

Hydrogeologie

Resistivity Methods

Jakob Wilk

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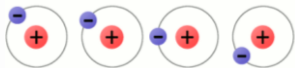


Introduction

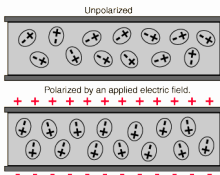


Basic Idea

- Measure electrical conductivities or resistivities using artificial fields.



Electric
current transport



Dielectric
current transport



Electrolytic
current transport

Main Fields of Application

- Delimiting lithologic units and fault zones
- Determining depth and properties of aquifers
- Monitoring the impermeability of dams
- Exploration and monitoring of residual waste sites
- Monitoring the spread of pollutants
- Detecting potential slip surfaces (e. g., clay layers) in landslide-prone slopes

Mostly:



Electric Field and Potential

- An electric field \vec{E} exerts a force

$$\vec{F} = q\vec{E}$$

on a charge q .

- In absence of time-dependent magnetic fields, the electric field can be represented by the gradient of the electric potential U :

$$\vec{E}(\vec{x}) = -\nabla U(\vec{x}) = -\begin{pmatrix} \frac{\partial}{\partial x_1} U(\vec{x}) \\ \frac{\partial}{\partial x_2} U(\vec{x}) \\ \frac{\partial}{\partial x_3} U(\vec{x}) \end{pmatrix}$$



$$\vec{F}(\vec{x}) = -q\nabla U(\vec{x})$$

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$$\vec{F}(\vec{x}) = -q\nabla U(\vec{x})$$

Ohm's Law

Force on free electrons in a conductor



Drift of electrons in direction of the force, velocity proportional to the force



Current density (charge density \times drift velocity)

$$\vec{j}(\vec{x}) = -\sigma \nabla U(\vec{x})$$

- Named after Georg Simon Ohm, 1789–1854.
- The constant of proportionality σ is a property of the material and is denoted electrical conductivity.

Introduction

Conductivity and Resistivity

Conductivity σ

$$[\sigma] = \frac{1}{\Omega\text{m}} = \frac{\text{S}}{\text{m}}, \quad \Omega = \text{Ohm} = \frac{\text{V}}{\text{A}}, \quad \text{S} = \text{Siemens} = \frac{\text{A}}{\text{V}}$$

Resistivity $\rho = \frac{1}{\sigma}$

$$[\rho] = \Omega\text{m}$$

Conductance and resistance refer to objects and not to materials and are measured in S and Ω , respectively.

(Semi)Conductors	ρ [Ωm]
copper	1.7×10^{-8}
iron	10^{-7}
silicium	2300

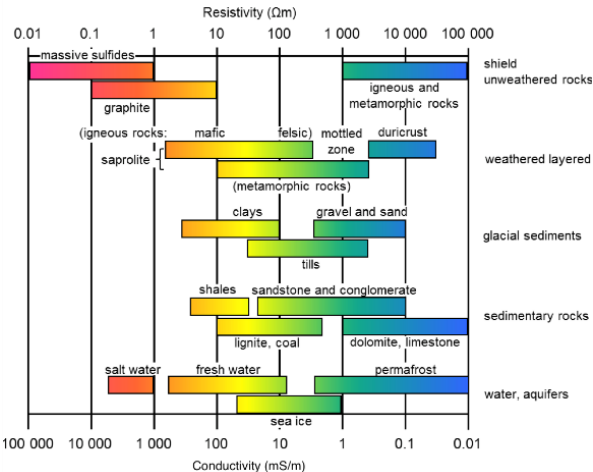
Nonconductors	ρ [Ωm]
porcelain	10^{12}
rubber	10^{13}
silica glass	7.5×10^{17}

Conductivity / Resistivity of Rocks and Soils

- Rock forming minerals have very low conductivities.
- Many ores have considerably higher conductivities.
- The conductivity of pure water is rather low, but strongly increases by solving salts.

Solution	ρ [Ωm]
distilled water	10000
ocean water	0.5
10 % copper sulfate	0.3
10 % sodium chlorite	0.08
10 % sulfuric acid	0.025
10 % hydrochloric acid	0.015

Conductivity / Resistivity of Rocks and Soils



Rock	% H ₂ O	P [Ohm m]
Siltstone	0.54	1.5*10 ⁴
shield unweathered rocks	0.38	5.6*10 ⁹
Coarse grain SS	0.39	9.6*10 ⁵
	0.18	10 ⁸
weathered layered	1.0	4.3*10 ³
Medium grain SS	0.1	1.4*10 ³
Dolomite	1.3	6*10 ⁹
	0.96	8*10 ⁸
Granite	0.31	4.4*10 ³
sedimentary rocks	0.19	1.8*10 ⁶
Basalt	0.95	4*10 ⁴
water, aquifers	0	1.3*10 ⁹
Graywacke SS	1.16	4.7*10 ³
	0.45	5.8*10 ⁴
Peridotite	0.1	3*10 ⁹
	0	1.8*10 ⁷

Source: Telford 1990 Applied Geophysics

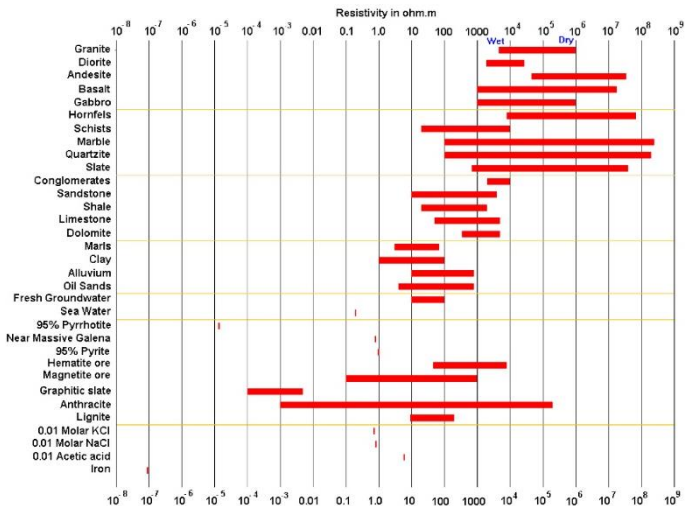
Conductivity / Resistivity of Rocks and Soils

Material	ρ [Ωm]
halite	$10^5 - 10^7$
dry sand	10^5
water satur. sand	1000 - 10000
quartzite	$3000 - 10^5$
ice	$1000 - 10^5$
granite	300 - 30000
sandy soils	150 - 7000
loamy soils	50 - 9000
clayey soils	20 - 4000

Material	ρ [Ωm]
limestone	100 - 7000
marsh	30 - 700
glacial moraine	10 - 300
clay shale	10 - 1000
marl	5 - 200
loam	3 - 300
dry clay	30 - 1000
wet clay	1 - 30
silt	10 - 1000

Source: Beblo (Ed.), Umweltgeophysik

Conductivity / Resistivity of Rocks and Soils



Source: Loke, Tutorial: 2-D and 3-D electrical imaging surveys

Conductivity / Resistivity of Rocks and Soils

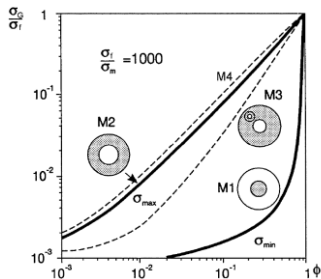
Thus, the total conductivity of a rock or a soil strongly depends on

- porosity
- water saturation
- connectivity of the pore space
- pureness of the contained water (in return depends on the properties of the rock/soil)

$$\sigma_G = a \phi^n S^m \sigma_F$$

with:

material conductivity (σ_G),
material properties (a, n, m),
porosity (ϕ),
conductivity of formation water (σ_F)
and pores actually containing water (S)



Hashin and Shtrikman (1962)

Introduction

Conductivity / Resistivity of Rocks and Soils

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Question

Which are the main dependencies of the hydraulic conductivity of an aquifer?

Introduction

Conductivity / Resistivity of Rocks and Soils

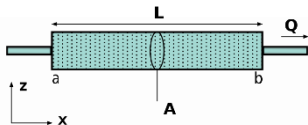
Thus, the total conductivity of a rock or a soil strongly depends on

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- pureness of the contained water (in return depends on the properties of the rock/soil)

Question

Which are the main dependencies of the hydraulic conductivity of an aquifer?

$$Q = \frac{kA}{\mu L} \Delta p$$



Dynamic viscosity (μ), Permeability (κ) and Pressure drop (Δp)

ER – Basic Terms

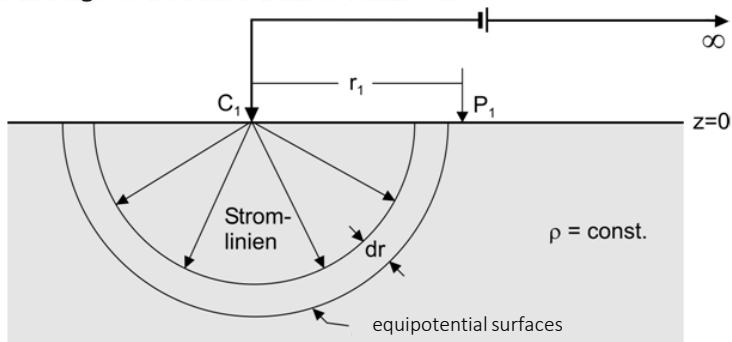
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The Principle of Subsurface Resistivity Measurement

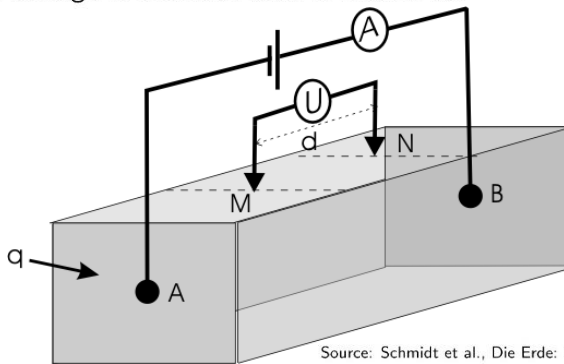
- 1 Two current electrodes A and B are plugged into the ground, and a voltage is applied, generating a current I from A to B.
- 2 Two potential electrodes M and N are plugged into the ground, and the voltage U between both is measured.



(from script Electromagnetic Methods in Geophysics H.Brasse SS2011)

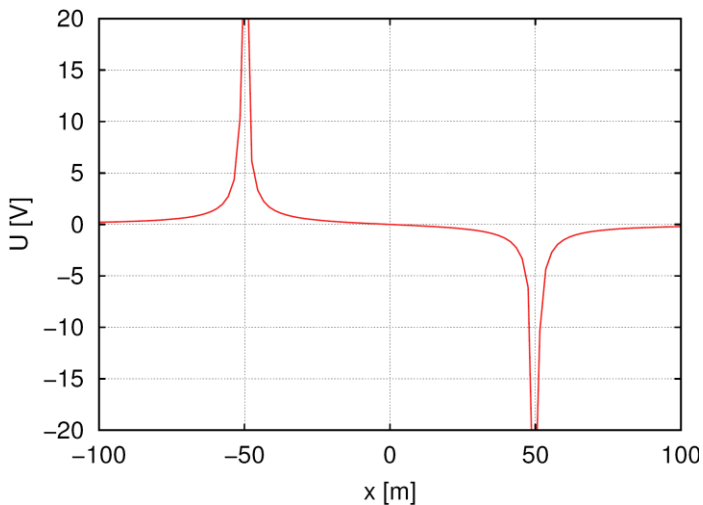
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- 1 Two current electrodes A and B are plugged into the ground, and a voltage is applied, generating a current I from A to B.
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Source: Schmidt et al., Die Erde: Der dynamische Planet (CD-ROM)

The Potential between the Electrodes



$\rho = 1000 \Omega\text{m}$, $I = 100 \text{ mA}$, offset = 100 m

Solutions of the Potential Equation in a Homogeneous Medium

Potential of a point source at the origin feeding a current I :

$$U(\vec{x}) = \frac{\rho I}{4\pi |\vec{x}|}$$

Potential of a point source at the point \vec{x}_A if the current is distributed in a half space only:

$$U(\vec{x}) = \frac{\rho I}{2\pi |\vec{x} - \vec{x}_A|}$$

Feeding in a current I at \vec{x}_A and extracting I at \vec{x}_B :

$$\begin{aligned} U(\vec{x}) &= \frac{\rho I}{2\pi |\vec{x} - \vec{x}_A|} - \frac{\rho I}{2\pi |\vec{x} - \vec{x}_B|} \\ &= \frac{\rho I}{2\pi} \left(\frac{1}{|\vec{x} - \vec{x}_A|} - \frac{1}{|\vec{x} - \vec{x}_B|} \right) \end{aligned}$$

Arbitrary Electrode Configuration in a Homogeneous Half-Space

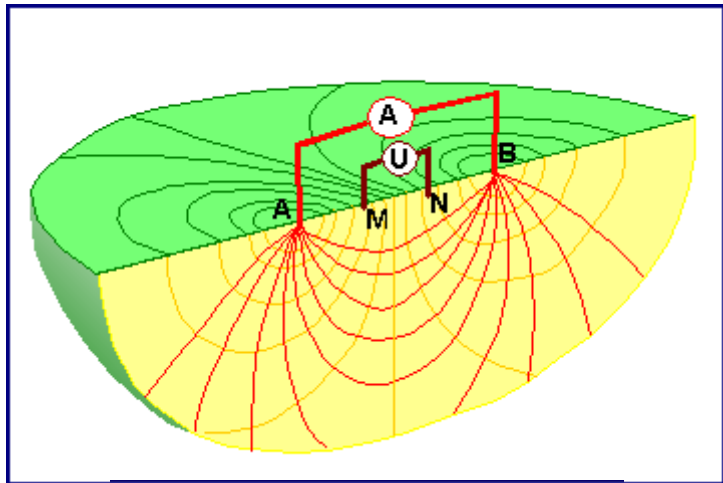
- Voltage between M and N is the difference of the potentials at \vec{x}_M and \vec{x}_N :

$$\begin{aligned}U &= U(\vec{x}_M) - U(\vec{x}_N) \\&= \frac{\rho l}{2\pi} \left(\frac{1}{|\vec{x}_M - \vec{x}_A|} - \frac{1}{|\vec{x}_M - \vec{x}_B|} - \frac{1}{|\vec{x}_N - \vec{x}_A|} + \frac{1}{|\vec{x}_N - \vec{x}_B|} \right) \\&= \frac{\rho l}{2\pi} \left(\frac{1}{r_{MA}} - \frac{1}{r_{MB}} - \frac{1}{r_{NA}} + \frac{1}{r_{NB}} \right)\end{aligned}$$

where $r_{...}$ are the distances between the respective electrodes.

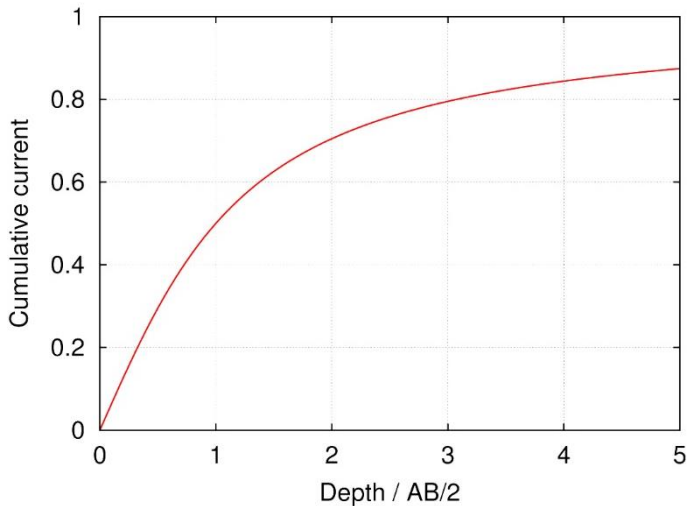
- Mostly, all electrodes are placed on a straight line.

Dipole Field in a Homogeneous Half-Space



Source: Schmidt et al., Die Erde: Der dynamische Planet (CD-ROM)

Penetration Depth of the Current



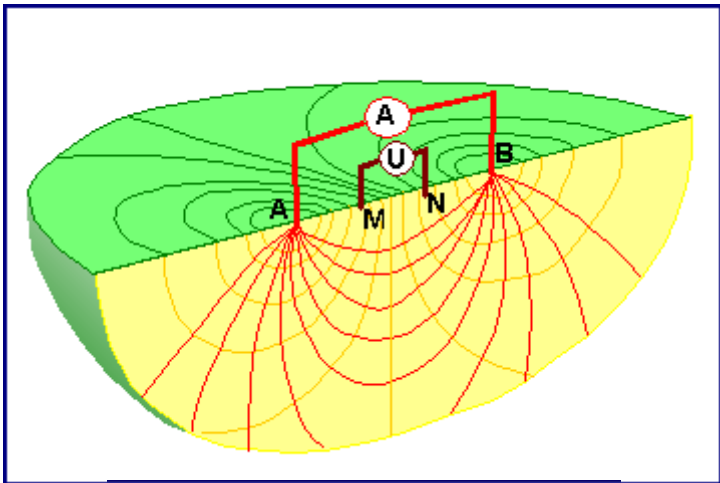
ER – Measuring Principle

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Dipole Field in a Homogeneous Half-Space



Source: Schmidt et al., Die Erde: Der dynamische Planet (CD-ROM)

Arbitrary Electrode Configuration in a Homogeneous Half-Space

- Voltage between M and N is the difference of the potentials at \vec{x}_M and \vec{x}_N :

$$\begin{aligned}U &= U(\vec{x}_M) - U(\vec{x}_N) \\&= \frac{\rho l}{2\pi} \left(\frac{1}{|\vec{x}_M - \vec{x}_A|} - \frac{1}{|\vec{x}_M - \vec{x}_B|} - \frac{1}{|\vec{x}_N - \vec{x}_A|} + \frac{1}{|\vec{x}_N - \vec{x}_B|} \right) \\&= \frac{\rho l}{2\pi} \left(\frac{1}{r_{MA}} - \frac{1}{r_{MB}} - \frac{1}{r_{NA}} + \frac{1}{r_{NB}} \right)\end{aligned}$$

where $r_{...}$ are the distances between the respective electrodes.

- Mostly, all electrodes are placed on a straight line.

Arbitrary Electrode Configuration in a Homogeneous Half-Space

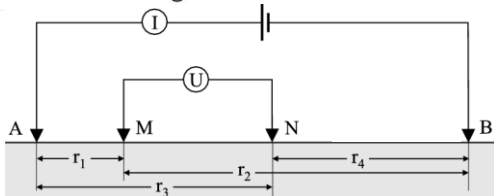
The resistivity of a homogeneous half-space can be determined according to

$$\rho = K \frac{U}{I}$$

with the geometric factor

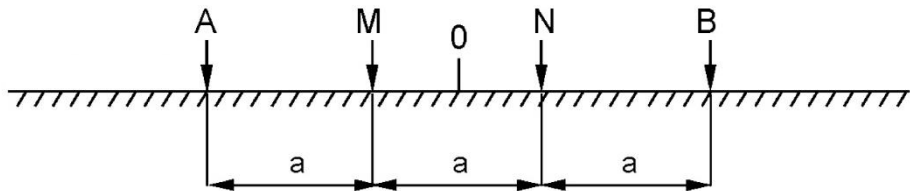
$$K = \frac{2\pi}{\frac{1}{r_{MA}} - \frac{1}{r_{MB}} - \frac{1}{r_{NA}} + \frac{1}{r_{NB}}}$$

of the selected electrode configuration.



(from script Electromagnetic
Methods in Geophysics H.Brasse
SS2011)

The Wenner (α) Configuration



Source: Wikipedia

$$K = 2\pi a$$

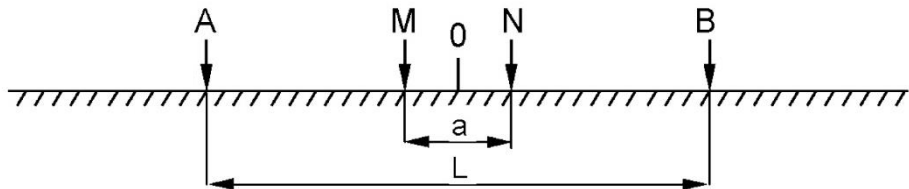
Widely used for horizontal profiling (a fixed)

Variants of the Wenner Configuration

Configuration	Electrode sequence	Geometric factor
Wenner α	A-M-N-B	$K = 2\pi a$
Wenner β	A-B-M-N	$K = 6\pi a$
Wenner γ	A-M-B-N	$K = 3\pi a$

Wenner α is the standard configuration (Wenner without further specification).

The Schlumberger Configuration



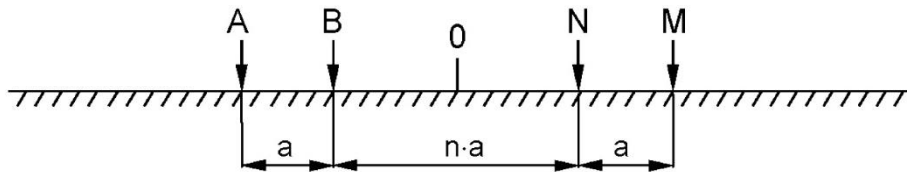
Source: Wikipedia

$$K = \frac{\pi (L^2 - a^2)}{4a} \approx \frac{\pi L^2}{4a} \quad \text{für } L \gg a$$

Widely used for vertical sounding (a fixed, L variable)

Caution: Sometimes L is used for $AB/2$ instead of the total offset AB .

The Dipole-Dipole Configuration

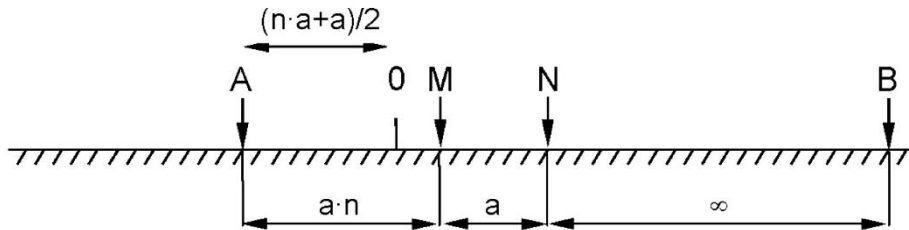


Source: Wikipedia

$$K = \pi n(n+1)(n+2) a$$

Particularly suitable for profiling of small-scale structures, but a requires high power input.

The Pole-Dipole Configuration



Source: Wikipedia

$$K = 2\pi n(n+1)a$$

Particularly suitable for investigating horizontal contrasts.

Surveys



Source: <http://www.gfinstruments.cz>



Source: <http://www.lgm.de>

Types of Resistivity Measurements

Results obtained for large offsets AB are more sensitive to the resistivities at greater depth than results obtained for small offsets.

Vertical sounding: same location, but different offsets

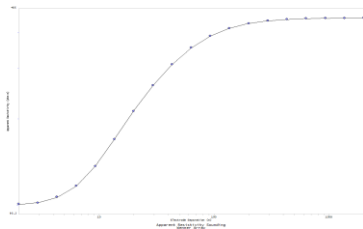
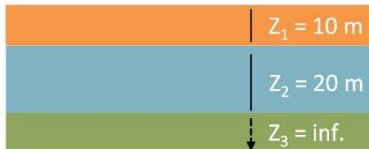
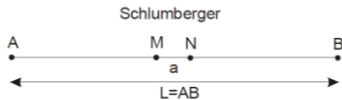
Horizontal profiling: constant electrode configuration used at different positions

Resistivity tomography: variable location and variable electrode spacing



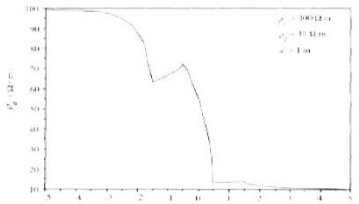
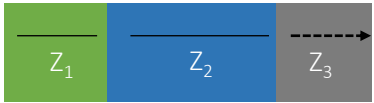
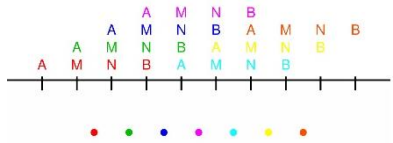
Various types of electrode configurations more or less suitable for different purposes

Surveys – Vertical Sounding



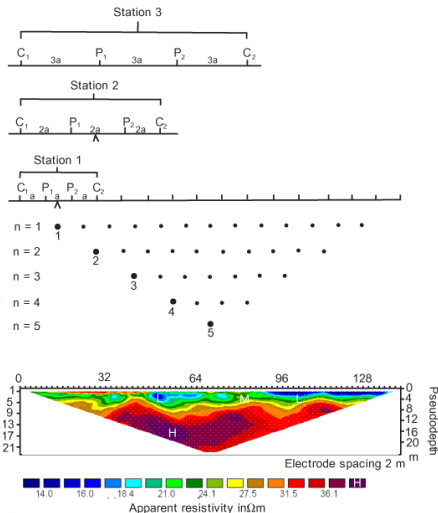
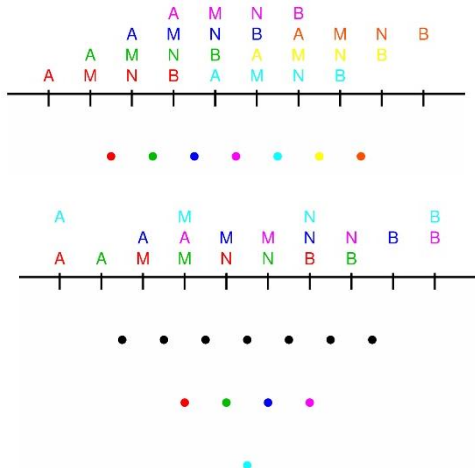
Measuring Principle

Surveys – Horizontal Profiling/Constant Separation Traversing (CST)

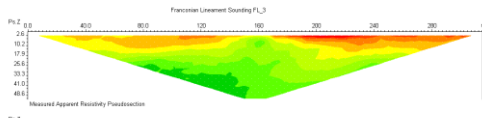
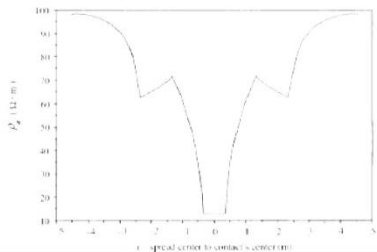


Source: Teaching material A. Henk

Surveys – Resistivity Tomography (ERT)



Surveys – ERT vs. Horizontal Profiling (CST)

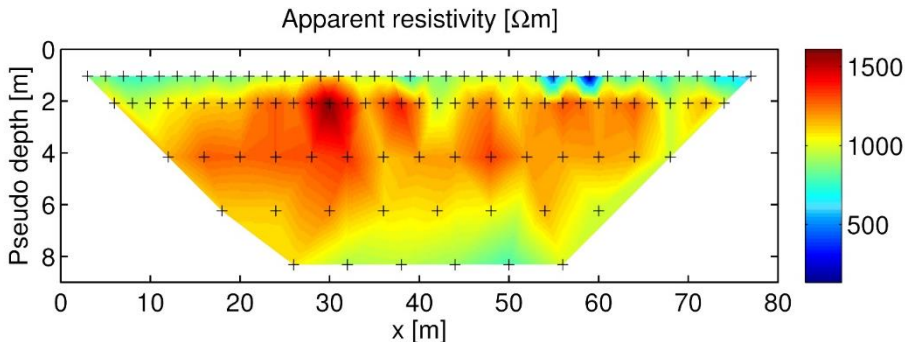


1D profile and 2d inversion of Wenner line over the Franconian line (Source: H.Brasse 2003)

Surveys – Resistivity Tomography (ERT)

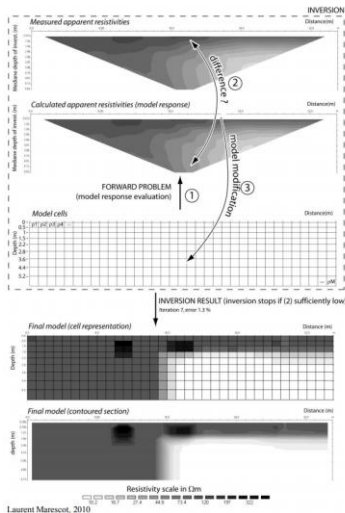
- Several (up to some hundred) electrodes are plugged into the ground, either on a profile line or distributed in two dimensions.
- A programmable channel selector replays a defined sequence of usage of the electrodes as current or potential electrode pairs.
- The method is also called electric tomography, in particular if the electrodes are distributed in two dimensions.

Surveys – Resistivity Tomography (ERT)



The Wenner (α) configuration is most widely used, but all other configurations are also possible.

Surveys – Resistivity Tomography (ERT)



Electric
tomography
inversion

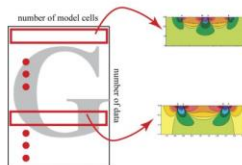
$$\Delta \mathbf{m} = [\mathbf{G}^T \mathbf{G} + \mathbf{W}]^{-1} \mathbf{G}^T \Delta \mathbf{d}$$

with:

$$\Delta \mathbf{m} = \log(\rho)$$

$$\Delta \mathbf{d} = \log(\rho_a^{meas}) - \log(\rho_a^{calc})$$

$$G_{ij} = \frac{\partial g(m)_i}{\partial m_j} \text{ is the sensitivity matrix}$$



Surveys – Electrodes

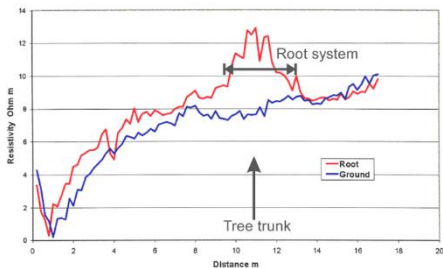
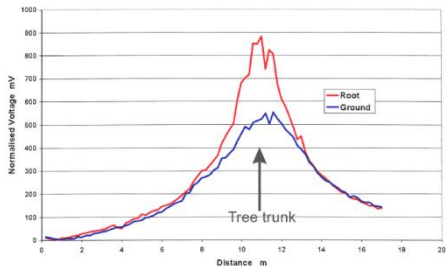
Current and potential electrodes are technically identical. Criteria (in particular for the potential electrodes):

Contact resistance to the ground should be low.

Contact voltage should be small.

- Usage of nonpolarizable electrodes, e. g., copper core in CuSO_4 solution in a porous clay cylinder.
- Simple steel electrodes can be used with modern central units that are able to compensate contact voltages automatically.

Surveys – Horizontal Profiling



Comparison between planting the current electrode directly into a tree trunk versus into the ground near the tree's base: left - normalized potential, right – normalized resistivity profile

Surveys – Electrodes

- Power source (constant current),
- voltmeter, and
- channel selector (for multi-electrode equipment)

are mostly combined in one unit.

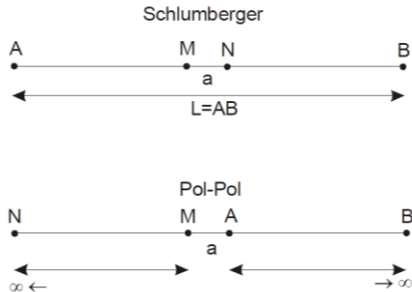
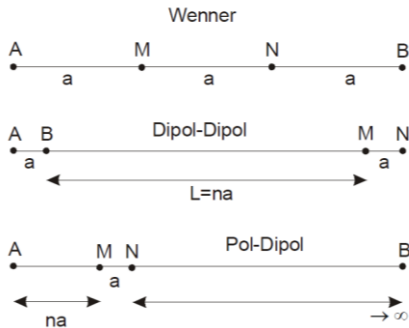
Power up to about 1000 W

Currents mostly between 10 mA and 1 A

Voltages (between the current electrodes) up to some 1000 V

Types of current: DC, low-frequency AC or switched DC with changing polarity

Surveys



ER - Data Analysis I

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Apparent Resistivity

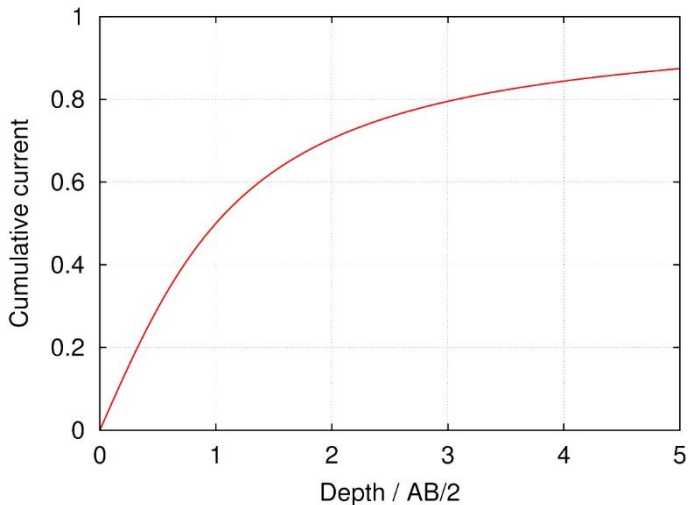
In a inhomogeneous medium,

$$\rho_a = K \frac{U}{I}$$

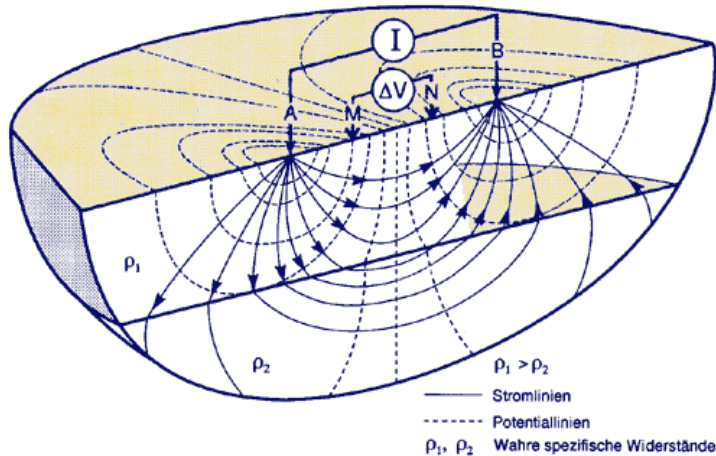
is called the apparent resistivity obtained from one measurement.

- ρ_a is the resistivity of a homogeneous medium that would yield the same result for the considered electrode configuration.
- ρ_a is not the real resistivity at any depth.
- The larger the offset is, the bigger is the contribution of deep regions to ρ_a .

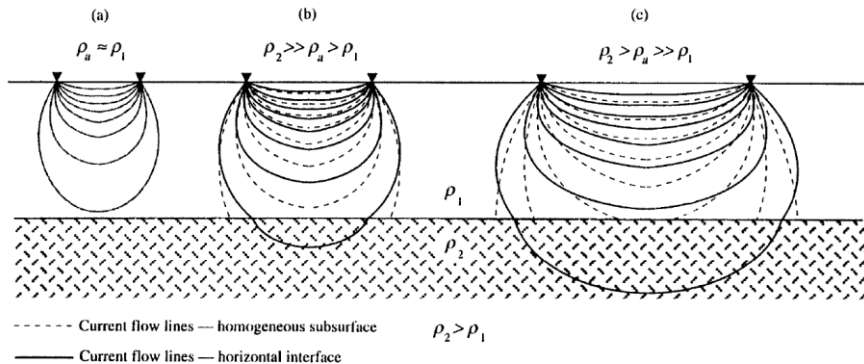
Penetration Depth of the Current



Vertical Sounding in the Two-Layer Case



Vertical Sounding in the Two-Layer Case



apparent electric resistivity in a two layer scenario

Vertical Sounding in the Two-Layer Case

Situation: Two homogeneous regions separated by a horizontal interface.

Target properties:

ρ_1 = resistivity of the upper layer

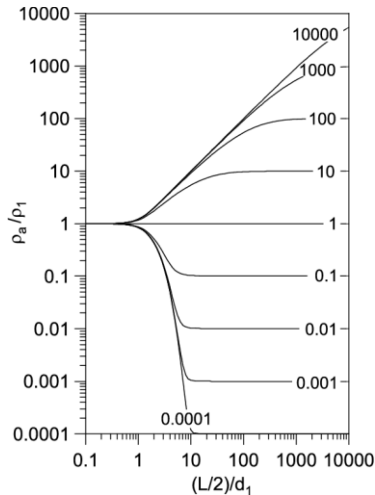
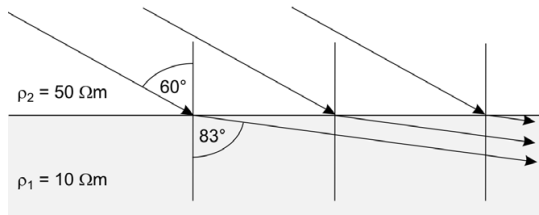
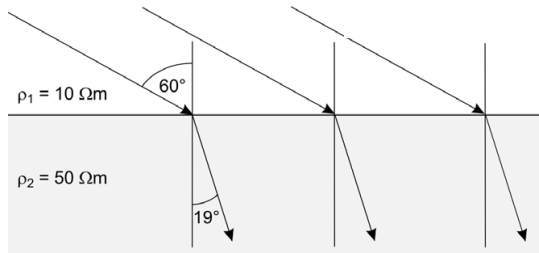
ρ_2 = resistivity of the lower region

d = thickness of the upper layer

Procedure: ρ_a is measured for several offsets AB (Wenner or Schlumberger configuration).

Data analysis can be performed graphically because $\frac{\rho_a}{\rho_1}$ only depends on $\frac{\rho_2}{\rho_1}$ and $\frac{AB/2}{d}$.

Vertical Sounding in the Two-Layer Case



Source: U.S. Environmental Protection Agency

Scaling Behaviour

Rescaling the resistivities: If $\rho(\vec{x})$ is changed by the same factor λ everywhere, ρ_a changes by the same factor λ .

Spatial scaling: Stretching the entire system (including the positions of the electrodes) horizontally and vertically by a factor λ :

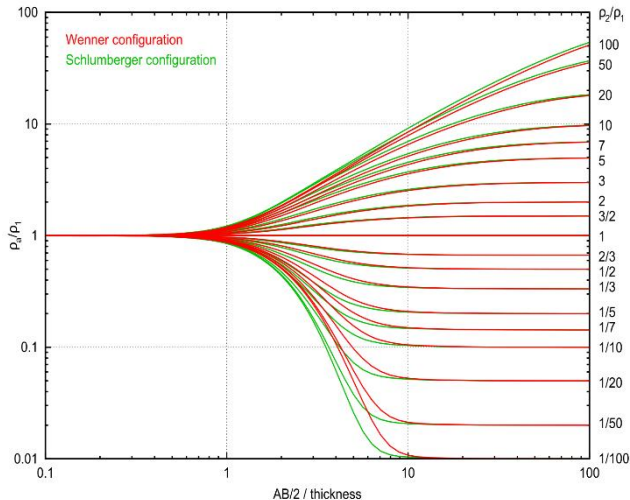
- If I is kept constant, all potentials change by the factor $\frac{1}{\lambda}$.
- K changes by the factor λ .



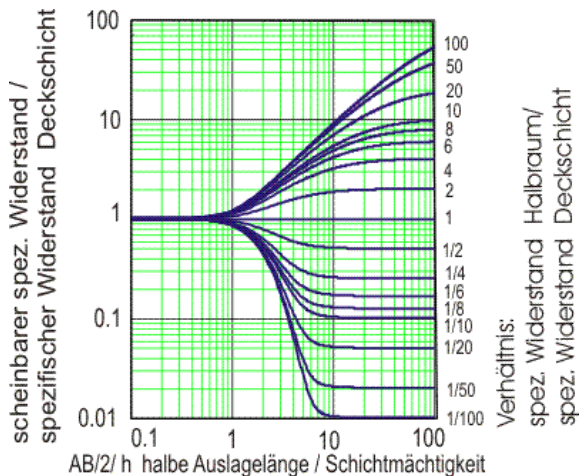
ρ_a remains the same.

Consequence for the two-layer case: For any given electrode configuration at variable offset, $\frac{\rho_a}{\rho_1}$ depends only on $\frac{\rho_2}{\rho_1}$ and $\frac{AB}{d}$ (or $\frac{AB/2}{d}$ or $\frac{a}{d}$).

Wenner and Schlumberger Configurations in the Two-Layer Case

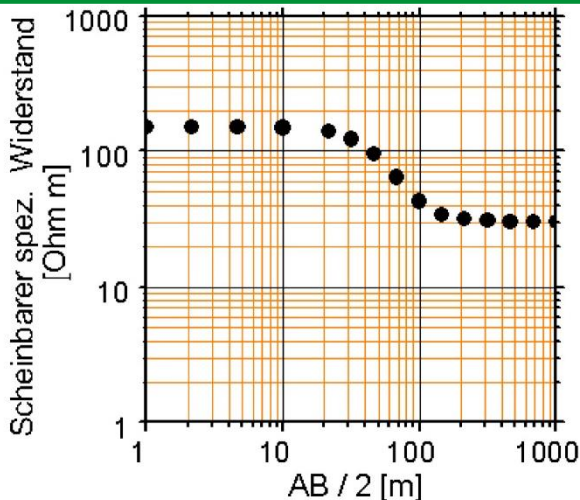


Graphical Data Analysis in the Two-Layer Case



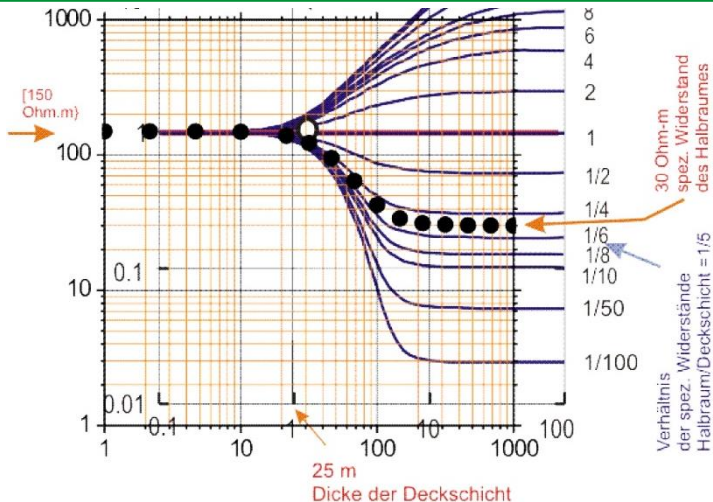
Source: Schmidt et al., Die Erde: Der dynamische Planet (CD-ROM)

Graphical Data Analysis in the Two-Layer Case



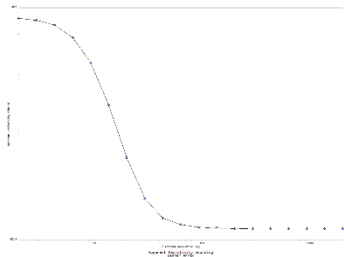
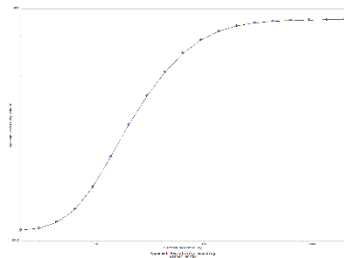
Source: Schmidt et al., Die Erde: Der dynamische Planet (CD-ROM)

Graphical Data Analysis in the Two-Layer Case



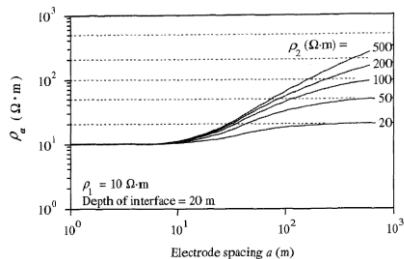
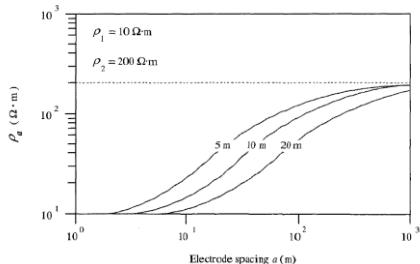
Source: Schmidt et al., Die Erde: Der dynamische Planet (CD-ROM)

Analysis in the Two-Layer Case



- material with greater resistivity lies below the interface
- material with greater resistivity lies above the interface

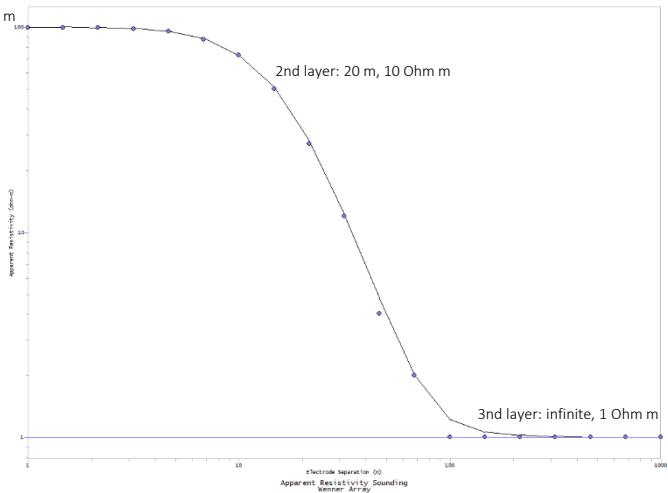
Analysis in the Two-Layer Case



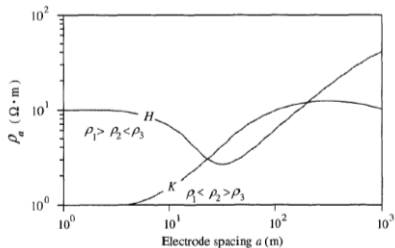
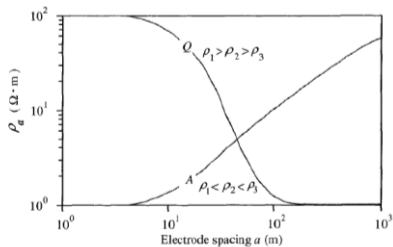
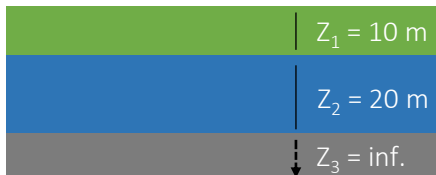
- constant resistivity
- constant depth

Analysis in the Two-Layer Case

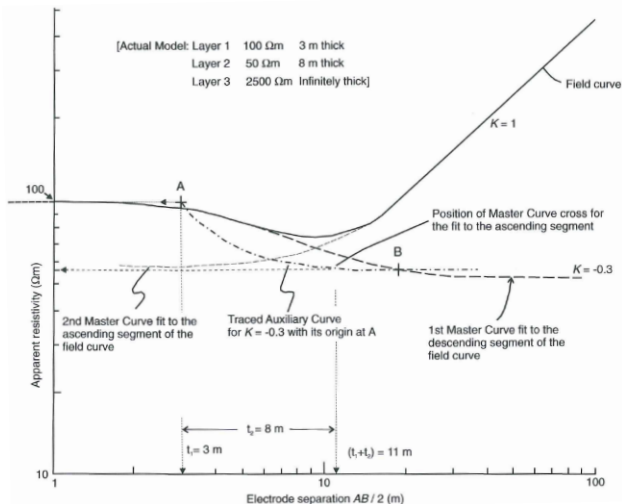
1st layer: 10 m, 100 Ohm m



Analysis in the Three-Layer Case



Analysis in the Three-Layer Case

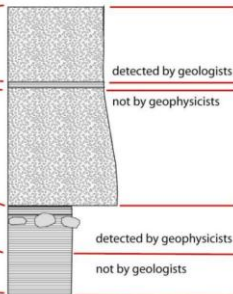


Multi-Layer Case

Real world



geological model



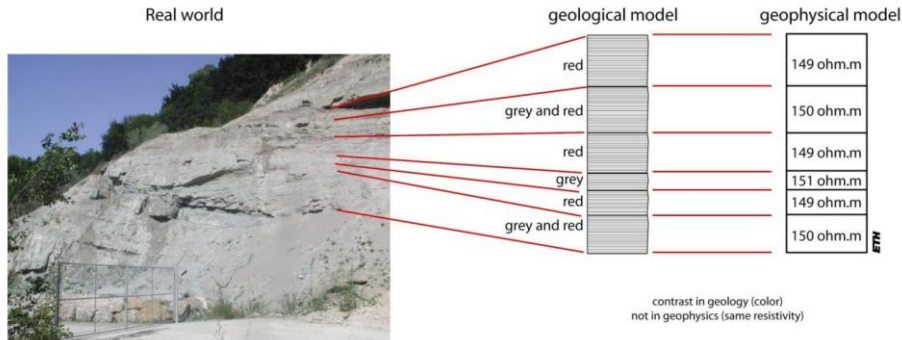
geophysical model

	150 ohm.m
	90 ohm.m
	50 ohm.m

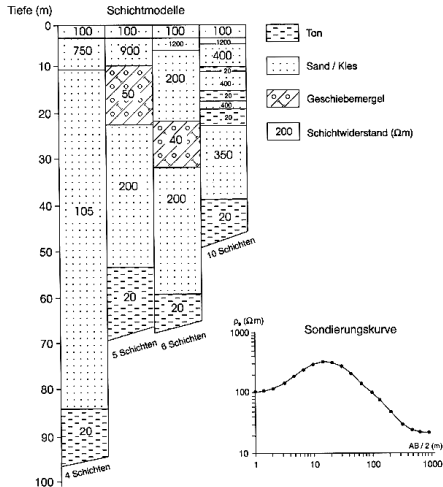
Source: Laurent Marescot, 2010

Multi-Layer Case

To characterize different material using geophysics, a **contrast** must exist (i.e. a difference in the physical properties)



Multi-Layer Case



2-Layer Case

- The result is more or less unique if a sufficient range of offsets is covered.
- The procedure can also be applied to gently dipping interfaces.
- This method has only historical and educational meaning. Practically, numerical inversion is preferred.

N-Layer Case

- Must be inverted numerically. Resistivities and thicknesses of the layers are adjusted to obtain the best fit to the measured apparent resistivities.
- The uppermost layer has a strong influence on the result.
- A deep, thin layer with a high contrast in resistivity may have a similar effect as a thicker layer with a lower contrast in resistivity.
- In the standard inversion procedure of vertical sounding, the number of layers is given, and thicknesses and resistivities are adjusted. Different numbers of layers may lead to strongly different results.



Quantitative analysis often hinges on independent information, e. g., from seismics or boreholes.

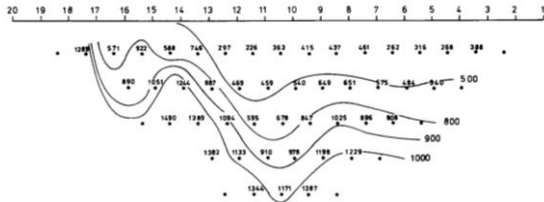
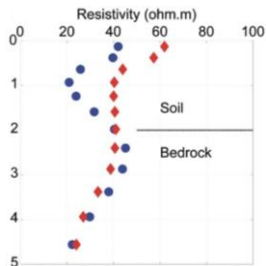
ER - Data Analysis II

Jakob Wilk

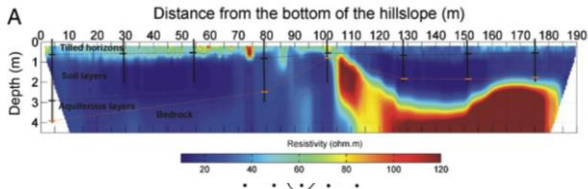
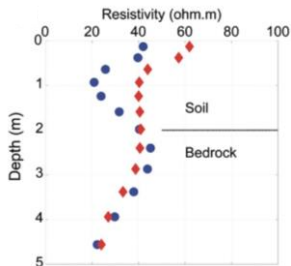
Institute of Earth and Environmental Science



Penetration Depth of Current

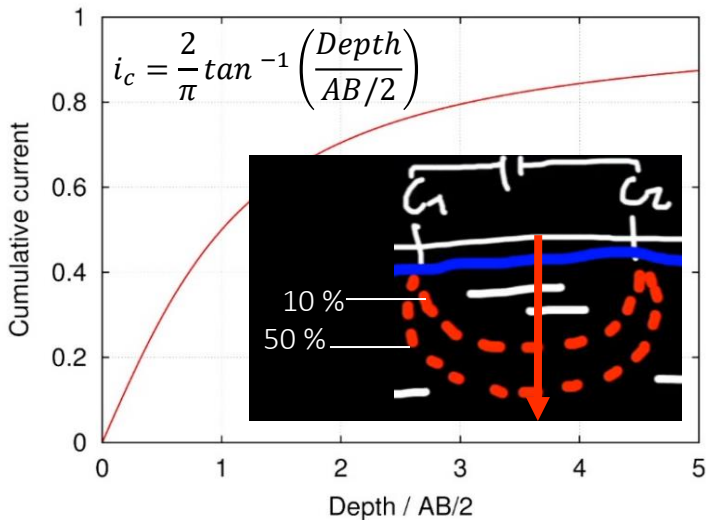


Penetration Depth of Current

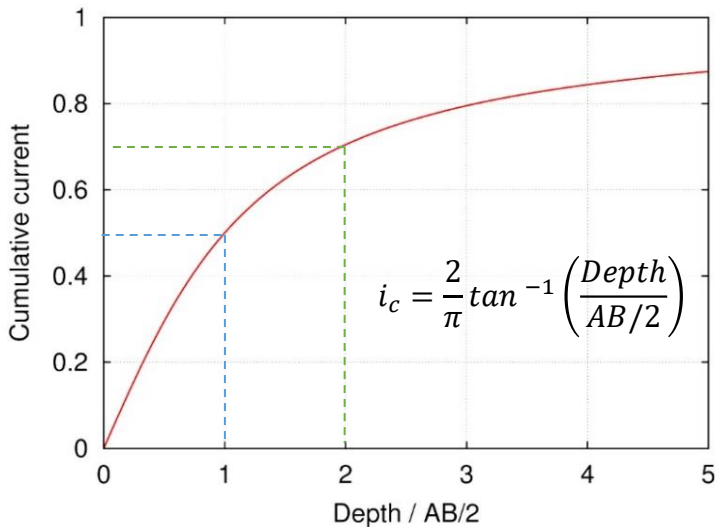


G. Coulouma et al. / Geoderma 170 (2012) 39–47

Penetration Depth of Current



Penetration Depth of Current



Penetration Depth of Current

Half of the current penetrates deeper than half of the total offset ($AB/2$), but

- the entire current must also pass shallow regions, and
- the potential electrodes are at the surface.



Typical depth of investigation is lower than $AB/2$.

Principle of the Sensitivity Analysis and Depth of Investigation Characteristic formula

$$\Delta\rho \sim \rho I$$

$$F(z) = \frac{2I_A}{\pi} \frac{z}{(r_{AM}^2 + 4z^2)^{1.5}}$$

$$F_4(z) = \frac{2zI_A}{\pi} \left[(r_{AM}^2 + 4z^2)^{-1.5} - (r_{AN}^2 + 4z^2)^{-1.5} \right. \\ \left. - (r_{BM}^2 + 4z^2)^{-1.5} + (r_{BN}^2 + 4z^2)^{-1.5} \right]$$

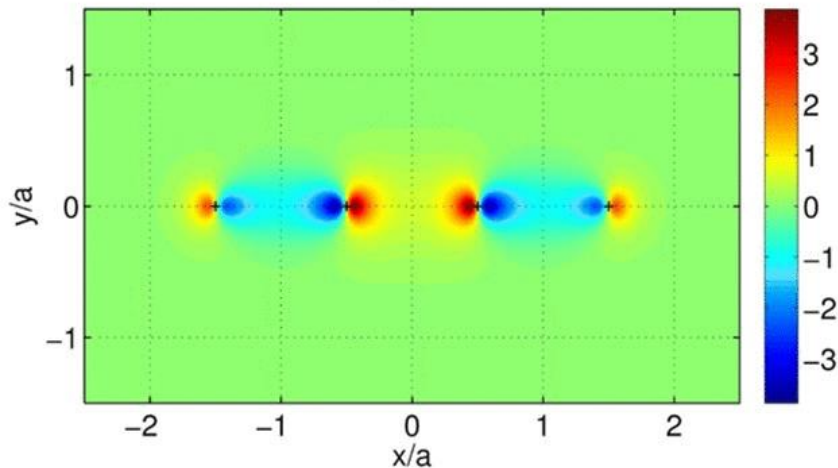
Source: Butler, 2015 after Roy and Apparo, 1971

Principle of the Sensitivity Analysis and DIC formula

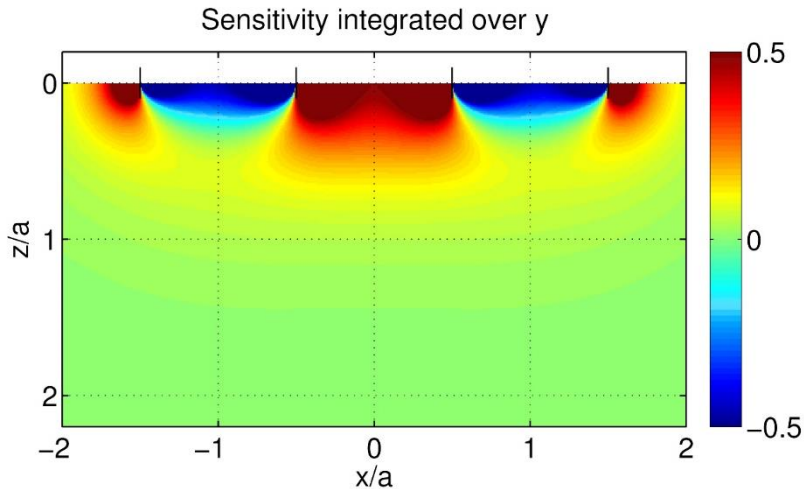
- Assume a given configuration of electrodes in a homogeneous medium with a resistivity ρ .
- Assume that ρ is increased (decreased) by a small amount $\delta\rho$ in a small region around a given point \vec{x} in the subsurface.
- Determine how this small change affects the voltage between M and N if the current between A and B is given.

Sensitivity of the Wenner Configuration

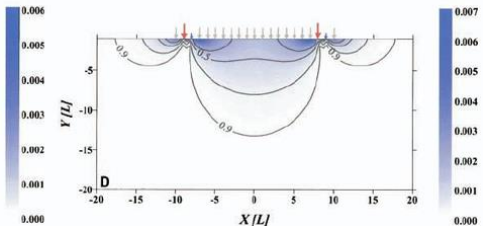
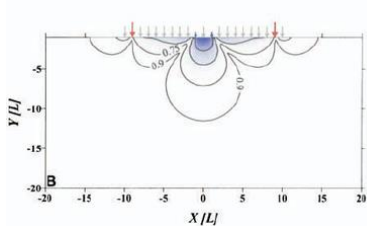
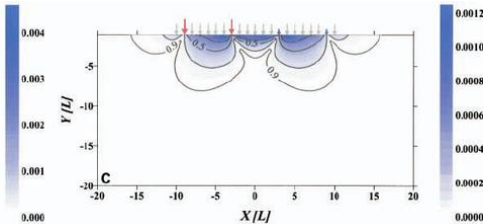
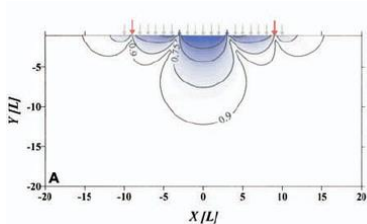
Sensitivity at $z/a = 0.1$



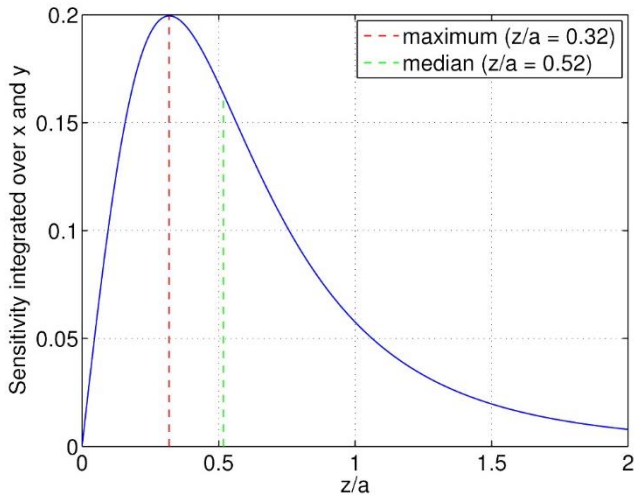
Sensitivity / Signal contribution section of the Wenner Configuration



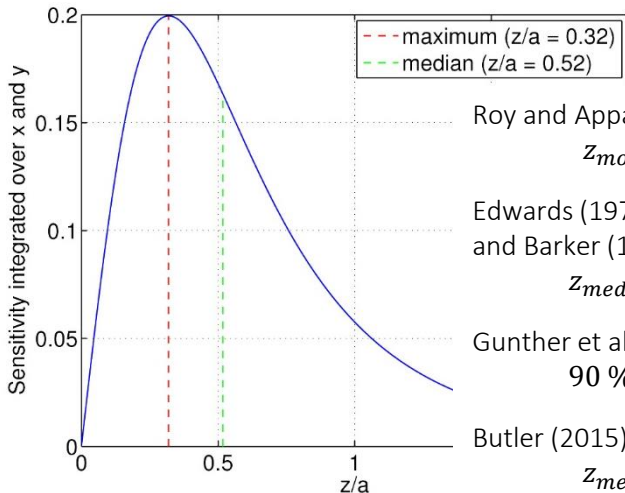
Sensitivity maps for Wenner, Schlumberger, double-dipole, & partial



Sensitivity of the Wenner Configuration



Sensitivity of the Wenner Configuration



Roy and Apparao (1971):

z_{mode}

Edwards (1977)
and Barker (1989):

z_{median}

Gunther et al. (2006):
90 %

Butler (2015):

z_{mean}

Sensitivity of the Wenner Configuration

- Sensitivity is always highest at low depth, in particular close to the electrodes M and N.
- Sensitivity changes its sign at low depths.
- Horizontally integrated sensitivity is highest at $z \approx 0.32 a$.
- Median of the horizontally integrated sensitivity distribution is at $z \approx 0.52 a$.

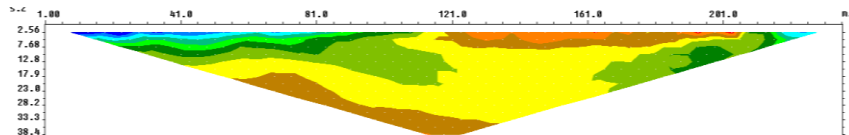


Regions with $z < 0.52 a$ and $z > 0.52 a$ contribute equally to the sensitivity in total.

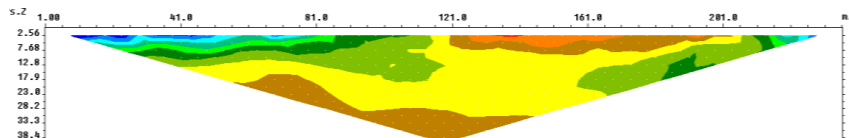


$0.52 a$ is often assumed as the typical depth of investigation.

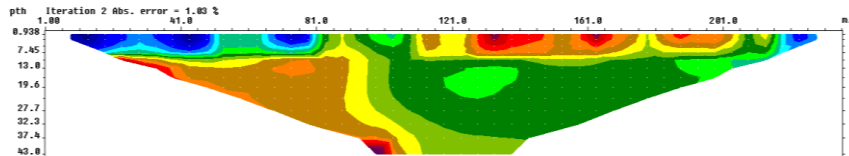
Pseudosections



Measured Apparent Resistivity Pseudosection



Calculated Apparent Resistivity Pseudosection

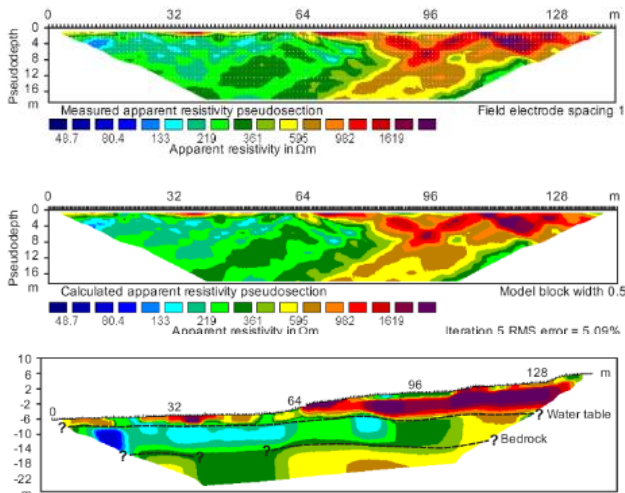


Inverse Model Resistivity Section

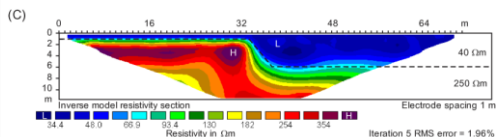
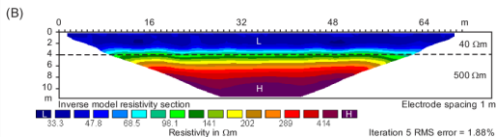
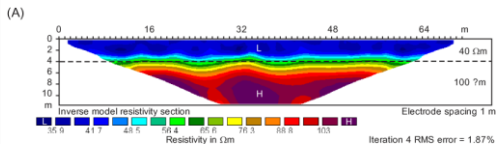


Unit electrode spacing 5.00 m.

Pseudosections

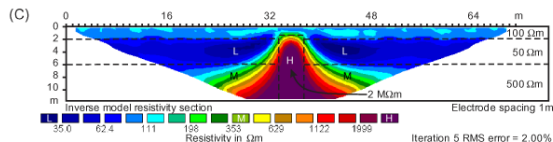
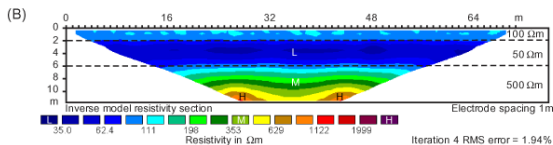
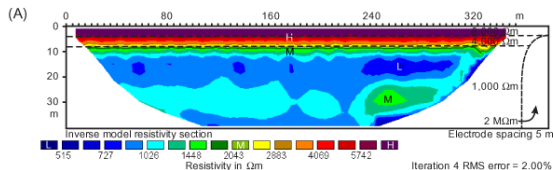


Pseudosections



1. Normal fault
2. High contrast two-layer case
3. Low contrast two-layer case

Pseudosections



1. Section with a lateral Discontinuity
2. Lateral homogenous three-layer section