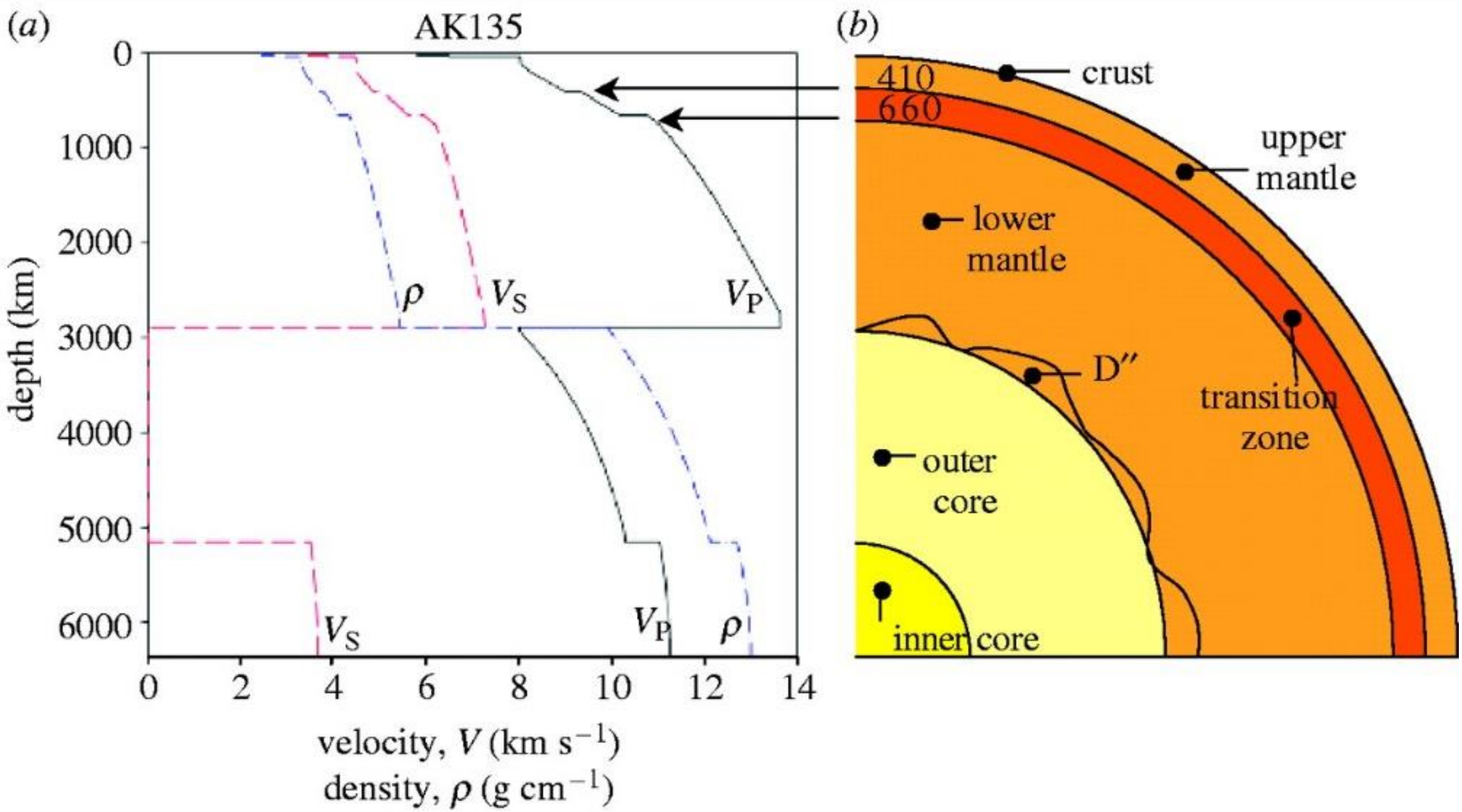


Source: Clauser, Einführung in die Geophysik, figs. 3.15 and 3.16

Travel Time Curves

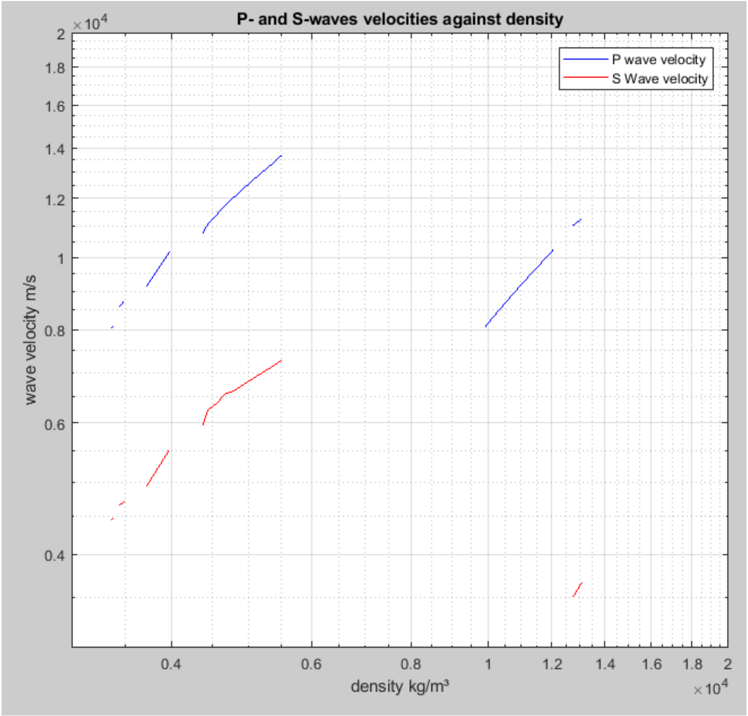


Sheet



(a)
AK135
0
1000
2000
3000
4000
5000
6000
depth (km)
ρ
VS
VP
VS
VP
ρ
0
2
4
6
8
10
12
14
velocity, V (km s−1)
density, ρ (g cm−1)
(b)
410
660
crust
upper
mantle
lower
mantle
D″
transition
zone
outer
core
inner core

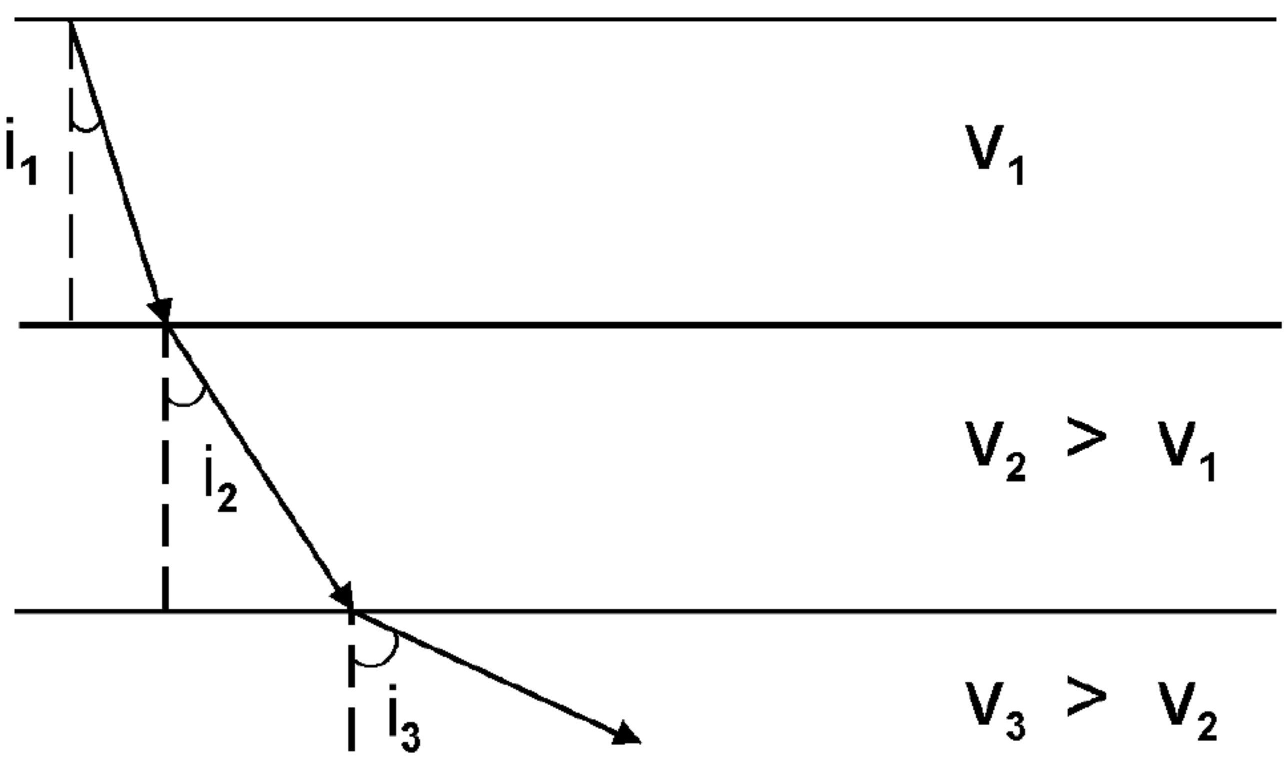
```
% Loading data.

data=csvread ('prem.csv');

radius=data(:,1); % Earth radius.
density=data(:,2); %Density.
pwave=data(:,3); % P-wave velocity.
swave=data(:,4); % S-wave velocity.


% Adding a value of NaN for discontinuities


for i=2:numel(radius)
    A=[radius(i-1) radius(i)]
    if A(1)==A(2)
        density(i)=NaN;
        pwave(i)=NaN;
        swave(i)=NaN;
    end
end

% Plotting data with NaN values.

figure
loglog(density,pwave,'b-')
hold on
loglog(density,swave,'r-')

% Labeling figure and axis.

legend('P wave velocity','S Wave velocity')
title('P- and S-waves velocities against density')
xlabel ('density kg/m³')
ylabel ('wave velocity m/s')
grid on
set(gcf, 'color', [0.8 0.8 0.8])
axis ([3*10^3 2*10^4 3*10^3 2*10^4])
```

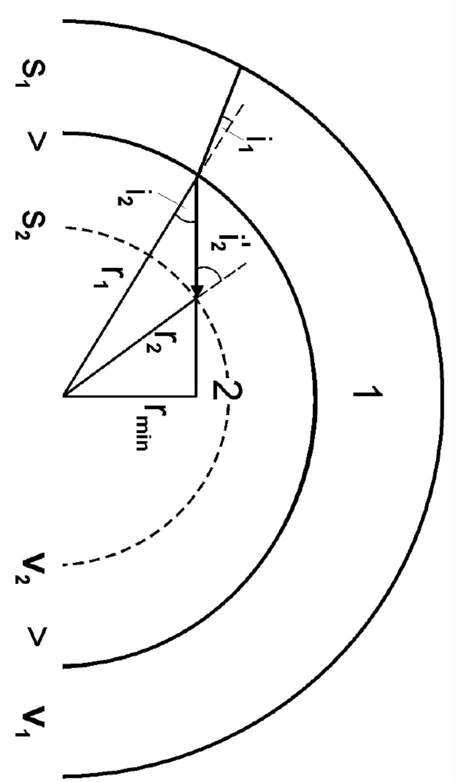


Sheet

- Wiechert-Herglotz inversion:

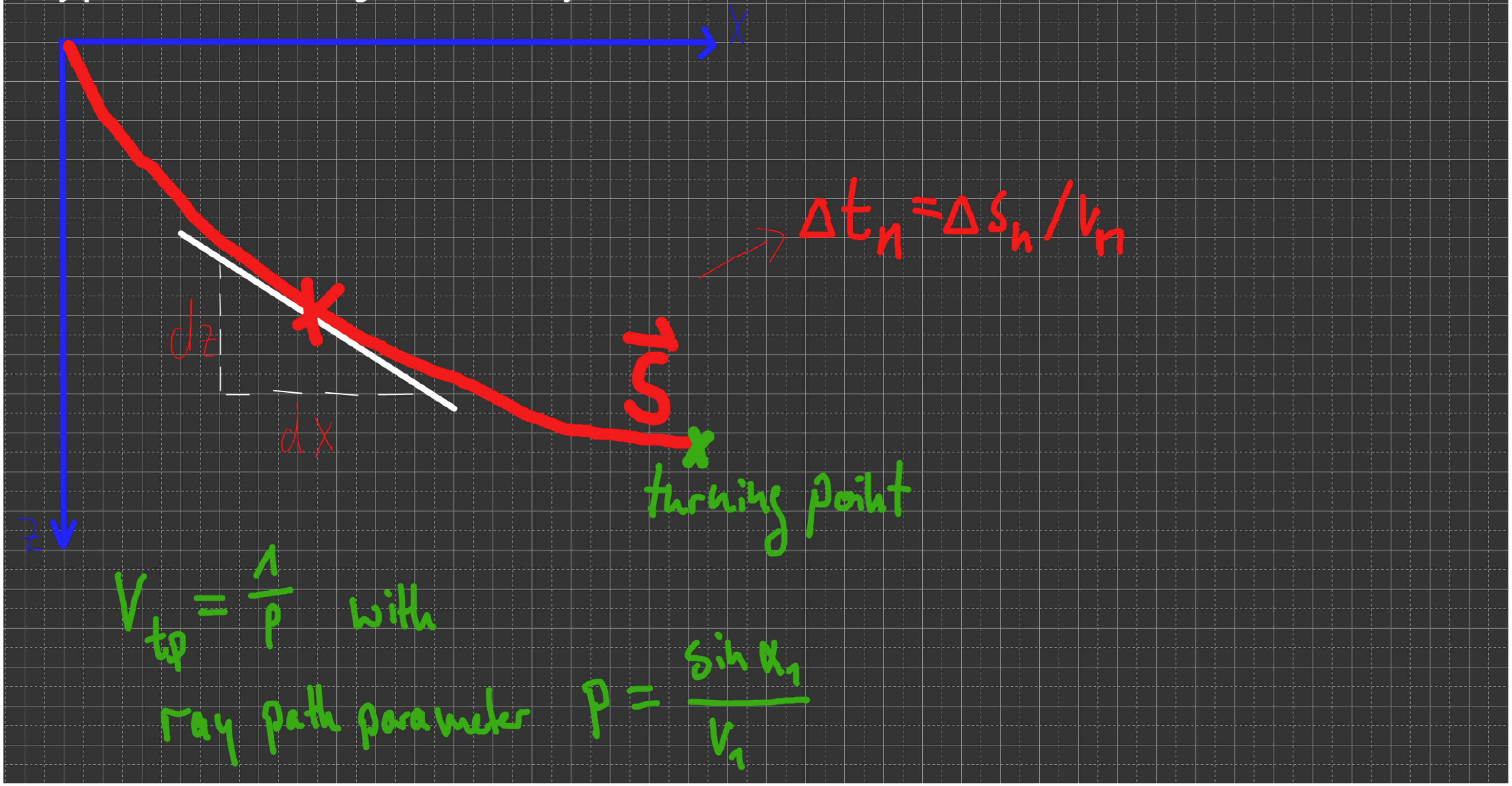
Propagation in the case of velocity v = f(z) increasing monotonously with depth z .





Sheet

Ray path vor v increasing monotonously with f(z)



Sheet

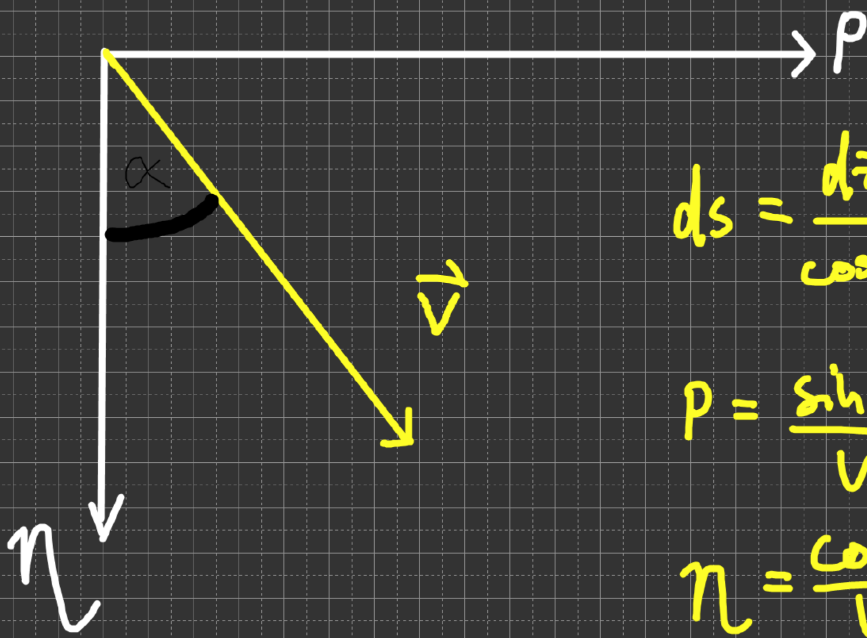
to compute the state of the system at a later time OR depth (ds):

$$x_{expl} = x_0 + \frac{p}{\eta} \cdot ds$$

with η = vertical slowness $\Longrightarrow$

$$t_{expl} = t_0 + ds \cdot \frac{v_{expl}^{-1}}{\sqrt{1 - p^2 \cdot v_{expl}^2}}$$

Sheet



$$ds = \frac{dz}{\cos\alpha}$$

$$p = \frac{\sin\alpha}{v}$$

$$\eta = \frac{\cos\alpha}{v}$$