Mass Movements
Figures

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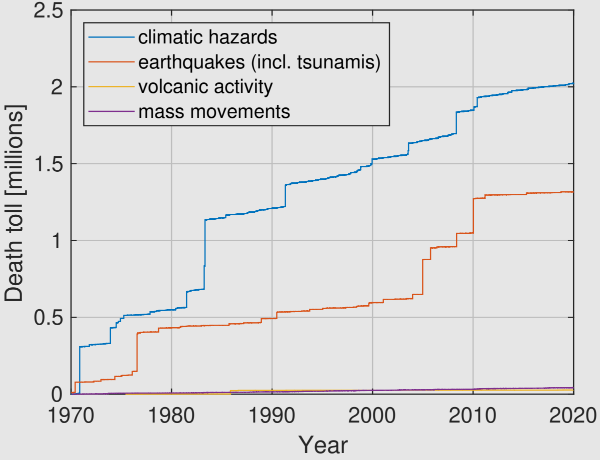
Mass Movements as a Geohazard

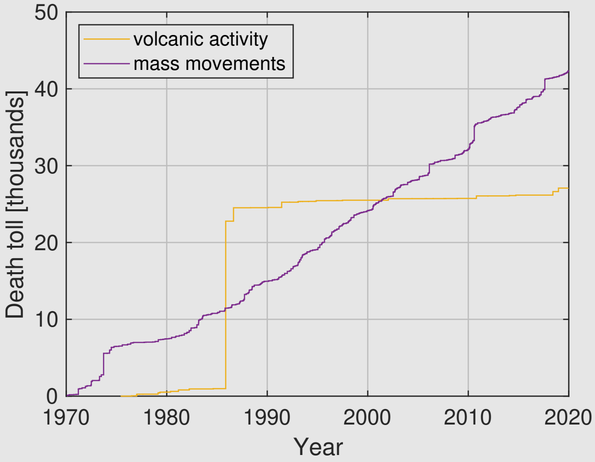
Worldwide Death Toll Since 1970



Data: Centre for Research on the Epidemiology of Disasters

Mass Movements as a Geohazard

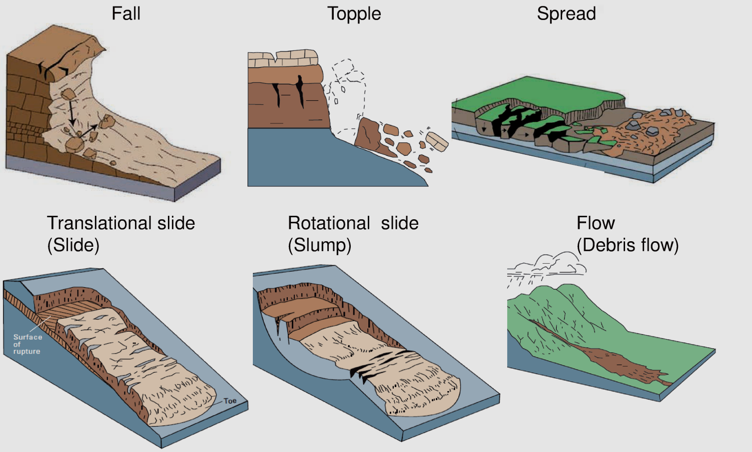
Worldwide Death Toll Since 1970



Data: Centre for Research on the Epidemiology of Disasters

Classification of Mass Movements According to Varnes

Classification by the Type of Movement



Modified from: Shanmugam & Wang, Journal of Palaeogeography, 2015, doi 10.3724/SP.J.1261.2015.00071

Classification of Mass Movements According to Varnes

Classification by the Material

Rock: Hard or firm mass that was intact and in its natural place before the initiation of movement.

Soil: An aggregate of solid particles, generally of minerals and rocks, that either was transported or was formed by the weathering of rock in place. Gases or liquids filling the pores of the soil form part of the soil.

Earth: Material in which 80 % or more of the particles are smaller than 2 mm, the upper limit of sand sized particles.

Mud: Material in which 80 % or more of the particles are smaller than 0.06 mm, the upper limit of silt sized particles.

Debris: Contains a significant proportion of coarse material; 20 % to 80 % of the particles are larger than 2 mm.

Examples From the Alps

Rockslide at Randa (Matter Valley, Switzerland, 1991, $V \approx 30\,\text{milion m}^3$)

Source: Wikipedia

Photo: S. Hergarten

Examples From the Alps

Flims Rockslide ($\approx$ 9150 b.p., $V \geq 8$ km^3)

Photo: K. Stüwe & R. Homberger (www.alpengeologie.org)

Wutach Gorge, Black Forest (2017)

Photo: M. Geyer (www.geotourist-freiburg.de)

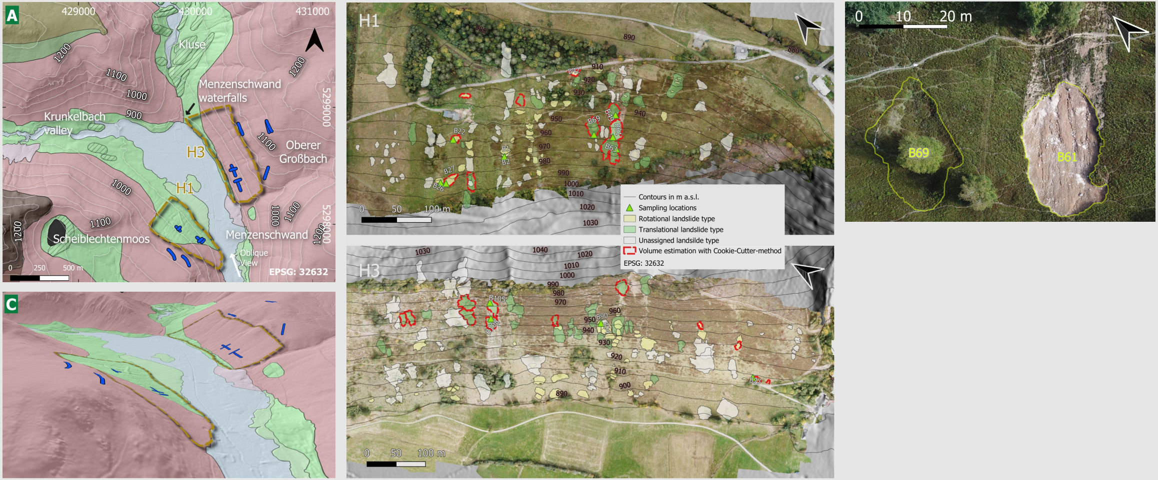
Regional Examples

Freiburg, Main Railway Track (2016)

Photo: T. Kunz (Badische Zeitung)

Regional Examples

Menzenschwand, Black Forest

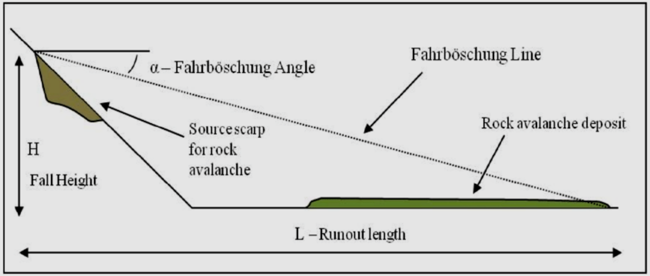


Source: Büschelberger et al., Earth Surf. Process. Landforms, 2022, doi 10.1002/esp.5237

Fahrboeschung and Talweg

The Fahrboeschung Concept

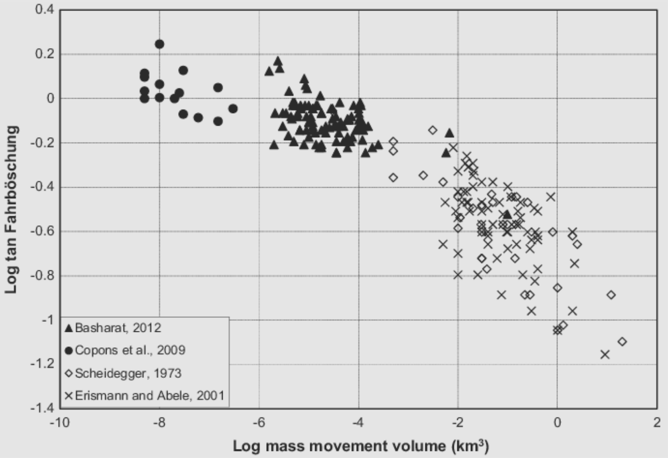
- Dates back to Albert Heim (1932).
- Mostly applied to rockfalls and rock avalanches, but also to mud flows and debris flows.
- Ratio of fall height δH and runout length L.



Source: de Graaf & Bowman, 12th International Symposium on Landslides, 2016

Fahrboeschung and Talweg

Dependence of the Fahrboeschung on Volume



Source: Basharat & Rohn, Nat. Hazards, 2015, doi 10.1007/s11069-015-1590-4

Fahrboeschung and Talweg

Physical Interpretation of the Fahrboeschung

Consider a particle moving on a 1D topography $H(x)$ with a given coefficient of kinetic (dynamic, sliding) friction ξ, starting from $x = 0$ at $t = 0$.

Friction force:

$$F_{\mathrm{f}} \;=\; \xi\, mg \qquad \beta$$

if dynamic effects are neglected with the slope angle β according to

$$\tan\beta \;=\; -\frac{\partial H}{\partial x}$$

Fahrboeschung and Talweg

Physical Interpretation of the Fahrboeschung

Energy consumed by friction:

$$E_\mathrm{f} \;=\; \int F_\mathrm{f} \,\square\, dt \;=\; \xi\, mg \int \square\;\square\;\beta\, dt \;=\; \xi\, mg\, x$$

Energy balance:

$$mg\, H(x) + \square + E_\mathrm{f} \;=\; \text{const} \;=\; \square$$

$$\Downarrow$$

$$\square \;=\; \sqrt{\square \left(\square - \square - \square \right)}$$

Fahrboeschung and Talweg

Definition and Mathematical Description of the Talweg

Consider a given topography $H(x_1, x_2)$. The talweg (also thalweg) is the line (from a given point) following the direction of the steepest descent.

The talweg line $\vec{s}(t) = \binom{s_1(t)}{s_2(t)}$ (in map view) can be described by the ordinary differential equation

$$\frac{d}{dt}\vec{s}(t) \sim \boxed{} H(\vec{s}(t))$$

where t is the curve parameter (not necessarily time).

Definition and Mathematical Description of the Talweg

The factor of proportionality does not affect the talweg line, but only the meaning of t; any positive (not necessarily constant) value can be used.

Convenient choice:

$$\frac{d}{dt}\vec{s}(t) \;=\; -\,\frac{H(\vec{s}(t))}{H(\vec{s}(t))}$$

Description of a Particle Moving Along the Surface

Variables:

$$\vec{s}(t) = \begin{pmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{pmatrix}, \quad \vec{v}(t) = \begin{pmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{pmatrix}$$

Constraints:

$$s_3(t) = H(s_1(t), s_2(t))$$

$$\Downarrow$$

$$v_3(t) = \begin{pmatrix} v_1(t) \\ v_2(t) \end{pmatrix} \cdot \quad$$

$$\Downarrow$$

$$\vec{v} \cdot \vec{n} = 0 \quad \text{with} \quad \vec{n} = $$

Lumped-Mass Models

Acceleration

Total acceleration acting on a particle at the surface:

$$\frac{d\vec{v}}{dt} = \vec{a} = \vec{g} + \vec{a}_n + \vec{a}_f = \vec{g} + a_n\vec{n} - \xi a_n \vec{e}$$

where

$$\vec{g} = \begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix}$$

$$\vec{n} = \frac{1}{\quad} \begin{pmatrix} -\nabla H \\ 1 \end{pmatrix}$$

$$\vec{e} = \frac{\quad}{\quad}$$

Simplified 2-D Model

Approximation: $|\nabla H| \ll 1$

$$\Downarrow$$

$$\vec{v} \approx \begin{pmatrix} v_1 \\ v_2 \\ 0 \end{pmatrix} \quad \text{and} \quad \vec{n} \approx \begin{pmatrix} -\nabla H \\ 1 \end{pmatrix}$$

$$\Downarrow$$

$$\frac{d}{dt}\begin{pmatrix} v_1 \\ v_2 \\ 0 \end{pmatrix} = \vec{g} + a_n\vec{n} - \xi a_n \vec{e} = \begin{pmatrix} \quad \\ \quad \\ \quad \end{pmatrix}$$

$$\Downarrow$$

$$a_n = \quad$$

3D Lumped-Mass Model

Differential equations:

$$\frac{d}{dt}\vec{s} = \vec{v}$$
$$\frac{d}{dt}\vec{v} = \vec{g} + a_n\vec{n} - \xi a_n\vec{e}$$

Explicit Euler scheme ($t \to t + \delta t$):

$$\vec{s} = \vec{s} + \delta t\, \vec{v}$$
$$\vec{v} = \vec{v} + \delta t\, (\vec{g} + a_n\vec{n} - \xi a_n\vec{e})$$

Where are the problems?

3D Lumped-Mass Model

Modification 1:

1. $\vec{s} = \vec{s} + \delta t\, \vec{v}$ as in explicit Euler scheme.
2. Set $s_3 = H(s_1, s_2)$.
3. Assume that $\delta t = \frac{|\vec{s} - \vec{s}|}{|\vec{v}|}$ instead of the original δt for the rest of this time step.

Modification 2: mixed scheme for $\vec{v}$

$$\vec{v} \;=\; \vec{v} + \delta t\,(\vec{g} + a_n \vec{n} - \xi a_n \vec{e})$$

Basic Structure

Define yield point as a function of the components of σ. Material behaves elastically as long as

$$f(\sigma) < f_{\text{yield}}.$$

Brittle failure or plastic deformation if

$$f(\sigma) = f_{\text{yield}}.$$

Types of Yield Criteria

Anisotropic criteria predict threshold and plane of failure; mostly used for brittle materials

Criterion	Properties of σ
Tresca	maximum shear stress
Mohr-Coulomb	shear stress and normal stress

Isotropic criteria predict only the threshold of failure; mostly used for ductile materials

Criterion	Properties of σ
von Mises	von-Mises stress
Drucker-Prager	von-Mises stress and mean stress

The Mohr-Coulomb Criterion

Compressive normal stresses increase the strength against shear failure:

$$\sigma_s^{\text{crit}} = \mp \xi\, \sigma_n + C$$

with

$$\begin{aligned} \xi &= \text{coefficient of internal friction} = \tan\phi \\ \phi &= \text{angle of internal friction} \\ C &= \text{cohesion} \end{aligned}$$

First term is formally the same as solid-state friction.

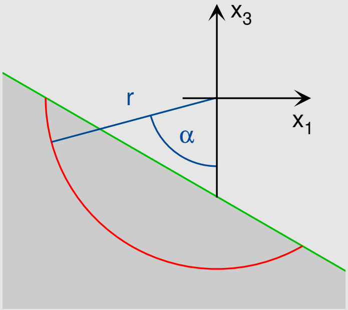
The Mohr-Coulomb Criterion

Typical parameter values:

	C	ϕ
surface rocks	10 MPa	30–50°
soils	0–100 kPa	20–40°
sand	0	27–45°
snow	0–500 Pa	15–30°

Rotational Slides

Geometry and Coordinates



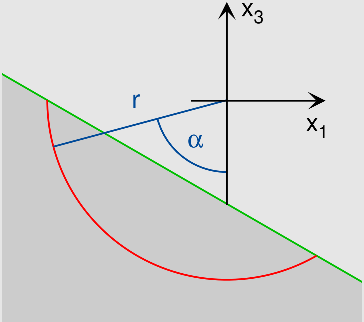
Area Element

Size of an area element:

$$\delta A = w\, r\, \delta\alpha = w \frac{\delta x}{\cos\alpha}$$

with

$$w = \text{width in } x_2 \text{ direction}$$
$$\delta\alpha = \text{angle increment}$$
$$\delta x = \text{increment in } x_1 \text{ direction}$$



In integral form:

$$\int \ldots\, dA = w\, r \int \ldots\, d\alpha = w \int \frac{\ldots}{\cos\alpha}\, dx$$

Overall Factor of Safety

Continuous form:

$$M = r \int \sigma_s dA = r^2 w \int \sigma_s d\alpha = r\, w \int \frac{\sigma_s}{\cos\alpha} dx$$

As a discrete sum:

$$M \approx r \sum_i \sigma_{si} \delta A_i \approx r^2 w \sum_i \sigma_{si} \delta\alpha_i \approx r\, w \sum_i \frac{\sigma_{si}}{\cos\alpha_i} \delta x_i$$

Overall FoS:

$$\mathrm{FoS} = \frac{M^{\mathrm{crit}}}{M} = \frac{\int \frac{\sigma_s^{\mathrm{crit}}}{\cos\alpha} dx}{\int \frac{\sigma_s}{\cos\alpha} dx} \approx \frac{\sum_i \frac{\sigma_{si}^{\mathrm{crit}}}{\cos\alpha_i} \delta x_i}{\sum_i \frac{\sigma_{si}}{\cos\alpha_i} \delta x_i}$$

Fellenius' Method

- Introduced by W. Fellenius 1929
- Earliest and simplest model for rotational slope failure taking into account the variation in σ_n and thus $\sigma_\mathrm{s}^\mathrm{crit}$ along the slip circle
- Also called ordinary method of slices (OMS)
- Originally developed in terms of torques for a discrete set of vertical slices
- Can also be derived from a simplified stress tensor

$$\boldsymbol{\sigma} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\rho g h \end{pmatrix}$$

Fellenius' Method

$$\sigma_\mathrm{n} = -\rho g h \cos^2\alpha$$
$$\sigma_\mathrm{s} = \rho g h \cos\alpha \sin\alpha$$
$$\sigma_\mathrm{s}^\mathrm{crit} = C - \sigma_\mathrm{n} \tan\phi = C + \rho g h \cos^2\alpha \tan\phi$$

$$\Downarrow$$

Local FoS

$$\mathrm{FoS_{loc}} = \frac{\sigma_\mathrm{s}^\mathrm{crit}}{\sigma_\mathrm{s}} = \frac{C + \tan\phi\, \rho g h \cos^2\alpha}{\rho g h \sin\alpha \cos\alpha} = \frac{\tan\phi}{\tan\alpha} + \frac{C}{\rho g h \cos\alpha \sin\alpha}.$$

Fellenius' Method

$$
\begin{aligned}
M &= r\,w \int \rho g h \sin\alpha \, dx \;\approx\; r\,w \sum_i \rho g h_i \sin\alpha_i \, \delta x_i \\
M^{\text{crit}} &= r\,w \int \left(\frac{C}{\cos\alpha} + \tan\phi \, \rho g h \cos\alpha \right) dx \;\approx\; r\,w \sum_i \left(\frac{C}{\cos\alpha_i} + \tan\phi \, \rho g h_i \cos\alpha_i \right) \delta x_i
\end{aligned}
$$

$$\Downarrow$$

$$
\text{FoS} = \frac{\int \left(\frac{C}{\cos\alpha} + \tan\phi \, \rho g h \cos\alpha \right) dx}{\int \rho g h \sin\alpha \, dx} \approx \frac{\sum_i \left(\frac{C}{\cos\alpha_i} + \tan\phi \, \rho g h_i \cos\alpha_i \right) \delta x_i}{\sum_i \rho g h_i \sin\alpha_i \, \delta x_i}
$$

Bishop's Method

- Introduced by A. W. Bishop 1955
- Most widely used model for rotational slope failure
- Originally developed in terms of torques for a discrete set of vertical slices
- Can also be derived from a simplified, inconsistent stress tensor

$$\boldsymbol{\sigma} = \begin{pmatrix} 0 & 0 & \tau \\ 0 & 0 & 0 \\ 0 & 0 & -\rho g h \end{pmatrix}$$

with an arbitrary stress τ

Rotational Slides

Bishop's Method

$$\sigma_\mathrm{n} = -\rho g h \cos^2\alpha + \tau \cos\alpha \sin\alpha$$

$$\sigma_\mathrm{s} = \rho g h \cos\alpha \sin\alpha + \tau \cos^2\alpha$$

$$\sigma_\mathrm{s}^\mathrm{crit} = C - \sigma_\mathrm{n} \tan\phi$$

$$\Downarrow$$

$$\mathrm{FoS}_\mathrm{loc} = \frac{\sigma_\mathrm{s}^\mathrm{crit}}{\sigma_\mathrm{s}} = \frac{C + \tan\phi\left(\rho g h \cos^2\alpha - \tau \cos\alpha \sin\alpha\right)}{\rho g h \cos\alpha \sin\alpha + \tau \cos^2\alpha}$$

$$\Downarrow$$

$$\tau = \frac{C + \rho g h \qquad + \mathrm{FoS}_\mathrm{loc}\, \rho g h \qquad}{\qquad + \mathrm{FoS}_\mathrm{loc} \qquad}$$

Bishop's Method

↓

$$\sigma_{\mathrm{s}}^{\mathrm{crit}} = \frac{C + \tan\phi\,\rho g h}{1 + \frac{\tan\phi\,\tan\alpha}{\mathrm{FoS}_{\mathrm{loc}}}}$$

- Combine this $\sigma_{\mathrm{s}}^{\mathrm{crit}}$ with σ_{s} from Fellenius' method.
- Assume that $\mathrm{FoS}_{\mathrm{loc}}$ in the expression for $\sigma_{\mathrm{s}}^{\mathrm{crit}}$ is the overall FoS.

↓

$$\mathrm{FoS} = \frac{\int \frac{C+\tan\phi\,\rho g h}{\cos\alpha + \frac{\tan\phi\,\sin\alpha}{\mathrm{FoS}}}\,dx}{\int \rho g h\,\sin\alpha\,dx} \approx \frac{\sum_i \frac{C+\tan\phi\,\rho g h_i}{\cos\alpha_i + \frac{\tan\phi\,\sin\alpha_i}{\mathrm{FoS}}}\,\delta x_i}{\sum_i \rho g h_i\,\sin\alpha_i\,\delta x_i}$$

Bishop's Method

Occurrence of FoS at the right-hand side can be treated using a fixed-point iteration.

- Converges rapidly
- Useful initial guess: FoS of Fellenius method

In case we need it:

$$\mathrm{FoS_{loc}} = \frac{\sigma_\mathrm{s}^\mathrm{crit}}{\sigma_\mathrm{s}} = \frac{\frac{C+\tan\phi\,\rho g h}{1+\frac{\tan\phi\,\tan\alpha}{\mathrm{FoS}}}}{\rho g h\,\cos\alpha\,\sin\alpha} = \frac{\frac{C}{\rho g h}+\tan\phi}{\left(\cos\alpha+\frac{\tan\phi\,\sin\alpha}{\mathrm{FoS}}\right)\sin\alpha}$$

Falling

Particle Motion

Neglect air drag and interactions between particles

$\Downarrow$

parabolic traces

$$\begin{aligned}\vec{v}(t+\delta t) &= \vec{v}(t) + \delta t\,\vec{g} \\ \vec{s}(t+\delta t) &= \vec{s}(t) + \delta t\,\vec{v}(t) + \end{aligned}$$

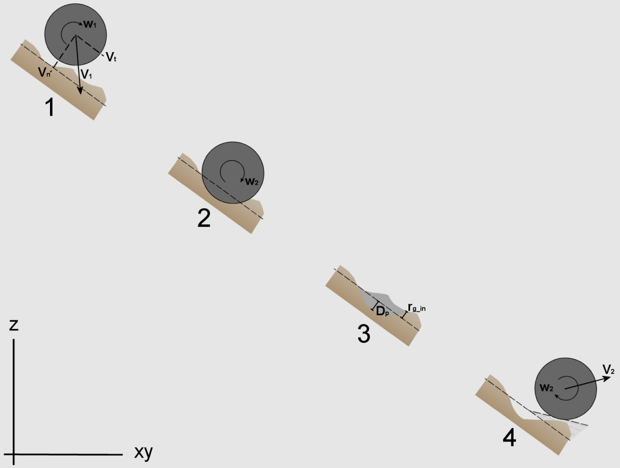
valid for any t and δt

Find δt so that

$$s_3(t+\delta t) = H(s_1(t+\delta t), s_2(t+\delta t))$$

Falling

Rebound at the Surface



Source: Dorren, Rockyfor3D (v5.2) revealed, ecorisQ paper, 2016

Rebound at the Surface

Simplest approach: normal and tangential components of the velocity are reduced by different factors

$$\begin{aligned} v_n &\rightarrow -R_n\, v_n \\ v_t &\rightarrow R_t\, v_t \end{aligned}$$

where

$$\begin{aligned} R_n &= \text{coefficient of restitution normal to the surface,} \\ &\quad\ \text{depends on the material} \\ R_t &= \text{coefficient of restitution parallel to the surface,} \\ &\quad\ \text{mainly depends on the roughness of the surface} \end{aligned}$$

Coefficient of Restitution Normal to the Slope

Soiltype	General description of the underground	mean R_n value	R_n value range
0	River, or swamp, or material in which a rock could penetrate completely	0	0
1	Fine soil material (depth > ~100 cm)	0.23	0.21 - 0.25
2	Fine soil material (depth < ~100 cm), or sand/gravel mix in the valley	0.28	0.25 - 0.31
3	Scree (Ø < ~10 cm), or medium compact soil with small rock fragments, or forest road	0.33	0.30 - 0.36
4	Talus slope (Ø > ~10 cm), or compact soil with large rock fragments	0.38	0.34 - 0.42
5	Bedrock with thin weathered material or soil cover	0.43	0.39 - 0.47
6	Bedrock	0.53	0.48 - 0.58
7	Asphalt road	0.35	0.32 – 0.39

Source: Dorren, Rockyfor3D (v5.2) revealed, ecorisQ paper, 2016

Coefficient of Restitution Parallel to the Slope

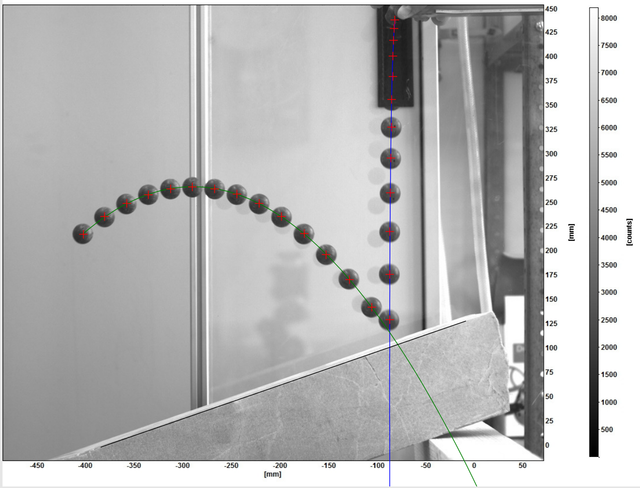
Difficult to estimate, e. g.,

$$R_t = \frac{1}{1 + \frac{MOH + D_p}{R}}$$

with

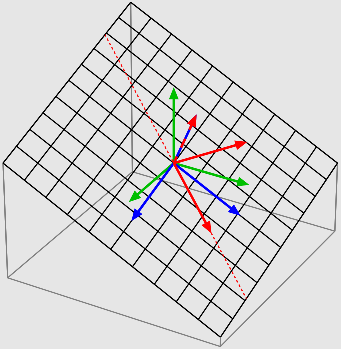
$$\begin{aligned} MOH &= \text{representative obstacle height} \\ D_p &= \text{depth of penetration} \\ R &= \text{radius of the particle} \end{aligned}$$

Measuring Coefficients of Restitution in Laboratory



Applications of the Talweg Concept to Mass Movements

- Simplest "realistic" path of downward movement; relation $\frac{H}{L} = \xi$ remains valid with $L =$ track length (not a straight line).
- Construction of locally aligned coordinate systems for granular flow models based on continuum mechanics



Savage-Hutter model (1989)
avalanche model RAMMS
Cartesian coordinate system
(Hergarten & Robl, NHESS, 2015)

Explanation of the Quadratic Friction Law